CAMBRIDGE

Cambridge University Press
978-1-107-18233-2 — Discrete Harmonic Analysis
Tullio Ceccherini-Silberstein , Fabio Scarabotti , Filippo Tolli
Table of Contents
More Information

Contents

Preface *page* xi

Part I Finite Abelian groups and the DFT

1 Finite Abelian groups 3
1.1 Preliminaries in number theory 3
1.2 Structure theory of finite Abelian groups: preliminary results 10
1.3 Structure theory of finite Abelian groups: the theorems 18
1.4 Generalities on endomorphisms and automorphisms of finite Abelian groups 26
1.5 Endomorphisms and automorphisms of finite cyclic groups 30
1.6 The endomorphism ring of a finite Abelian p-group 35
1.7 The automorphisms of a finite Abelian p-group 40
1.8 The cardinality of $\mathrm{Aut}(A)$ 42

2 The Fourier transform on finite Abelian groups 46
2.1 Some notation 46
2.2 Characters of finite cyclic groups 48
2.3 Characters of finite Abelian groups 50
2.4 The Fourier transform 53
2.5 Poisson's formulas and the uncertainty principle 59
2.6 Tao's uncertainty principle for cyclic groups 62

3 Dirichlet's theorem on primes in arithmetic progressions 74
3.1 Analytic preliminaries 74
3.2 Preliminaries on multiplicative characters 84
3.3 Dirichlet L-functions 89

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-1-107-18233-2 — Discrete Harmonic Analysis
Tullio Ceccherini-Silberstein , Fabio Scarabotti , Filippo Tolli
Table of Contents
More Information

3.4 Euler's theorem 96
3.5 Dirichlet's theorem 99

4 Spectral analysis of the DFT and number theory 101
4.1 Preliminary results 101
4.2 The decomposition into eigenspaces 107
4.3 Applications: some classical results by Gauss and Schur 115
4.4 Quadratic reciprocity and Gauss sums 116

5 The Fast Fourier Transform 129
5.1 A preliminary example 129
5.2 Stride permutations 131
5.3 Permutation matrices and Kronecker products 139
5.4 The matrix form of the FFT 151
5.5 Algorithmic aspects of the FFT 161

Part II Finite fields and their characters

6 Finite fields 167
6.1 Preliminaries on ring theory 167
6.2 Finite algebraic extensions 171
6.3 The structure of finite fields 176
6.4 The Frobenius automorphism 177
6.5 Existence and uniqueness of Galois fields 178
6.6 Subfields and irreducible polynomials 183
6.7 Hilbert Satz 90 187
6.8 Quadratic extensions 192

7 Character theory of finite fields 197
7.1 Generalities on additive and multiplicative characters 197
7.2 Decomposable characters 201
7.3 Generalized Kloosterman sums 203
7.4 Gauss sums 210
7.5 The Hasse-Davenport identity 213
7.6 Jacobi sums 217
7.7 On the number of solutions of equations 222
7.8 The FFT over a finite field 227

Part III Graphs and expanders

8 Graphs and their products 235
8.1 Graphs and their adjacency matrix 235

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-1-107-18233-2 — Discrete Harmonic Analysis
Tullio Ceccherini-Silberstein , Fabio Scarabotti , Filippo Tolli
Table of Contents
More Information

8.2 Strongly regular graphs 241
8.3 Bipartite graphs 245
8.4 The complete graph 247
8.5 The hypercube 248
8.6 The discrete circle 250
8.7 Tensor products 252
8.8 Cartesian, tensor, and lexicographic products of graphs 258
8.9 Wreath product of finite graphs 265
8.10 Lamplighter graphs and their spectral analysis 268
8.11 The lamplighter on the complete graph 270
8.12 The replacement product 273
8.13 The zig-zag product 277
8.14 Cayley graphs, semidirect products, replacement products, and zig-zag products 279

9 Expanders and Ramanujan graphs 283
9.1 The Alon-Milman-Dodziuk theorem 284
9.2 The Alon-Boppana-Serre theorem 295
9.3 Nilli's proof of the Alon-Boppana-Serre theorem 300
9.4 Ramanujan graphs 307
9.5 Expander graphs 309
9.6 The Margulis example 311
9.7 The Alon-Schwartz-Shapira estimate 320
9.8 Estimates of the first nontrivial eigenvalue for the Zig-Zag product 327
9.9 Explicit construction of expanders via the Zig-Zag product 338

Part IV Harmonic analysis on finite linear groups

10 Representation theory of finite groups 343
10.1 Representations, irreducibility, and equivalence 343
10.2 Schur's lemma and the orthogonality relations 349
10.3 The group algebra and the Fourier transform 361
10.4 Group actions and permutation characters 372
10.5 Conjugate representations and tensor products 380
10.6 The commutant of a representation 390
10.7 A noncommutative FFT 397

11 Induced representations and Mackey theory 399
11.1 Induced representations 399
11.2 Frobenius reciprocity 409

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-1-107-18233-2 — Discrete Harmonic Analysis
Tullio Ceccherini-Silberstein , Fabio Scarabotti , Filippo Tolli
Table of Contents
More Information

11.3 Preliminaries on Mackey's theory 413
11.4 Mackey's formula for invariants 414
11.5 Mackey's lemma 419
11.6 The Mackey-Wigner little group method 421
11.7 Semidirect products with an Abelian group 424

12 Fourier analysis on finite affine groups and finite Heisenberg groups 426
12.1 Representation theory of the affine group Aff($\mathbb{F}_q$) 426
12.2 Representation theory of the affine group Aff($\mathbb{Z}/n\mathbb{Z}$) 432
12.3 Representation theory of the Heisenberg group $H_3(\mathbb{Z}/n\mathbb{Z})$ 437
12.4 The DFT revisited 443
12.5 The FFT revisited 447
12.6 Representation theory of the Heisenberg group $H_3(\mathbb{F}_q)$ 457

13 Hecke algebras and multiplicity-free triples 460
13.1 Preliminaries and notation 460
13.2 Hecke algebras 462
13.3 Commutative Hecke algebras 466
13.4 Spherical functions: intrinsic theory 469
13.5 Harmonic analysis on the Hecke algebra $\mathcal{H}(G, K, \chi)$ 474

14 Representation theory of GL(2, $\mathbb{F}_q$) 482
14.1 Matrices associated with linear operators 482
14.2 Canonical forms for $\mathfrak{M}_2(\mathbb{F})$ 484
14.3 The finite case 488
14.4 Representation theory of the Borel subgroup 492
14.5 Parabolic induction 494
14.6 Cuspidal representations 501
14.7 Whittaker models and Bessel functions 512
14.8 Gamma coefficients 522
14.9 Character theory of GL(2, $\mathbb{F}_q$) 527
14.10 Induced representations from GL(2, $\mathbb{F}_q$) to GL(2, $\mathbb{F}_{q^m}$) 533
14.11 Decomposition of tensor products 540

Appendix Chebyshëv polynomials 543

Bibliography 555
Index 563

© in this web service Cambridge University Press www.cambridge.org