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1
Introduction

Compact objects are the end points of stellar evolution and comprise white dwarfs, neutron stars, and black holes. There is a crucial difference from ordinary stars: compact stars do not burn nuclear fuel, so they are not supported by thermal pressure against the pull of gravity.

White dwarfs are supported by the degeneracy pressure of electrons. Neutron stars are supported by the repulsive interactions between nucleons. Black holes are completely collapsed objects, they contain a singularity. They are fully described by the Einstein equations in vacuum, so no matter is needed for the black hole solution.

All compact objects are essentially static over the lifetime of the universe and small in size compared to other astronomical objects such as ordinary stars or galaxies. Exceptions are mini black holes, which are subject to Hawking evaporation and evaporate in a finite time. And supermassive black holes have sizes comparable to ordinary stars.

Compact stars involve a huge range of densities. Also they involve all four fundamental forces: the strong, the weak, the electromagnetic force, and gravity, in many cases under extreme conditions. Table 1.1 lists the properties of compact objects together with the ones of our Sun and Earth. The mean mass density is defined as

$$\bar{\rho} = \frac{3M}{4\pi R^3}, \tag{1.1}$$

with the mass M and the radius R, and the surface gravity as

$$g = \frac{GM}{R^2}, \tag{1.2}$$

with the gravitational constant $G = 6.67408(31) \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$. For white dwarfs, the mass and radius is known in a few cases. For neutron stars, masses can be determined quite accurately. The range in neutron star radius comes from theoretical constraints, as its precise determination from observations at the time

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Table 1.1 *Comparison of compact objects.*

	Mass	Radius (km)	Compactness	Surface gra
Earth	$5.9724(3) \times 10^{24}$ kg	6 378.1[a]	6.9×10^{-10}	9.80
Sun	$1.98848(9) \times 10^{30}$ kg	695 700[c]	2.1×10^{-6}	27
White dwarf	$(0.5 - 1)\, M_\odot$	5,000–10,000	$(0.7\text{–}3) \times 10^{-4}$	$\sim$
Neutron star	$(1\text{–}2)\, M_\odot$	10–15	0.1–0.3	$\sim$
Stellar mass black hole	$\sim 10\, M_\odot$	~ 30	0.5	$\sim$
Supermassive black hole	$(10^6\text{–}10^{10})\, M_\odot$	$3 \times 10^6\text{–}3 \times 10^{10}$	0.5	$10^{3.}$

[a] Nominal Earth equatorial radius
[b] Defined as standard acceleration due to gravity
[c] Nominal Solar equatorial radius

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of writing is still rather model dependent. The satellite mission NICER measured the radius of the neutron star PSR J0030 + 0451 to be $12.71^{1.19}_{-1.14}$ km (Riley, T. E. et al. 2019. A NICER View of PSR J0030+451: Millisecond Pulsar Parameter Estimation. Astrophys. J. **887**, L21) and $13.02^{+1.24}_{-1.06}$ km (Miller, M. C. et al. 2019. PSR J0030+0451 Mass and Radius from NICER Data and Implications for the Properties of Neutron Star Matter. Astrophys. J. **887**, L24). For black holes, two classes are listed in Table 1.1 that are known from astrophysical observations. There exist several candidates for stellar mass black holes in our galaxy with rough mass estimates.

The radius listed for black holes is the Schwarzschild radius, named after Karl Schwarzschild:

$$R_s = \frac{2GM}{c^2}. \tag{1.3}$$

We will define the ratio of the gravitational mass to the radius as the compactness of a star

$$C = \frac{GM}{Rc^2}, \tag{1.4}$$

which is a measure how close the radius of the star is to the size of a black hole of the same mass. For nonrotating black holes, the compactness is one half. For a maximally rotating Kerr black hole, named after Roy Kerr, the event horizon is located at $R = GM/c^2$, so that its compactness is equal to 1. The mean density for black holes is undefined as no matter is present. The surface gravity for black holes is taken from the expression for Schwarzschild black holes at the horizon defined as $\kappa = 1/(4GM)$, which amusingly coincides with the Newtonian definition when using the Schwarzschild radius in Eq. (1.2). The surface gravity of a black hole is the local acceleration experienced by an observer hoovering at the black hole horizon. One should keep in mind, however, that the acceleration at the black hole horizon seen by a distant observer diverges.

