

Introduction to Engineering Heat Transfer

This new text integrates fundamental theory with modern computational tools such as EES, MATLAB, and FEHT to equip students with the essential tools for designing and optimizing real-world systems and the skills needed to become effective practicing engineers. Real engineering problems are illustrated and solved in a clear step-by-step manner. Starting from first principles, derivations are tailored to be accessible to undergraduates by separating the formulation and analysis from the solution and exploration steps to encourage a deep and practical understanding. Numerous exercises are provided for homework and self-study and include standard hand calculations as well as more advanced project-focused problems for the practice and application of computational tools. Appendices include reference tables for thermophysical properties, and answers to selected homework problems from the book. Complete with an online package of guidance documents on EES, MATLAB, and FEHT software, sample code, lecture slides, video tutorials, and a test bank and full solutions manual for instructors, this is an ideal text for undergraduate heat transfer courses and a useful guide for practicing engineers.

G. F. Nellis is Professor of Mechanical Engineering at the University of Wisconsin, Madison. His teaching expertise has been recognized through awards including the Polygon Engineering Council Outstanding Professor of Mechanical Engineering Award (2013 and 2007), the Pi Tau Sigma Distinguished Professor of Mechanical Engineering Award (2016, 2012, 2009, and 2006), and the J. G. Woodburn award for Excellence in Teaching (2008). He is a Fellow of the American Society of Heating, Refrigeration, and Air-Conditioning Engineers.

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“This excellent text on heat transfer continues the tradition of the strong analytical treatment of conduction and convection heat transfer, buttressed by strong EES, FEHT, and MATLAB examples . . . The emphasis on examples is substantial, and the use of the software is tastefully introduced in ways that emphasize the solution instead of the software . . . This edition is well organized, succinctly written, and well supported by software aids. The book is also a valuable reference for those in a wide variety of disciplines desiring to self-learn heat transfer. All the essential elements of a heat transfer course are well represented in this volume.”

Ernest W. Tollner, University of Georgia

“No other text spells out real-world problems with computer-based solutions as clearly as this one. This text will allow readers to translate quickly heat transfer lessons learned into interesting applied solutions.”

Thomas Merrill, Rowan University

“I’ve practiced heat transfer for 30 years as an engineer in industry, a scientist at a national lab, and an academic. Midway through my career, I studied Nellis and Klein’s pedagogically pioneering text. It was only then that I obtained a firm grasp of the subject matter. Feedback from students in my classes on their book has been remarkably terrific.”

Marc Hodes, Tufts University

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Contents

Preface	page xv
Sample Program of Study	xvii
Nomenclature	xix
I Introduction	1
1.1 Relevance of Heat Transfer	1
1.2 Relationship to Thermodynamics	2
1.3 Problem Solving Methodology	5
1.4 Heat Transfer Mechanisms	6
1.4.1 Conduction	6
1.4.2 Convection	7
1.4.3 Radiation	8
1.5 Thermophysical Properties	15
1.5.1 Real Fluids	15
1.5.2 Ideal Gas Model	17
1.5.3 Incompressible Substance Model	21
1.6 Conclusions and Learning Objectives	24
Reference	25
Problems	25
Projects	30
2 One-Dimensional, Steady-State Conduction	32
2.1 Conduction Heat Transfer	32
2.1.1 Fourier’s Law	32
2.1.2 Thermal Conductivity	35
2.2 Steady-State 1-D Conduction without Generation	35
2.2.1 Introduction	35
2.2.2 The Plane Wall	36
<i>Define a Differential Control Volume</i>	37
<i>Carry out an Energy Balance on the Control Volume</i>	37
<i>Take the Limit as $dx \rightarrow 0$</i>	38
<i>Substitute Rate Equations into the Differential Equation</i>	38
<i>Define Boundary Conditions</i>	39
2.2.3 The Resistance Concept	46
2.2.4 Radial Conduction	54
<i>Radial Conduction in a Cylinder</i>	54
<i>Radial Conduction in a Sphere</i>	57
2.2.5 Other Resistance Formulae	59
<i>Convection Resistance</i>	59
<i>Contact Resistance</i>	60
<i>Radiation Resistance</i>	62
2.3 Steady-State 1-D Conduction with Generation	75
2.3.1 Introduction	75
2.3.2 Uniform Thermal Energy Generation in a Plane Wall	78
2.3.3 Uniform Thermal Energy Generation in Radial Geometries	93
<i>Cylindrical Geometry</i>	93
<i>Spherical Geometry</i>	94

