String Theory Methods for Condensed Matter Physics

The discovery of a duality between Anti–de Sitter spaces (AdS) and Conformal Field Theories (CFT) has led to major advances in our understanding of quantum field theory and quantum gravity. String theory methods and AdS/CFT correspondence maps provide new ways to think about difficult condensed matter problems. String theory methods based on the AdS/CFT correspondence allow us to transform problems so they have weak interactions and can be solved more easily. They can also help map problems to different descriptions, for instance, mapping the description of a fluid using the Navier-Stokes equations to the description of an event horizon of a black hole using Einstein’s equations. This textbook covers the applications of string theory methods and the mathematics of AdS/CFT to areas of condensed matter physics. Bridging the gap between string theory and condensed matter, this is a valuable textbook for students and researchers in both fields.

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String Theory Methods for Condensed Matter Physics

HORAȚIU NĂSTASE
Universidade Estadual Paulista, São Paulo
To the memory of my mother,
who inspired me to become a physicist
Contents

Preface xvii
Acknowledgments xviii
Introduction xix

Part I Condensed Matter Models and Problems 1

1 Lightning Review of Statistical Mechanics, Thermodynamics, Phases, and Phase Transitions 3
  1.1 Note on Conventions 3
  1.2 Thermodynamics 3
  1.3 Phase Transitions 7
  1.4 Statistical Mechanics and Ensembles 8
  1.5 Distributions 12
  Exercises 14

2 Magnetism in Solids 16
  2.1 Types of Magnetism: Diamagnetism, Paramagnetism, (Anti)Ferromagnetism, Ferrimagnetism 16
  2.2 Langevin Diamagnetism 19
  2.3 Paramagnetism of Bound Electrons (Langevin-Brillouin) 20
  2.4 Pauli Paramagnetism 21
  2.5 Van Vleck Paramagnetism 22
  2.6 Ferromagnetism 23
  2.7 Ferrimagnetism and Antiferromagnetism 25
  Exercises 27

3 Electrons in Solids: Fermi Gas vs. Fermi Liquid 28
  3.1 Free Fermi Gas 28
  3.2 Fermi Surfaces 33
  3.3 Fermi Liquid 38
  Exercises 39

4 Bosonic Quasi-Particles: Phonons and Plasmons 42
  4.1 Phonons in 1+1 and 3+1 Dimensions 42
  4.2 Optical and Acoustic Branches; Diatomic Lattices 45
  4.3 Electron-Phonon Interaction 46
## Contents

4.4 Plasmons 48
4.5 Thomas-Fermi Screening 49
Exercises 51

5 Spin-Charge Separation in 1+1–Dimensional Solids: Spinons and Holons 52
5.1 One-Dimensional (Fermionic) Hubbard Model and the Luttinger Liquid 52
5.2 Bosonization 54
5.3 Spin-Charge Separation: Spinons and Holons 58
5.4 Sine-Gordon vs. Massive Thirring Model Duality 59
Exercises 61

6 The Ising Model and the Heisenberg Spin Chain 62
6.1 The Ising Model in 1+1 and 2+1 Dimensions 62
6.2 Mean Field Approximation 65
6.3 Kramers-Wannier Duality at $H = 0$ 67
6.4 The Heisenberg Model and Coordinate Bethe Ansatz 68
Exercises 73

7 Spin Chains and Integrable Systems 75
7.1 Classical Integrable Systems 75
7.2 Quantum Mechanical Integrable Systems in 1+1 Dimensions 76
7.3 Algebraic and Coordinate Bethe Ansatz 78
7.4 Generalizations 81
Exercises 83

8 The Thermodynamic Bethe Ansatz 84
8.1 Massive Relativistic 1+1–Dimensional Integrable Systems and Bethe Ansatz Equations 84
8.2 Thermodynamic Limit 86
8.3 Thermodynamics and TBA Equations 87
Exercises 89

9 Conformal Field Theories and Quantum Phase Transitions 91
9.1 Thermal Phase Transitions and Conformal Field Theory 91
9.2 Conformal Field Theory in Two and $d$ Dimensions 93
9.3 Quantum Conformal Field Theory 94
9.4 Quantum Phase Transitions 97
9.5 Insulator-Superconducting Phase Transition 99
Exercises 102

