GEOPHYSICAL WAVES AND FLOWS

Waves and flows are pervasive on and within Earth. This book presents a unified physical and mathematical approach to waves and flows in the atmosphere, oceans, rivers, volcanoes and the mantle, emphasizing the common physical principles and mathematical methods that apply to a variety of phenomena and disciplines. It is organized into seven parts: introductory material; kinematics, dynamics and rheology; waves in non-rotating fluids; waves in rotating fluids; non-rotating flows; rotating flows; and silicate flows. The chapters are supplemented by 47 ‘fundaments’, containing knowledge that is fundamental to the material presented in the main text, organized into seven appendices: mathematics; dimensions and units; kinematics; dynamics; thermodynamics; waves; and flows. This book is a valuable reference for graduate students and researchers seeking an introduction to the mathematics of waves and flows in the Earth system, and can serve as a supplementary textbook for a number of courses in geophysical fluid dynamics.

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Theory and Applications in the Atmosphere, Hydrosphere and Geosphere

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Preface

This monograph is the outgrowth of a set of notes that were prepared some years ago for a course that in fact never was taught. The notes lay fallow until the occasion of my 75th birthday, which inspired me to try to clean up my computer files. Upon discovering these notes, I consulted David Furbish regarding using these as the basis of a monograph on geophysical waves and flows. His encouraging response gave me the confidence to attempt to do so. This is the outcome of that endeavor.

The goal of that course was – and of the current monograph is – to present a unified approach to geophysical waves and flows, starting from the simplest case and progressively adding complicating factors in a systematic manner. Simplest are sound waves that occur in air and water. Similar, but somewhat more complicated, are compressive body waves (P waves), transverse waves (S waves) and edge waves (Rayleigh and Love waves) in Earth's mantle. Seemingly similar, but dynamically distinct, is the fluid edge wave (think of ocean waves), occurring in both deep and shallow water. It is a short step from shallow-water waves to flows in a horizontal channel. Next, if the channel is sloping, we encounter gravitationally driven flow and this leads naturally to the study of turbulent flows. These topics are found in Chapters 9–13 and 19–23 of this monograph.

In order to approach the analysis of waves and flows properly with a sound theoretical basis, we need to begin from square one, quantifying the manner in which a continuous body can move (kinematics), the nature of the forces that make it do so (dynamics) and the form of its response (rheology). These essential topics are covered in Chapters 3–7. Of course, rotation of Earth affects many types of waves and flows, so this subject is introduced in Chapter 8, with waves affected by rotation investigated in Chapters 15–18 and flows affected by rotation investigated in Chapters 25–28. It is interesting to compare and contrast the readily visible flows within the atmosphere and oceans to silicate flows occurring within Earth's mantle and in volcanoes; these latter flows are investigated in Chapters 29–35.

The book attempts to present a wide range of waves and flows fairly simply, while retaining mathematical rigor. A minimum prerequisite of prior knowledge is an understanding of multivariate calculus and ordinary differential equations; a previous knowledge of partial differential equations, complex variables and linear algebra is desirable but not necessary. The book is written in an informal manner; my guide in this approach has been to imagine writing for a precocious grandchild: Noah Bliss. Finally, I wish to acknowledge the significant assistance provided by Paul Roberts, who read the manuscript with an eagle eye and provided me with many suggestions for improvement.

General Description of Contents

The main part of this book consists of 35 chapters organized into seven parts, consisting of I: introductory material, II: kinematics, dynamics and rheology, III: waves in non-rotating fluids, IV: waves in rotating fluids, V: one-dimensional flows, VI: rotating flows and VII: silicate flows. The chapters are supplemented by 47 "fundaments," containing knowledge that is fundamental to the material presented in the main text, organized into seven appendices: A: mathematics, B: dimensions and units, C: kinematics, D: dynamics, E: thermodynamics, F: waves and G: flows. These fundaments are intended to aid the reader who may have some gaps in background or to serve as a resource. The 35 chapters and 47 fundaments may be thought of as the warp and woof of a grand tapestry, with related topics joined together by several hundred footnotes.

Part I consists of an introductory chapter discussing a number of broad issues, including mathematical modeling, continuum mechanics, energy and planetary cooling and the concept of a continuous body, and a chapter containing a number of more detailed preliminaries, including establishment of a reference coordinate system. Part II deals with kinematics, dynamics and rheology, beginning in Chapter 3 with an analysis of the kinematics of deformation and flow, and followed in Chapter 4 with an investigation of dynamics and the stress tensor. Next, in Chapter 5 we are introduced to some thermodynamics relevant to geophysical waves and flows and in Chapter 6 to the fundamentals of shear rheology. Chapter 7 contains an introduction to the concept of a static reference state and the equations governing perturbations of this state. This part concludes with an introduction to rotating fluids in Chapter 8.