White dwarfs, neutron stars, and stellar mass black holes have masses similar to that of our Sun. The sizes of white dwarfs are similar to that of Earth, while those of neutron stars are much smaller and are comparable to the size of a city on Earth. Supermassive black holes are as massive as globular clusters, with typical masses of $10^6 M_\odot$, and can nearly reach the masses of small galaxies. For comparison, the mass of our galaxy is estimated to be $10^{11} M_\odot$. From the first detection of gravitational waves in 2015, we know that stellar mass black holes exist with masses of $36^{+5}_{-4} M_\odot$, $29^{+4}_{-4} M_\odot$, and $62^{+4}_{-4} M_\odot$ (Abbott et al., 2016a). Several black hole mergers have been measured since then, observing black holes in the mass range of about $M = 10$–$80 M_\odot$ (Abbott et al., 2018a). Masses of many supermassive black holes sitting at the center of galaxies are known, in particular the one in our own galaxy with a mass of $(4.3 \pm 0.5) \times 10^6 M_\odot$ (Gillessen et al., 2009). In 2019, a picture of the supermassive black hole at the center of galaxy M87 was taken by

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the Event Horizon Telescope, showing the accretion disk around the supermassive black hole and its shadow. The mass of black hole M87* was determined to be $(6.5 \pm 0.7) \times 10^9 M_\odot$ (Akiyama et al., 2019).

The radii of black holes span from a size slightly larger than that of our Sun to the size of our solar system – about 100 AU (by definition $1\,\mathrm{AU} = 149\,597\,870\,700\,\mathrm{m}$, which is about the average distance of the Earth to our Sun). The compactness shows how close the star is in size to half its own Schwarzschild radius. The closer the compactness gets to 1, the more important will the effects be from general relativity. White dwarfs have a rather small compactness of about 10^{-4}. Neutron stars have a compactness that is not too far from 1, so it is essential to describe neutron stars within general relativity. By looking at the mean density, one realizes that compact stars, white dwarfs, and neutron stars involve huge mass densities that are several orders of magnitude larger than our Sun and Earth. In fact, one teaspoon of neutron star material is equal to the mass of all humans together.[1] Also, the surface gravity is several orders of magnitude larger than our Earth and Sun and is similar to those of astrophysical black holes.

It is interesting to look at the origin of the names, the etymology, of compact objects.

White dwarfs were first characterized by their appearance. As they have high effective temperatures at their surfaces, they appear much 'whiter' than ordinary stars. The first white dwarf that was discovered was Sirius B, the companion to the brightest star in the sky, Sirius A, or simply Sirius. The deduced radius of Sirius B was much smaller than that of ordinary stars (see Table 1.1). Therefore, the name 'white dwarf' was given and is still used today.

Neutron stars were first characterized by their properties postulated before their actual discovery. The term 'neutron stars' was introduced for the first time by Walter Baade and Fritz Zwicky in the proceedings of the American Physical Society meeting at Stanford, December 15–16, 1933:

> With all reserve we advance the view that supernovae represent the transitions from ordinary stars into *neutron stars*, which in their final stages consist of extremely closely packed neutrons. (Baade and Zwicky, 1934a)

However, the concept of neutron stars was envisioned earlier by Lev Landau (Landau, 1932), probably as early as 1931 in a discussion with Niels Bohr and Leon Rosenfeld in Copenhagen. We refer the interested reader to the excellent historical treatise (Yakovlev et al., 2013). Neutron stars consist mainly of neutrons in the interior, due to the inverse β-decay of $e^- + p \rightarrow n + \nu_e$. Hence, neutron stars can be considered as 'giant nuclei' with mass densities comparable to and exceeding

[1] Just imagine having all humans packed onto a single teaspoon!

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that of the nucleus of an atom of $\rho_{\text{nuclei}} = 2.5 \times 10^{17}\,\text{kg}\,\text{m}^{-3}$. However, while ordinary stable nuclei have mass numbers of up to $A = 208$ for ^{208}Pb, neutron stars accommodate mass numbers of the order of $A = 10^{57}$. Neutron stars do not shine as ordinary stars. However, neutron stars have been observed in the optical, in X-rays, and in the radio band. Young neutron stars have been observed directly and they have the highest surface temperatures known for astrophysical objects. They radiate mainly in X-rays with temperatures in the range of $1{,}000{,}000^\circ$C. Pulsars are rotation-powered neutron stars and are regularly observed by radio telescopes.