10 Classical vs. Quantum Hall Effect 104
10.1 Classical Hall Effect 104
10.2 Hall Effect Experimental Results 105
10.3 Integer Quantum Hall Effect 106
### Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.4</td>
<td>Edge Currents and Laughlin’s Gauge Principle</td>
<td>109</td>
</tr>
<tr>
<td>10.5</td>
<td>The Fractional Quantum Hall Effect</td>
<td>112</td>
</tr>
<tr>
<td></td>
<td>Exercises</td>
<td>115</td>
</tr>
<tr>
<td>11</td>
<td>Superconductivity: Landau-Ginzburg, London, and BCS</td>
<td>117</td>
</tr>
<tr>
<td>11.1</td>
<td>Superconductivity Properties</td>
<td>117</td>
</tr>
<tr>
<td>11.2</td>
<td>Landau’s Theory of Second Order Phase Transitions</td>
<td>119</td>
</tr>
<tr>
<td>11.3</td>
<td>Thermodynamics of the Superconducting Phase Transition</td>
<td>122</td>
</tr>
<tr>
<td>11.4</td>
<td>Landau-Ginzburg Theory and Gross-Pitaevskii Equation</td>
<td>124</td>
</tr>
<tr>
<td>11.5</td>
<td>Electrodynamics of the Superconductor: London Equation</td>
<td>127</td>
</tr>
<tr>
<td>11.6</td>
<td>BCS Theory</td>
<td>129</td>
</tr>
<tr>
<td>11.7</td>
<td>Vortices in Type II Superconductors</td>
<td>133</td>
</tr>
<tr>
<td>11.8</td>
<td>Josephson Junctions</td>
<td>134</td>
</tr>
<tr>
<td>11.9</td>
<td>High $T_c$ Superconductors</td>
<td>137</td>
</tr>
<tr>
<td></td>
<td>Exercises</td>
<td>140</td>
</tr>
<tr>
<td>12</td>
<td>Topology and Statistics: Berry and Chern-Simons, Anyons, and Nonabelions</td>
<td>141</td>
</tr>
<tr>
<td>12.1</td>
<td>Berry Phase and Connection</td>
<td>141</td>
</tr>
<tr>
<td>12.2</td>
<td>Effective Chern-Simons Theories in Solids</td>
<td>143</td>
</tr>
<tr>
<td>12.3</td>
<td>Anyons and Fractional Statistics</td>
<td>145</td>
</tr>
<tr>
<td>12.4</td>
<td>Nonabelian Anyons (Nonabelions) and Statistics</td>
<td>148</td>
</tr>
<tr>
<td>12.5</td>
<td>Topological Superconductors</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>Exercises</td>
<td>153</td>
</tr>
<tr>
<td>13</td>
<td>Insulators</td>
<td>154</td>
</tr>
<tr>
<td>13.1</td>
<td>Fermionic Hubbard Model and Metal-Insulator Transition</td>
<td>155</td>
</tr>
<tr>
<td>13.2</td>
<td>Defects and Anderson Localization</td>
<td>158</td>
</tr>
<tr>
<td>13.3</td>
<td>Mott Insulators</td>
<td>160</td>
</tr>
<tr>
<td>13.4</td>
<td>Topological Insulators</td>
<td>161</td>
</tr>
<tr>
<td></td>
<td>Exercises</td>
<td>166</td>
</tr>
<tr>
<td>14</td>
<td>The Kondo Effect and the Kondo Problem</td>
<td>167</td>
</tr>
<tr>
<td>14.1</td>
<td>The Kondo Hamiltonian and Impurity-Conduction Spin Interaction</td>
<td>167</td>
</tr>
<tr>
<td>14.2</td>
<td>Electron-Impurity Scattering and Resistivity Minimum; Kondo Temperature</td>
<td>169</td>
</tr>
<tr>
<td>14.3</td>
<td>Strong Coupling and the Kondo Problem</td>
<td>173</td>
</tr>
<tr>
<td>14.4</td>
<td>Heavy Fermions and the Kondo Lattice</td>
<td>175</td>
</tr>
<tr>
<td></td>
<td>Exercises</td>
<td>177</td>
</tr>
<tr>
<td>15</td>
<td>Hydrodynamics and Transport Properties: From Boltzmann to Navier-Stokes</td>
<td>178</td>
</tr>
<tr>
<td>15.1</td>
<td>Boltzmann Equations</td>
<td>178</td>
</tr>
<tr>
<td>15.2</td>
<td>Applications of the Boltzmann Equation</td>
<td>179</td>
</tr>
<tr>
<td>15.3</td>
<td>Nonviscous Hydrodynamics</td>
<td>181</td>
</tr>
<tr>
<td>15.4</td>
<td>Viscous Hydrodynamics and Navier-Stokes</td>
<td>183</td>
</tr>
</tbody>
</table>
## Contents