2.3.4	Spatially Nonuniform Generation	101
2.4	Numerical Solutions	105
2.4.1	Introduction	105
2.4.2	Developing the Finite Difference Equations	106
2.4.3	Solving the Equations with EES	110
2.4.4	Solving the Equations with Matrix Decomposition	113
2.4.5	Solving the Equations with Gauss–Seidel Iteration	119
2.4.6	Temperature-Dependent Properties	123
	<i>Implementation in EES</i>	125
	<i>Implementation using Matrix Decomposition</i>	127
	<i>Implementation using Gauss–Seidel Iteration</i>	131
2.5	Conclusions and Learning Objectives	132
	References	133
	Problems	133
	Conduction without Generation: Concepts and Analytical Solutions	133
	Thermal Resistance Problems	138
	Conduction with Generation: Concepts and Analytical Solutions	154
	Numerical Solution Concepts	165
	Numerical Solutions	166
	Projects	171
3	Extended Surface Problems	185
3.1	Extended Surfaces	185
3.1.1	The Extended Surface Approximation	185
3.1.2	The Biot Number	186
3.2	Analytical Solutions to Extended Surface Problems	188
3.2.1	Deriving the ODE and Boundary Conditions	188
3.2.2	Solving the ODE	189
3.2.3	Applying the Boundary Conditions	191
3.2.4	Hyperbolic Trigonometric Functions	194
3.2.5	Solutions to Linear Homogeneous ODEs	195
3.3	Fins	201
3.3.1	Fin Efficiency	202
3.3.2	Convection from the Fin Tip	207
3.3.3	Fin Resistance	213
3.3.4	Finned Surfaces	217
3.4	Numerical Solutions to Extended Surface Problems	223
3.5	Conclusions and Learning Objectives	228
	Problems	229
	The Extended Surface Approximation and the Biot Number	229
	Analytical Solutions to Extended Surface Problems	231
	Fins and Finned Surfaces	238
	Numerical Solutions to Extended Surface Problems	241
	Projects	246
4	Two-Dimensional, Steady-State Conduction	255
4.1	The Governing Differential Equation and Boundary Conditions	255
4.2	Shape Factors	258
4.2.1	Definition of Shape Factor	258
4.2.2	Shape Factor Resistance	261
4.2.3	The Meaning of a Shape Factor	263

4.3	Finite Difference Solution	268
4.3.1	Introduction	268
4.3.2	Developing the Finite Difference Equations	268
4.3.3	Solving the Equations with EES	273
4.3.4	Solving the Equations with Matrix Decomposition	277
4.3.5	Solving the Equations with Gauss–Seidel Iteration	283
4.4	Finite Element Solution	288
4.4.1	Introduction	288
4.4.2	Specifying the Problem	289
4.4.3	Specifying the Mesh and Solving	289
4.4.4	Examination of the Solution	290
	<i>Mesh Convergence</i>	290
	<i>Engineering Judgment</i>	291
4.5	Conclusions and Learning Objectives	293
	References	293
	Problems	293
	The Governing Differential Equation and Boundary Conditions	293
	Shape Factors	296
	Finite Difference Solutions	298
	Finite Element Method using FEHT	302
	Projects	308
5	Lumped Transient Problems	310
5.1	The Lumped Capacitance Assumption	310
5.1.1	The Biot Number	310
5.1.2	The Lumped Capacitance Time Constant	311
5.2	Analytical Solutions	316
5.2.1	Deriving the Differential Equation	317
5.2.2	Solving the Differential Equation	318
	<i>Step Change in Ambient Temperature</i>	320
	<i>Ramped Ambient Temperature</i>	320
5.3	Numerical Solutions	332
5.3.1	Introduction	332
5.3.2	The Euler Method	333
5.3.3	Predictor–Corrector Methods	338
5.3.4	Implicit Methods	340
5.3.5	Using ODE Solvers	343
	<i>EES’ Integral Command</i>	343
	<i>MATLAB’s ODE Solvers</i>	345
5.4	Conclusions and Learning Objectives	352
	Problems	353
	The Lumped Capacitance Approximation and the Biot Number	353
	Analytical Solutions	355
	Numerical Solutions	362
	Projects	367
6	Transient Conduction	375
6.1	Conceptual Tools	375
6.1.1	Diffusive Energy Transport	375
6.1.2	The Diffusive Time Constant	380
6.1.3	The Semi-Infinite Resistance	385