Black holes were characterized by their theoretically derived appearance and properties. Neither matter nor light can escape from black holes, so they will appear 'black' to an observer. Anything falling into a black hole will disappear. The naming seems to be obvious and is attributed to John Wheeler, who first introduced it into scientific literature. However, as he recalls in his autobiography, someone in the audience suggested the name to him during a talk on the discovery of pulsars in the fall of 1967 instead of 'gravitationally completely collapsed object' (Wheeler and Ford, 1998, p. 296).

Nowadays, black holes are indirectly observed as astrophysical phenomena in the form of, for example, active galactic nuclei or X-ray bursters. The former imply supermassive black holes at the center of galaxies, the latter, stellar mass black holes with a companion star. In both cases, the black holes accrete gas from the surroundings, which is then heated up. The heated matter radiates, so black holes can be observed through their impact on ordinary matter. The black hole in the center of our galaxy in the constellation Sagittarius is observed by the Keplerian motion of nearby stars, revealing a huge gravitational pull of a supermassive object within a small volume. The accretion disk around the supermassive black hole at the center of galaxy M87 was made directly visible by the very long baseline interferometry of the Event Horizon Telescope collaboration.

White dwarfs and neutron stars are strongly connected with stellar evolution. Both are the endpoints of stellar evolution. Whether a white dwarf or a neutron star is formed at the end of the life of a star depends on the initial mass of the star, to be more precise, the mass of the star when it reaches stable hydrogen burning. The luminosity of the stars can be plotted against the observed surface temperature, which is equivalent to the spectral classification scheme OBAFGKM, in the Hertzsprung–Russell diagram. One observes that most of the stars are located along a line called the main sequence of the Hertzsprung–Russell diagram. The stars burning hydrogen spent most of their time along the main sequence. The initial mass of a star is then called the zero age main sequence (ZAMS) mass.

Black holes could be produced in a variety of ways: as the endpoint of stellar evolution of sufficiently massive stars (including the first stars formed after the big

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bang), in the early universe forming primordial black holes ('mini black holes' with $M \sim 10^{15}$ g), or by the collision of compact stars in binary systems.

The formation of compact objects as endpoints of stellar evolution can be summarized as follows. As pointed out earlier, the final endpoint of stellar evolution depends on the ZAMS mass.

$M < 0.01 M_\odot$: This is the realm of planets, which are not sufficiently massive to generate any nuclear burning at all. For comparison, the mass of Jupiter is about $M_J \approx 0.001 M_\odot$.

$0.01 M_\odot \leq M < 0.08 M_\odot$: The star is not sufficiently massive for stable nuclear burning in the core. There is some burning of deuterons, which gives the failed star a reddish-brownish color. This is the realm of brown dwarfs.

$0.08 M_\odot \leq M < 0.4 M_\odot$: Stable hydrogen burning is present in the core of the star. The star will eventually exhaust its hydrogen supply and will end up in a hydrogen-helium white dwarf.

$0.4 M_\odot \leq M < 8 M_\odot$: The star is sufficiently massive to start helium burning during its evolution. Heavier elements are produced so that the final white dwarfs contain carbon and oxygen. Shells of matter will be blown into interstellar space and will be illuminated by the ultraviolet glow of the white dwarf, generating a so-called planetary nebula. Our Sun will eventually end as such a carbon-oxygen white dwarf.

$8 M_\odot \leq M < 10 M_\odot$: Carbon burning will be possible during the final stages of stellar evolution. A degenerate neon-oxygen-magnesium core forms, which will eventually collapse to a neutron star. The initially hot proto-neutron star emits neutrinos. The outgoing shock wave releases energy visible in a so-called core-collapse supernova.

$10 M_\odot \leq M < 25 M_\odot$: Above $10 M_\odot$ ZAMS mass silicon burning sets in as the final burning stage of the star, forming a degenerate iron core. The iron core collapses to a neutron star, which is visible as a core-collapse supernova.