15.5 Relativistic Generalization 184
Exercises 187

### Part II Elements of General Relativity and String Theory 189

16 The Einstein Equation and the Schwarzschild Solution 191
16.1 Special Relativity 191
16.2 The Geometry of General Relativity 191
16.3 Einstein’s Theory: Definition 194
16.4 Kinematics of Gravity 195
16.5 Dynamics of Gravity: The Einstein Equation 197
16.6 The Newtonian Limit 199
16.7 The Schwarzschild Solution 200
Exercises 202

17 The Reissner-Nordstrom and Kerr-Newman Solutions and Thermodynamic Properties of Black Holes 204
17.1 The Reissner-Nordstrom Solution: Solution with Electric Charge 204
17.2 Horizons, BPS Bound, and Extremality 205
17.3 General Stationary Black Hole: The Kerr-Newman Solution 207
17.4 Quantum Field Theory at Finite Temperature and Hawking Radiation 208
17.5 The Four Laws of Black Hole Thermodynamics 211
Exercises 215

18 Extra Dimensions and Kaluza-Klein 216
18.1 Kaluza-Klein Theory 216
18.2 The Kaluza-Klein Metrics 217
18.3 Fields with Spin 220
18.4 The Original Kaluza-Klein Theory 222
Exercises 225

19 Electromagnetism and Gravity in Various Dimensions, Consistent Truncations 226
19.1 Electromagnetism in Higher Dimensions 226
19.2 Gravity in Higher Dimensions 227
19.3 Spheres in Higher Dimensions 227
19.4 The Electric Field of a Point Charge in Higher Dimensions 228
19.5 The Gravitational Field of a Point Mass in Higher Dimensions 229
19.6 Consistent Truncations 232
Exercises 235

20 Gravity Plus Matter: Black Holes and $p$-Branes in Various Dimensions 236
20.1 Schwarzschild Solution in Higher Dimensions 236
20.2 Reissner-Nordstrom Solution in Higher Dimensions 237
20.3 $p$-Branes 240
## Contents

20.4 P-Brane Solutions 242
Exercises 245

21 Weak/Strong Coupling Dualities in $1+1$, $2+1$, $3+1$, and $d+1$ Dimensions 247
21.1 $1+1$ Dimensions: T-Duality in the Path Integral 248
21.2 $2+1$ Dimensions: Particle-Vortex Duality 250
21.3 $3+1$ Dimensions: Maxwell Duality 251
21.4 $d+1$ Dimensions: Poincaré Duality 252
Exercises 254

22 The Relativistic Point Particle and the Relativistic String 256
22.1 The Relativistic Point Particle 256
22.2 First Order Action for the Point Particle 258
22.3 Relativistic Strings 260
22.4 First Order Action for the Relativistic String 262
22.5 Gauge Fixing and Constraints 265
Exercises 267

23 Lightcone Strings and Quantization 269
23.1 Conformal Invariance 269
23.2 Light-Cone Gauge and Mode Expansions 270
23.3 Constraints and Hamiltonian 272
23.4 Quantization of the String 274
23.5 Light-Cone Gauge Quantization 276
23.6 String Spectrum 278
Exercises 280

24 D-Branes and Gauge Fields 282
24.1 D-Branes 282
24.2 The D-Brane Action: DBI 283
24.3 The WZ Term 288
24.4 Quantization of Open Strings on $Dp$-Branes 289
Exercises 293

25 Electromagnetic Fields on D-Branes: Supersymmetry and $\mathcal{N} = 4$ SYM, T-Duality of Closed Strings 295
25.1 Low-Energy D-Brane Action 295
25.2 Supersymmetry 296
25.3 $\mathcal{N} = 4$ Super Yang-Mills in $3+1$ Dimensions 297
25.4 Nonlinear Born-Infeld Action 298
25.5 Closed Strings on Compact Spaces 300
25.6 T-Duality of the Spectrum and of the Background 302
Exercises 305
# Contents