6.2	Analytical Solution	388
6.2.1	The Differential Equation	388
6.2.2	Semi-Infinite Body Solutions	392
6.2.3	Bounded Problem Solutions	398
	<i>The Plane Wall – Exact Solution</i>	398
	<i>The Plane Wall – Approximate Solution</i>	404
	<i>The Cylinder – Exact Solution</i>	407
	<i>The Cylinder – Approximate Solution</i>	409
	<i>The Sphere – Exact Solution</i>	409
	<i>The Sphere – Approximate Solution</i>	410
6.3	1-D Numerical Solutions	414
6.3.1	Introduction	414
6.3.2	The State Equations	415
6.3.3	The Euler Method	418
6.3.4	Predictor–Corrector Methods	423
6.3.5	Implicit Methods	425
	<i>Implementation with EES</i>	425
	<i>Implementation with Matrix Decomposition</i>	426
	<i>Implementation with Gauss–Seidel Iteration</i>	428
6.3.6	Using ODE Solvers	430
	<i>EES’ Integral Command</i>	430
	<i>MATLAB’s ODE Solver</i>	432
6.4	2-D Numerical Solutions	435
6.4.1	Introduction	435
6.4.2	The Finite Difference Solution	435
	<i>Deriving the State Equations</i>	435
	<i>Integrating through Time</i>	438
6.4.3	The Finite Element Solution	441
	<i>Specifying the Problem</i>	442
	<i>Specifying the Mesh and Solving</i>	443
	<i>Mesh Convergence</i>	444
6.5	Conclusions and Learning Objectives	445
	References	446
	Problems	446
	Conceptual Tools	446
	The Differential Equation and Boundary Conditions	453
	Semi-Infinite Solutions	456
	Plane Wall, Cylinder, and Sphere Solutions	457
	1-D Transient Numerical Solutions	460
	Finite Element Method using FEHT	467
	Projects	469
7	Convection	477
7.1	The Laminar Boundary Layer	477
7.1.1	The Velocity Boundary Layer	477
7.1.2	The Thermal Boundary Layer	480
7.1.3	A Conceptual Model of Laminar Boundary Layer Growth	482
7.1.4	The Prandtl Number	483
7.1.5	A Conceptual Model of Shear Stress and the Heat Transfer Coefficient	484
7.1.6	The Reynolds Number	489
7.1.7	The Friction Coefficient and the Nusselt Number	490
7.1.8	The Reynolds Analogy	492

7.1.9	Local vs. Average Quantities	494
	<i>The Average Friction Coefficient</i>	494
	<i>The Drag Coefficient</i>	495
	<i>The Average Nusselt Number</i>	495
7.2	Turbulent Boundary Layer Concepts	498
7.2.1	Introduction	498
7.2.2	The Critical Reynolds Number	498
7.2.3	A Conceptual Model of the Turbulent Boundary Layer	501
7.3	The Boundary Layer Equations	504
7.3.1	Introduction	504
7.3.2	The Governing Equations for Viscous Fluid Flow	504
	<i>The Continuity Equation</i>	504
	<i>The Momentum Equations</i>	506
	<i>The Energy Conservation Equation</i>	509
7.3.3	The Boundary Layer Simplifications	510
	<i>The Continuity Equation</i>	511
	<i>The x-Momentum Equation</i>	512
	<i>The y-Momentum Equation</i>	513
	<i>The Energy Conservation Equation</i>	513
7.4	Dimensional Analysis in Convection	514
7.4.1	Introduction	514
7.4.2	The Dimensionless Boundary Layer Equations	516
	<i>The Dimensionless Continuity Equation</i>	516
	<i>The Dimensionless Momentum Equation</i>	517
	<i>The Dimensionless Energy Equation</i>	517
7.4.3	Correlations	518
	<i>The Friction and Drag Coefficients</i>	518
	<i>The Nusselt Number</i>	520
7.4.4	The Reynolds Analogy (Revisited)	524
7.5	Conclusions and Learning Objectives	525
References		526
Problems		526
	Laminar Boundary Layer Concepts	526
	Turbulent Boundary Layer Concepts	529
	The Boundary Layer Equations and Dimensional Analysis for Convection	530
	Projects	533
8	External Forced Convection	536
8.1	Methodology for using a Convection Correlation	536
8.2	Flow over a Flat Plate	538
8.2.1	The Friction Coefficient	538
	<i>Local Friction Coefficient for a Smooth Plate</i>	538
	<i>Local Friction Coefficient for a Rough Plate</i>	539
	<i>Average Friction Coefficient</i>	540
8.2.2	The Nusselt Number	543
	<i>Constant Temperature</i>	543
	<i>Unheated Starting Length</i>	553
	<i>Constant Heat Flux</i>	555
8.3	Flow across a Cylinder	558
8.3.1	The Drag Coefficient	560
8.3.2	The Nusselt Number	561
8.4	Flow across other Extrusions	568

