String Theory Methods for Condensed Matter Physics

The discovery of a duality between Anti–de Sitter spaces (AdS) and Conformal Field Theories (CFT) has led to major advances in our understanding of quantum field theory and quantum gravity. String theory methods and AdS/CFT correspondence maps provide new ways to think about difficult condensed matter problems. String theory methods based on the AdS/CFT correspondence allow us to transform problems so they have weak interactions and can be solved more easily. They can also help map problems to different descriptions, for instance, mapping the description of a fluid using the Navier-Stokes equations to the description of an event horizon of a black hole using Einstein's equations. This textbook covers the applications of string theory methods and the mathematics of AdS/CFT to areas of condensed matter physics. Bridging the gap between string theory and condensed matter, this is a valuable textbook for students and researchers in both fields.

Horaţiu Năstase is a Researcher at the Institute for Theoretical Physics at the State University of São Paulo, Brazil. To date, his career has spanned four continents. As an undergraduate he studied at the University of Bucharest and Copenhagen University. He later completed his Ph.D. at the State University of New York, Stony Brook, before moving to the Institute for Advanced Study, Princeton, where his collaboration with David Berenstein and Juan Maldacena defined the pp-wave correspondence. He has also held research and teaching positions at Brown University and the Tokyo Institute of Technology.

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HORAŢIU NĂSTASE
Universidade Estadual Paulista, São Paulo

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To the memory of my mother,
who inspired me to become a physicist

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Preface

Over the past 20 years, string theory has started to branch out and has tried to tackle difficult problems in several areas, mostly through the advent of the AdS/CFT correspondence. While at the beginning the sought-for applications were mostly in particle physics and cosmology, over the last 15 years or so we have seen a gradual increase in the number of applications to condensed matter theory, to the point that now various string theorists work completely in condensed matter. The purpose of this book is to provide an introduction to the various methods that have been developed in string theory for condensed matter applications, and to be accessible to graduate students just beginning to learn about either string theory or condensed matter theory, with the aim of leading them to where they can start research in the field. I assume a solid working knowledge of Quantum Field Theory, as can be obtained from a two-semester graduate course, and an advanced undergraduate course on Solid State physics, as well as some basic elements of General Relativity (but not necessarily a full course). Familiarity with string theory or modern condensed matter theory is helpful, but not necessary, since I try to be as self-consistent as possible. To that end, in Part I, I give an introduction to modern topics in condensed matter from the perspective of a string theorist. In Part II, I give a very basic introduction to general relativity and string theory, mostly for the benefit of people who haven't seen them before, as they are not very detailed. Parts III and IV then describe the string theory applications to the condensed matter problems of Part I. The goal is to give an introduction to the various tools available, but I will not try to be extensive in my treatment of any of them. Instead, my aim is to have a fair overview of all the methods currently available in the field. Part III describes tools that are by now standard: the pp wave correspondence, spin chains and integrability, AdS/CFT phenomenology ("AdS/CMT"), and the fluid-gravity correspondence. Part IV focuses on more advanced topics that are still being developed, like Fermi liquids and non-Fermi liquids, insulators, the quantum Hall effect, nonstandard statistics, etc.

Acknowledgments

I have to start by thanking all the people who have guided me along the path of becoming a physicist, without whom it would have been impossible to even think about writing this book. My mother, Ligia, who first inspired me in the love of physics, through her example as a physicist. My high school physics teacher, Iosif Sever Georgescu, who showed me that I could be successful at physics and make a career out of it. Poul Olesen at the Niels Bohr Institute, who during a student exchange period under his supervision introduced me to string theory. My Ph.D. advisor, Peter van Nieuwenhuizen, from whom I learned not just about supergravity and string theory, but about the rigor and beauty of theoretical physics, and the value of perseverance in calculations. Juan Maldacena at IAS from whom I learned, during my postdoc years, more about the AdS/CFT correspondence, the basis for most of the methods in this book.