$M > 25 M_\odot$: This mass limit is not well known and depends on the unknown maximum mass of the neutron star formed in the collapse of the degenerate iron core. The still hot neutron star is too massive to withstand the pull of gravity and collapses to a black hole. This could lead to a failed supernova or in the other extreme to a particularly bright supernova, a hypernova.

$M \sim 100 M_\odot$: This is about the upper mass limit for ordinary stars. They are so hot in the core that they produce electron-positron pairs that destabilize the star. The first stars formed in the early universe can be much more massive due to a lack of elements that are heavier than helium (metals in the jargon of astrophysicists).

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Table 1.2 *The endpoints of stellar evolution based on the zero age main sequence (ZAMS) mass, the initial mass of the star.*

ZAMS mass	0.04–8$M_\odot$	8–25$M_\odot$	$> 25M_\odot$
	White dwarf	Neutron star	Black hole

This picture has been simplified, as it neglects effects from magnetic fields and rotation, for example. As a short summary, Table 1.2 shows the compact objects that are the endpoints of stellar evolution based on the ZAMS mass of the star.

It should be clear from this that the different endpoints in stellar evolution are controlled by the ZAMS mass not the mass of the compact object itself. In standard stellar evolution, there is not a path from one compact object to the other. However, there are possibilities of a compact object to transform to another one for accreting systems, that is, for a white dwarf or a neutron star to accrete matter from a companion star. An accreting neutron star will eventually collapse to a black hole when reaching the maximum possible mass. For white dwarfs the issue is more subtle.

A white dwarf reaches a critical density for igniting thermonuclear burning when accreting matter. The standard scenario is that this burning is a runaway reaction, leading to an explosion of the whole white dwarf with no remnants. This explosion is well known as a type Ia supernova and used as a standard candle in cosmology. There is the possibility that the accreting white dwarf experiences a collapse instead of an explosion, a so-called accretion-induced collapse. The collapsing white dwarf would then form a neutron star during the collapse. For iron white dwarfs this would be an option. However, as mentioned earlier, the stellar burning stops before carbon fusion for stars ending in a white dwarf, so that the standard white dwarf contains carbon and oxygen but not iron in its core.

Exercises

(1.1) Calculate the Schwarzschild radius of the Sun, the Earth, and your own one.

(1.2) Calculate the gravitational acceleration for the Sun, white dwarfs, and neutron stars.

(1.3) Assume that every 100 years a supernova produces a neutron star in our galaxy. Estimate the present number of neutron stars in our galaxy.

(1.4) How many nucleons are in the Sun?

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2
General Relativity

This chapter is a brief introduction to some of the basic concepts of general relativity. It is not meant to provide a thorough introduction to general relativity and we refer the reader to the many excellent textbooks written on that subject. The main purpose of this chapter is to derive the results of general relativity necessary for our discussion of the properties of compact stars and its astrophysical applications in the later chapters. A separate chapter is devoted to the subject of gravitational waves (see Chapter 10).

2.1 Gravity and the Equivalence Principle

Let us introduce the concept of general relativity by looking at Newton's law and Newton's expression for the gravitational force. According to Newton's law a massive particle feels a kinetic force

$$F_{\text{kin}} = m_i \cdot a \tag{2.1}$$

that is proportional to the acceleration a with the coefficient m_i. We will denote the coefficient m_i as the inertial mass of the particle. According to Newton, the classical gravitational force

$$F_g = -m_g \cdot \nabla\phi \tag{2.2}$$

is proportional to the gradient of the gravitational potential ϕ with the coefficient m_g. We will denote the coefficient m_g as the gravitational mass of the particle. In experiments first performed by Galileo Galilei, it was found out that every object falls with the same acceleration in a gravitational field so that

$$a = -\nabla\phi \tag{2.3}$$

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independent of its inertial and gravitational mass. Hence, the inertial and the gravitational mass have to be the same

$$m_g = m_i. \tag{2.4}$$

Eq. (2.4) constitutes the weak equivalence principle (or WEP).

Weak equivalence principle: All particles experience the same acceleration in a gravitational field irrespective of their masses.