## 26 Dualities and M Theory

- 26.1 T-Duality of Open Strings  
- 26.2 T-Duality and Chan-Paton Factors  
- 26.3 Supergravity Actions  
- 26.4 S-Duality of Type IIB Theory  
- 26.5 M-Theory  
- 26.6 The String Duality Web  
Exercises  

## 27 The AdS/CFT Correspondence: Definition and Motivation

- 27.1 Conformal Invariance and Conformal Field Theories in $D > 2$ Dimensions  
- 27.2 AdS Space  
- 27.3 AdS/CFT Motivation  
- 27.4 AdS/CFT Definition and Limits  
- 27.5 State Map  
- 27.6 Witten (or GKPW) Construction  
- 27.7 Generalizations  
Exercises  

## Part III Applying String Theory to Condensed Matter Problems

## 28 The pp Wave Correspondence: String Hamiltonian from $\mathcal{N} = 4$ SYM

- 28.1 PP Waves  
- 28.2 The Penrose Limit in Gravity  
- 28.3 The Penrose Limit of AdS/CFT  
- 28.4 The String Hamiltonian on the pp Wave  
- 28.5 Bosonic SYM Operators for String Theory Modes  
- 28.6 Discretized String Action from $\mathcal{N} = 4$ SYM  
Exercises  

## 29 Spin Chains from $\mathcal{N} = 4$ SYM

- 29.1 Cuntz Oscillators, Eigenstates, and Eigenenergies from the pp Wave  
- 29.2 The $\text{SO}(6)$ Spin Chain from $\mathcal{N} = 4$ SYM  
- 29.3 The $\text{SU}(2)$ Sector and the Dilatation Operator  
- 29.4 Bethe Ansatz Results  
Exercises  

## 30 The Bethe Ansatz: Bethe Strings from Classical Strings in AdS

- 30.1 Thermodynamic Limit of the Bethe Ansatz for the $\text{SU}(2)$ Sector  
- 30.2 Bethe Strings and their Equations  
- 30.3 Long Strings in AdS  
- 30.4 Classical String Equations and Bethe String Limit  
- 30.5 Massive Relativistic Systems from AdS Space and the TBA?  
Exercises  

# Contents

31 Integrability and AdS/CFT 373
- 31.1 Hints of Integrability 373
- 31.2 Integrability and the Yangian 374
- 31.3 Quantum Integrability for Spin Chains 375
- 31.4 Classical Integrability for Strings in AdS Space 377
- 31.5 Higher Loops in $\mathcal{N} = 4$ SYM and All-Loop Bethe Ansatz 378

Exercises 381

32 AdS/CFT Phenomenology: Lifshitz, Galilean, and Schrödinger Symmetries and their Gravity Duals 383
- 32.1 Lifshitz Symmetry and its Phenomenological Gravity Dual 383
- 32.2 Galilean and Schrödinger Symmetries and their Phenomenological Gravity Duals 385
- 32.3 String Theory Embeddings of Schrödinger Symmetry 387

Exercises 389

33 Finite Temperature and Black Holes 391
- 33.1 Finite Temperature and Hawking Radiation 391
- 33.2 Fermions and Spin Structures in Black Holes and Supersymmetry Breaking 392
- 33.3 Witten Prescription for Finite Temperature in AdS/CFT 394
- 33.4 The Witten Metric 397

Exercises 400

34 Hot Plasma Equilibrium Thermodynamics: Entropy, Charge Density, and Chemical Potential of Strongly Coupled Theories 402
- 34.1 $\mathcal{N} = 4$ SYM Plasma from AdS/CFT: Entropy, Energy Density, Pressure 402
- 34.2 Adding Chemical Potential and Charge Density 405
- 34.3 Phenomenological Plasma Models 407
- 34.4 Speed of Sound 410

Exercises 412

35 Spectral Functions and Transport Properties 413
- 35.1 Retarded Green’s Functions and Susceptibilities 413
- 35.2 Spectral Functions 415
- 35.3 Kubo Formulas 417
- 35.4 AdS/CFT in Minkowski Space at Finite Temperature 419
- 35.5 Other Transport Properties 421