This book is an expanded version of a course I gave at the IFT in São Paulo, so I would like to thank all the students who participated in the course for their input that helped shape the material.

I have to thank of course all my collaborators with whom I developed my research which led to my being able to write this book. I have also to thank my students and postdocs for their patience in dealing with my reduced time to work with them. A special thanks to my Ph.D. student Heliudson de Oliveira Bernardo, who helped me get rid of typos and errors in equations by carefully checking a previous version of the book. To my editor at Cambridge University Press, Simon Capelin, thank you for believing in me and helping me to get the book published, and to all the staff at CUP for making sure the book is up to standard.

Introduction

Before learning about string theory methods for condensed matter physics, we must define the problems we want to tackle and what string theory is. The interesting and difficult problems in condensed matter theory are in general strong coupling ones, though not always. In the first part of the book we will therefore learn about a variety of problems of topical interest, like Fermi liquids, spin chains, integrable systems, conformal field theories, quantum phase transitions, superconductivity, the quantum Hall effect, the Kondo problem, and hydrodynamics.

We then need to define string theory and its main low-energy tool, general relativity. After defining general relativity, we will focus on the objects of most interest, black holes and their extensions, that will be used extensively in the book. String theory is then defined, mostly as a perturbative theory of strings, a field theory on the 2-dimensional "worldsheets" spanned by the string. But we will also learn that there are various nonperturbative "dualities," where one description is mapped to another, in terms of other variables and valid in other regions of parameter space. We will also learn about the most important tool available for condensed matter applications, the AdS/CFT correspondence.

The AdS/CFT correspondence relates string theory, usually in its low-energy version of *supergravity*, a supersymmetric theory of gravity, and living in a curved background spacetime, to field theory in a flat spacetime of lower dimensionality. The correspondence is *holographic*, which means that in some sense, the physics in the higher dimension is projected onto a flat surface without losing information, at the price of encoding the information nonlocally.

The original incarnation of the AdS/CFT correspondence defined by Juan Maldacena related a certain field theory, 4-dimensional "$\mathcal{N} = 4$ SYM," to string theory in a curved background, $AdS_5 \times S^5$. In that case, although Maldacena's presentation emphasized it is a conjecture, the overwhelming number of checks that were performed since then makes it a pedantic point to still call it a conjecture; to all intents and purposes it is proven. Similar comments apply to the first (to my knowledge) application to condensed matter, the "pp wave correspondence" and the generalization to spin chains, started by the work of Berenstein, Maldacena, and myself, as an off-shoot of the original correspondence.

However, more common applications to condensed matter usually are of two types: "top-down" constructions, where one starts with a duality defined in string theory, with fewer checks than in the original case, but nevertheless defined beyond the level of conjecture. In that case, however, it is usually less clear why the field theory can be applied to the condensed matter problem of interest. Or "bottom-up" constructions, where one starts with a field theory we would like to describe, for condensed matter reasons, but then one guesses a possible holographic dual gravitational (string) theory, in which case the duality itself is less than clear. The point of view most commonly stated is that either way, one finds nice gravitational ways to parametrize the condensed matter problem, which makes it less important whether we can actually rigorously prove that the gravitational description is rigorously equivalent to the condensed matter problem.

There are other cases as well. Sometimes it is possible to view the condensed problem in a different way by simply embedding it into string theory. This is the case of Fermi gas constructions, particle-vortex duality, and Chern-Simons and Quantum Hall Effect constructions pre-dating holographic models. Yet in other cases like the fluid/gravity correspondence, which includes a mapping of the Navier-Stokes equations of viscous fluid hydrodynamics to the Einstein equations near the event horizon of a black hole, there are ways to argue for the map, but it is not clear that the new description is better than the original one, just different.

There are a number of reviews on various string theory applications to condensed matter, but they each mostly focus on one particular aspect and assume a lot of background. These include [1], [2], [3], and [4]. The book [5] presents many, but not all, of the applications described here and spends less time on the background material needed to understand it.