We begin our study of waves in Part III, focusing on waves unaffected by rotation. Chapter 9 contains an introduction to waves, with an investigation of sound waves in § 9.3. The various types of waves that can occur in elastic bodies are studied in Chapter 10, with seismic waves in Earth's mantle discussed in § 10.5. Deep-water waves are considered in Chapter 11, which includes a discussion of the nature of ocean waves in § 11.5. Following this, we investigate linear shallow-water waves in Chapter 12 and nonlinear shallow-water waves in Chapter 13. Our survey of non-rotating waves concludes in Chapter 14 with analyses of capillary, interfacial and internal-gravity waves.

Our study of rotating waves in Part IV begins in Chapter 15 with geostrophic, inertial and Rossby waves. In Chapter 16 we investigate how rotation affects surface, interfacial and internal-gravity waves. The last two chapters of this part deal with ocean waves that rely solely on rotation for their existence: equatorial Kelvin and Rossby waves studied in Chapter 17 and coastal Kelvin and topographic Rossby waves studied in Chapter 18.

Part V contains six chapters devoted to simple one-dimensional flows, beginning in Chapter 19 with an orientation to the topic and a review of the relevant governing equations.

Steady and unsteady flows in a uniform horizontal channel are investigated in Chapters 20 and 21, respectively. We introduce gravitational forcing of channel flows, by allowing the channel to tilt downward, in Chapter 22. This leads us to the topic of turbulence, which is investigated in some detail in Chapter 23, leading to a simple model of turbulent diffusion. This model is applied to some simple turbulent flows (katabatic winds, avalanches and cumulonimbus clouds) in Chapter 24.

Flows in rotating fluids are investigated in Part VI, beginning with a detailed study of laminar and turbulent Ekman layers in Chapter 25. Atmospheric flows, including the general circulation, thermal winds and the jet stream, are investigated in Chapter 26, and oceanic currents, including the Sverdrup balance, western-boundary currents and thermo-haline circulation, are investigated in Chapter 27. This part concludes in Chapter 28 with a survey of vortices and brief analyses of the structure and dynamics of tornadoes and hurricanes.

In Part VII our attention returns to the “solid” Earth with an investigation of various silicate flows. We begin in Chapter 29 with a survey of the equations governing silicate flows and estimates of parameter values within the mantle. Earth’s mantle is convecting in an effort to cool the mantle and core. The nature of this flow is reviewed in Chapter 30, then the modes of mantle convection involved in cooling the mantle and core are investigated in Chapters 31 and 32. In Chapter 33 we survey the various types of volcanic flows, then in Chapter 34 investigate the nature of flow in volcanic conduits and conclude in Chapter 35 with a study of lava sheet flow on the surface.

The fundaments are organized into seven appendices. The first, Appendix A, deals with mathematical issues, including vector algebra, vector calculus, curvilinear coordinates, Taylor series, Fourier series and integrals, classification of partial differential equations, a listing of the Greek alphabet and introductions to scalar and vector potentials and the stream function. Appendix B deals with dimensions and units, including introductions to dimensional analysis and the SI system of units, and contains several tables giving values of parameters relevant to geophysical waves and flows. Kinematic topics surveyed in Appendix C include non-inertial frames of reference and virtual forces, the material derivative, finite deformation and flow lines and points. Appendix D deals with dynamics and surveys viscoelastic behavior, silicate rheology, the constants of elasticity, surface tension, the general conservation law, the Euler and Bernoulli equations, kinetic and internal energies, thin-layer equations and variables, the Proudman–Taylor theorem and the vector vorticity equation. Perhaps the most important appendix is the fifth, Appendix E, surveying thermodynamics, including storage and transfers of energy, the mole, the first law of thermodynamics, thermodynamic potentials, variables and parameters, equations of state for density and entropy, ideal mixtures, the energy equation, ideal gases, thermodynamics of the atmosphere, phase equilibrium and thermodynamic efficiency. Appendix F contains fundaments on waves, including waves in three dimensions, Fourier representation of waves, Stokes waves, Kelvin’s ship waves, energy of deep-water waves, Laplace’s tidal equations and inertial waves. Finally, Appendix G contains three fundaments of flows: shear-flow instability, boundary-layer theory and a critique of models of open-channel flows.