Exercises 423

- 36.1 Electric Charge Transport in the Plasma and the Membrane Paradigm 424
- 36.2 Momentum Broadening, Langevin Diffusion, and its Gravity Dual 429
# Contents

36.3 Thermalization from Quasi-Normal Modes of the Dual Black Hole 434  
Exercises 437

37 The Holographic Superconductor 438  
37.1 Holographic Superconductor Ingredients 438  
37.2 Superconducting Black Holes 440  
37.3 Transport and Mass Gap 443  
37.4 Breitenlohner-Freedman Bound and Superconductor 445  
37.5 Holographic Josephson Junctions 446  
Exercises 452

38 The Fluid-Gravity Correspondence: Conformal Relativistic Fluids from Black Hole Horizons 453  
38.1 Relativistic Fluids 453  
38.2 Conformal Fluids 455  
38.3 Fluid Equations from Black Holes 457  
38.4 Viscosity over Entropy Density from Black Holes 458  
38.5 Membrane Paradigm: Relativistic Fluid Equations from Black Hole Horizons 460  
Exercises 465

39 Nonrelativistic Fluids: From Einstein to Navier-Stokes and Back 466  
39.1 The Navier-Stokes Scaling Limit 466  
39.2 Relativistic Hydrodynamics and Bjorken Flow 468  
39.3 Boost-Invariant Conformal Flows from Black Hole Horizon 472  
39.4 Einstein’s Equations Solutions from Solutions of Navier-Stokes Equation 473  
Exercises 475

Part IV Advanced Applications 477

40 Fermi Gas and Liquid in AdS/CFT 479  
40.1 Gravity Dual for Fermi Gas 479  
40.2 Fermi Liquid Construction for Gravity Dual 483  
40.3 “Electron Star” Gravity Dual, Its “Neutron Star” Origin, and Interpolation 486  
Exercises 492

41 Quantum Hall Effect from String Theory 493  
41.1 Quantum Hall Effect and Chern-Simons Theory in String Theory 493  
41.2 Quantum Hall Effect and Chern-Simons Models from Holography 499  
Exercises 506