WEP has been tested to a high accuracy in many different experiments. Modern torsion balance experiments have tested the WEP down to a level of 10^{-13} (Wagner et al., 2012). The satellite experiment MICROSCOPE has tested the weak equivalence principle at a level of 10^{-15} (Touboul et al., 2017). There are equivalent formulations of the WEP. For example, WEP implies that there is a preferred trajectory of particles through spacetime on which particles move that is determined just by gravity.

An extension of the WEP is the Einstein equivalence principle (EEP), which includes any form of matter, not only particles. Consider an observer in a sealed box performing experiments. From the equivalence of the inertial and the gravitational mass, an observer cannot distinguish whether an object is accelerated by the acceleration of the box or whether it is accelerated by the presence of a gravitational field. It is impossible to detect the existence of a gravitational field by local experiments. It is important to add the condition of locality to the experiments as gradients of the gravitational field will lead to tidal forces on larger scales that would be measurable. The observer will measure locally only the laws of physics as they would be in a spacetime without gravity, so that the kinematics are governed by special relativity. The EEP can be stated as follows (see e.g., Schutz, 2009):

Einstein equivalence principle: In small enough regions of spacetime, any physical experiment not involving gravity will have the same result if performed in a freely falling inertial frame as if performed in the flat spacetime of gravity.

We will heavily use EEP in the following sections. It allows the transfer of equations valid in special relativity to its general form in general relativity as both equations have to be of the same form in a small enough region of spacetime.

EEP has important implications for the way gravity couples to matter. As all physical laws, except gravity, are included in defining the EEP, it implies that gravity couples to the other forces, of nature in a special way. Let us be more specific here. An atom consists of electrons and a nucleus bound by electromagnetic forces, which is stabilized by quantum mechanics. The mass of an atom is not the sum of the mass of electrons and the nucleus but slightly less. The difference in mass

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is dubbed the mass defect or simply the binding energy of the atom. Seemingly, gravity couples not only to the masses of the constituents of the atom but it is also sensitive to the mass defect generated by the quantum electromagnetic forces. Hence, gravity couples to electromagnetic energy. The same story goes for the nucleus of an atom. The nucleus consists of protons and neutrons. The mass of the nucleus is not the sum of the masses of the protons and neutrons in the nucleus but somewhat less, which is just the mass defect or the binding energy of the nucleus. In the nucleus, the nuclear forces or the strong interactions are at work. So gravity couples to energy generated by strong interactions. The same reasoning should also apply for the other fundamental force of the standard model left, weak interactions. The masses of the elementary particle, quarks and leptons, are generated by the Higgs mechanism. So gravity couples also to energy (mass) generated by the weak interactions. The reader is invited to fancy at a system consisting of leptons only that is bound by weak interactions only, as, for example, a hypothetical neutrino ball, and perform the same reasoning for nuclei and strong interactions.

The equivalence of performing experiments in an accelerating system and in one at rest being exposed to a gravitational field can be exploited in Einstein's famous way via thought experiments. Hereby one imagines certain experimental situations to arrive at statements about physical laws. Let us imagine a rocket that is constantly accelerating in free space that shall be equal to the gravitational acceleration on Earth. According to EEP, experiments performed inside the rocket are equivalent to the ones performed in a rocket standing on Earth as the systems are exposed to the same acceleration. From this setting one can immediately derive two important consequences of EEP that form the basis of general relativity: the gravitational redshift of light and the gravitational bending of light in a gravitational field.

Imagine an electromagnetic wave with a fixed frequency (e.g., from a laser) being emitted from the bottom of the rocket and being measured at the top of the rocket. If the rocket is continuously accelerating, the speed of the rocket at the time of the detection will be correspondingly larger compared to the one at the time of emission for each crest (or trough) of the wave. The time of arrival of the wave crests of the light at the detector will be correspondingly larger than the one at constant speed. This effect leads to a decrease of the frequency of the wave being measured at the top of the rocket compared to the original frequency, that is, a redshift of the electromagnetic wave. Switching the emitter with the detector, so that the wave travels from the top of the rocket to its bottom, the electromagnetic wave will be blueshifted when measured at the bottom of the rocket. EEP now states that the experimental situation is equivalent to the one in a gravitational field with the same (gravitational) acceleration. Hence, an electromagnetic wave being emitted within a gravitational potential to the outside will be measured to have a lower frequency compared to original frequency by an outside observer. This is

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