8.5	Flow past a Sphere	573
8.6	Conclusions and Learning Objectives	578
	References	579
	Problems	579
	Flow over a Flat Plate	579
	Flow over Cylinders and other Extrusions	583
	Flow over a Sphere	585
	Projects	587
9	Internal Forced Convection	591
9.1	Internal Flow Concepts	591
9.1.1	Velocity and Momentum Considerations	591
	<i>Internal vs. External Flow</i>	591
	<i>The Developing Region vs. the Fully Developed Region</i>	591
	<i>The Mean Velocity, Hydraulic Diameter, and Reynolds Number</i>	593
	<i>The Laminar Hydrodynamic Entry Length</i>	594
	<i>Turbulent Internal Flow</i>	595
	<i>The Pressure Gradient</i>	597
	<i>The Friction Factor</i>	599
9.1.2	Thermal Considerations	601
	<i>The Developing Region vs. the Fully Developed Region</i>	601
	<i>The Mean Temperature and the Heat Transfer Coefficient</i>	601
	<i>The Laminar Thermal Entry Length</i>	605
	<i>Turbulent Internal Flow</i>	605
	<i>The Nusselt Number</i>	607
9.2	Internal Flow Correlations	609
9.2.1	Introduction	609
9.2.2	Flow Classification	609
9.2.3	The Friction Factor	610
	<i>Laminar Flow</i>	611
	<i>Turbulent Flow</i>	619
	<i>EES' Internal Flow Convection Library</i>	621
9.2.4	The Nusselt Number	627
	<i>Laminar Flow</i>	627
	<i>Turbulent Flow</i>	632
9.3	The Energy Balance	637
9.3.1	Introduction	637
9.3.2	The Energy Balance	637
9.3.3	Specified Heat Flux	639
	<i>Constant Heat Flux</i>	639
9.3.4	Specified Wall Temperature	644
	<i>Constant Wall Temperature</i>	644
9.3.5	Specified External Temperature	645
9.4	Conclusions and Learning Objectives	650
	References	650
	Problems	651
	Internal Flow Concepts	651
	Internal Flow Correlations	653
	The Energy Balance	657
	Projects	667