42 Quantum Critical Systems and AdS/CFT 507  
42.1 The ABJM Model and its Vacuum 507
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>42.2</td>
<td>Gravity Dual of the ABJM Model</td>
<td>510</td>
</tr>
<tr>
<td>42.3</td>
<td>Models for Quantum Critical Systems</td>
<td>511</td>
</tr>
<tr>
<td>42.4</td>
<td>Abelian Reduction of the ABJM Model to Landau-Ginzburg</td>
<td>512</td>
</tr>
<tr>
<td>42.5</td>
<td>Abelian Reduction to Landau-Ginzburg in Condensed Matter Systems?</td>
<td>514</td>
</tr>
<tr>
<td>42.6</td>
<td>Nonrelativistic Limit of Duality and the Jackiw-Pi Model</td>
<td>516</td>
</tr>
<tr>
<td></td>
<td>Exercises</td>
<td>521</td>
</tr>
<tr>
<td>43</td>
<td>Particle-Vortex Duality and ABJM vs. $AdS_4 \times \mathbb{CP}^3$ Duality</td>
<td>522</td>
</tr>
<tr>
<td>43.1</td>
<td>Particle-Vortex Dualities in the Path Integral</td>
<td>522</td>
</tr>
<tr>
<td>43.2</td>
<td>Particle-Vortex Duality Constraints on Condensed Matter Transport</td>
<td>525</td>
</tr>
<tr>
<td>43.3</td>
<td>Particle-Vortex Duality Embedded in the ABJM Model</td>
<td>528</td>
</tr>
<tr>
<td>43.4</td>
<td>Particle-Vortex Duality as Maxwell Duality in $AdS_4$</td>
<td>529</td>
</tr>
<tr>
<td>43.5</td>
<td>Nonrelativistic Limit and Duality with Jackiw-Pi Vortex</td>
<td>531</td>
</tr>
<tr>
<td></td>
<td>Exercises</td>
<td>533</td>
</tr>
<tr>
<td>44</td>
<td>Topology and Nonstandard Statistics from AdS/CFT</td>
<td>535</td>
</tr>
<tr>
<td>44.1</td>
<td>Berry Phase from AdS/CFT</td>
<td>535</td>
</tr>
<tr>
<td>44.2</td>
<td>Anyons in AdS/CFT</td>
<td>537</td>
</tr>
<tr>
<td>44.3</td>
<td>Nonabelian Statistics in AdS/CFT</td>
<td>540</td>
</tr>
<tr>
<td></td>
<td>Exercises</td>
<td>541</td>
</tr>
<tr>
<td>45</td>
<td>DBI Scalar Model for QGP/Black Hole Hydro- and Thermo-Dynamics</td>
<td>542</td>
</tr>
<tr>
<td>45.1</td>
<td>The Heisenberg Model and Quark-Gluon Plasma</td>
<td>542</td>
</tr>
<tr>
<td>45.2</td>
<td>DBI Scalar Model for Black Hole</td>
<td>544</td>
</tr>
<tr>
<td>45.3</td>
<td>Generalizations of the Heisenberg Model</td>
<td>548</td>
</tr>
<tr>
<td>45.4</td>
<td>Gravity Duals and the Heisenberg Model</td>
<td>549</td>
</tr>
<tr>
<td>45.5</td>
<td>QGP Hydrodynamics from DBI Scalar</td>
<td>551</td>
</tr>
<tr>
<td></td>
<td>Exercises</td>
<td>555</td>
</tr>
<tr>
<td>46</td>
<td>Holographic Entanglement Entropy in Condensed Matter</td>
<td>556</td>
</tr>
<tr>
<td>46.1</td>
<td>Entanglement Entropy in Quantum Field Theory</td>
<td>556</td>
</tr>
<tr>
<td>46.2</td>
<td>The Ryu-Takayanagi Prescription</td>
<td>558</td>
</tr>
<tr>
<td>46.3</td>
<td>Holographic Entanglement Entropy in Two Dimensions</td>
<td>559</td>
</tr>
<tr>
<td>46.4</td>
<td>Condensed Matter Applications</td>
<td>560</td>
</tr>
<tr>
<td></td>
<td>Exercises</td>
<td>563</td>
</tr>
<tr>
<td>47</td>
<td>Holographic Insulators</td>
<td>565</td>
</tr>
<tr>
<td>47.1</td>
<td>Momentum Dissipation through Random Disorder: Anderson-Type Insulator?</td>
<td>565</td>
</tr>
<tr>
<td>47.2</td>
<td>Momentum Dissipation through Holographic Lattice: Hubbard-Type Insulator?</td>
<td>569</td>
</tr>
<tr>
<td>47.3</td>
<td>Effective Massive Gravity Model for Lattice</td>
<td>571</td>
</tr>
<tr>
<td>47.4</td>
<td>Strong Momentum Dissipation in the IR: New Holographic Insulators and Mott-Type Insulators</td>
<td>575</td>
</tr>
</tbody>
</table>
47.5 Hydrodynamic Flow, Dirac Fluid, and Linear Resistivity 577
Exercises 580

48 Holographic Strange Metals and the Kondo Problem 582
48.1 Hyperscaling Violation and Holography 582
48.2 Holographic Strange Metal Constructions 585
48.3 Supersymmetric Kondo Effect via Holography 589
48.4 Toward the Real Kondo Problem 592
Exercises 594

References 595
Index 603
Preface

Over the past 20 years, string theory has started to branch out and has tried to tackle difficult problems in several areas, mostly through the advent of the AdS/CFT correspondence. While at the beginning the sought-for applications were mostly in particle physics and cosmology, over the last 15 years or so we have seen a gradual increase in the number of applications to condensed matter theory, to the point that now various string theorists work completely in condensed matter. The purpose of this book is to provide an introduction to the various methods that have been developed in string theory for condensed matter applications, and to be accessible to graduate students just beginning to learn about either string theory or condensed matter theory, with the aim of leading them to where they can start research in the field. I assume a solid working knowledge of Quantum Field Theory, as can be obtained from a two-semester graduate course, and an advanced undergraduate course on Solid State physics, as well as some basic elements of General Relativity (but not necessarily a full course). Familiarity with string theory or modern condensed matter theory is helpful, but not necessary, since I try to be as self-consistent as possible. To that end, in Part I, I give an introduction to modern topics in condensed matter from the perspective of a string theorist. In Part II, I give a very basic introduction to general relativity and string theory, mostly for the benefit of people who haven’t seen them before, as they are not very detailed. Parts III and IV then describe the string theory applications to the condensed matter problems of Part I. The goal is to give an introduction to the various tools available, but I will not try to be extensive in my treatment of any of them. Instead, my aim is to have a fair overview of all the methods currently available in the field. Part III describes tools that are by now standard: the pp wave correspondence, spin chains and integrability, AdS/CFT phenomenology (“AdS/CMT”), and the fluid-gravity correspondence. Part IV focuses on more advanced topics that are still being developed, like Fermi liquids and non-Fermi liquids, insulators, the quantum Hall effect, nonstandard statistics, etc.
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This book is an expanded version of a course I gave at the IFT in São Paulo, so I would like to thank all the students who participated in the course for their input that helped shape the material.

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Before learning about string theory methods for condensed matter physics, we must define the problems we want to tackle and what string theory is. The interesting and difficult problems in condensed matter theory are in general strong coupling ones, though not always. In the first part of the book we will therefore learn about a variety of problems of topical interest, like Fermi liquids, spin chains, integrable systems, conformal field theories, quantum phase transitions, superconductivity, the quantum Hall effect, the Kondo problem, and hydrodynamics.

We then need to define string theory and its main low-energy tool, general relativity. After defining general relativity, we will focus on the objects of most interest, black holes and their extensions, that will be used extensively in the book. String theory is then defined, mostly as a perturbative theory of strings, a field theory on the 2-dimensional “worldsheets” spanned by the string. But we will also learn that there are various nonperturbative “dualities,” where one description is mapped to another, in terms of other variables and valid in other regions of parameter space. We will also learn about the most important tool available for condensed matter applications, the AdS/CFT correspondence.

The AdS/CFT correspondence relates string theory, usually in its low-energy version of supergravity, a supersymmetric theory of gravity, and living in a curved background spacetime, to field theory in a flat spacetime of lower dimensionality. The correspondence is holographic, which means that in some sense, the physics in the higher dimension is projected onto a flat surface without losing information, at the price of encoding the information nonlocally.

The original incarnation of the AdS/CFT correspondence defined by Juan Maldacena related a certain field theory, 4-dimensional “$\mathcal{N} = 4$ SYM,” to string theory in a curved background, $AdS_5 \times S^5$. In that case, although Maldacena’s presentation emphasized it is a conjecture, the overwhelming number of checks that were performed since then makes it a pedantic point to still call it a conjecture; to all intents and purposes it is proven. Similar comments apply to the first (to my knowledge) application to condensed matter, the “pp wave correspondence” and the generalization to spin chains, started by the work of Berenstein, Maldacena, and myself, as an off-shoot of the original correspondence.

However, more common applications to condensed matter usually are of two types: “top-down” constructions, where one starts with a duality defined in string theory, with fewer checks than in the original case, but nevertheless defined beyond the level of conjecture. In that case, however, it is usually less clear why the field theory can be applied to the condensed matter problem of interest. Or “bottom-up” constructions, where one starts with a field theory we would like to describe, for condensed matter reasons, but then one guesses a possible holographic dual gravitational (string) theory, in which case the duality
itself is less than clear. The point of view most commonly stated is that either way, one finds nice gravitational ways to parametrize the condensed matter problem, which makes it less important whether we can actually rigorously prove that the gravitational description is rigorously equivalent to the condensed matter problem.

There are other cases as well. Sometimes it is possible to view the condensed problem in a different way by simply embedding it into string theory. This is the case of Fermi gas constructions, particle-vortex duality, and Chern-Simons and Quantum Hall Effect constructions pre-dating holographic models. Yet in other cases like the fluid/gravity correspondence, which includes a mapping of the Navier-Stokes equations of viscous fluid hydrodynamics to the Einstein equations near the event horizon of a black hole, there are ways to argue for the map, but it is not clear that the new description is better than the original one, just different.

There are a number of reviews on various string theory applications to condensed matter, but they each mostly focus on one particular aspect and assume a lot of background. These include [1], [2], [3], and [4]. The book [5] presents many, but not all, of the applications described here and spends less time on the background material needed to understand it.