10	Free Convection	672
10.1	Free Convection Flow	672
10.2	Dimensionless Parameters	672
10.2.1	The Characteristic Buoyancy Velocity	672
10.2.2	The Volumetric Thermal Expansion Coefficient	674
	<i>The Volumetric Thermal Expansion Coefficient of an Ideal Gas</i>	675
10.2.3	The Grashof Number and the Rayleigh Number	676
10.3	External Free Convection Correlations	676
10.3.1	Introduction	676
10.3.2	Plate	676
	<i>Heated or Cooled Vertical Plate</i>	676
	<i>Horizontal Plate – Heated Upward Facing or Cooled Downward Facing</i>	680
	<i>Horizontal Plate – Heated Downward Facing or Cooled Upward Facing</i>	681
	<i>Plate at an Arbitrary Angle</i>	688
10.3.3	Sphere	693
10.3.4	Cylinder	694
	<i>Horizontal Cylinder</i>	694
	<i>Vertical Cylinder</i>	694
10.4	Internal Free Convection Correlations	700
10.4.1	Introduction	700
10.4.2	Vertical Parallel Plate Channels	700
10.4.3	Enclosures	706
10.5	Combined Free and Forced Convection	707
10.6	Conclusions and Learning Objectives	711
	References	711
	Problems	712
	Free Convection Concepts	712
	Free Convection from Plates, Spheres, and Cylinders	712
	Free Convection in Channels	715
	Free Convection in Enclosures	715
	Combined Free and Forced Convection	717
	Projects	718
11	Boiling and Condensation	720
11.1	Relevance	720
11.2	Pool Boiling	721
11.2.1	Introduction	721
11.2.2	The Boiling Curve	721
11.2.3	Pool Boiling Correlations	724
11.3	Flow Boiling	730
11.3.1	Introduction	730
11.3.2	Flow Boiling Correlations	731
11.4	Film Condensation	736
11.4.1	Introduction	736
11.4.2	Correlations for Film Condensation	737
	<i>Vertical Wall</i>	737
	<i>Horizontal, Downward Facing Plate</i>	741
	<i>Horizontal, Upward Facing Plate</i>	742
	<i>Single Horizontal Cylinder</i>	742
	<i>Bank of Horizontal Cylinders</i>	742
	<i>Single Horizontal Finned Tube</i>	742

11.5 Flow Condensation	743
11.5.1 Introduction	743
11.5.2 Flow Condensation Correlations	744
11.6 Conclusions and Learning Objectives	745
References	746
Problems	747
Pool Boiling	747
Flow Boiling	748
Film Condensation	749
Flow Condensation	752
Projects	753
12 Heat Exchangers	754
12.1 Introduction to Heat Exchangers	754
12.1.1 Applications of Heat Exchangers	754
12.1.2 Heat Exchanger Classifications and Flow Configurations	755
12.1.3 Overall Energy Balance	759
12.1.4 Heat Exchanger Conductance	761
<i>Fouling Resistance</i>	762
12.1.5 Flow across Tube Banks	765
<i>The Friction Factor</i>	767
<i>The Nusselt Number</i>	768
12.1.6 Compact Heat Exchanger Correlations	773
12.2 The Heat Exchanger Problem	778
12.2.1 Introduction	778
12.2.2 The Counter-Flow Heat Exchanger Solution	778
12.3 The Log-Mean Temperature Difference Method	782
12.3.1 Introduction	782
12.3.2 Counter-Flow and Parallel-Flow Heat Exchangers	783
12.3.3 Shell-and-Tube and Cross-Flow Heat Exchangers	786
12.4 The Effectiveness– <i>NTU</i> Method	787
12.4.1 Introduction	787
12.4.2 Effectiveness, Number of Transfer Units, and Capacitance Ratio	787
12.4.3 Effectiveness– <i>NTU</i> Solution for a Counter-Flow Heat Exchanger	788
12.4.4 Effectiveness– <i>NTU</i> Solutions	791
12.4.5 Further Discussion of Heat Exchanger Effectiveness	792
<i>Behavior as C_R Approaches Zero</i>	793
<i>Behavior as NTU Approaches Zero</i>	795
<i>Behavior as NTU Becomes Infinite</i>	796
<i>Heat Exchanger Design</i>	797
12.5 Conclusions and Learning Objectives	803
References	803
Problems	804
Heat Exchanger Conductance, Tube Banks, and Compact Heat Exchangers	804
Log-Mean Temperature Difference Solution	806
Effectiveness– <i>NTU</i> Solution	807
Projects	817
13 Mass Transfer	827
13.1 Composition Relationships	827
13.1.1 Ideal Gas Mixtures	830

13.2 Mass Diffusion	830
13.2.1 Fick’s Law	830
13.2.2 The Diffusion Coefficient for Binary Mixtures	831
<i>Gas Mixtures</i>	831
<i>Liquid Mixtures</i>	833
<i>Solids</i>	834
13.2.3 Concentrations at Interfaces	834
<i>Gas Mixture in Contact with Pure Liquid or Solid</i>	834
<i>Liquid Mixture in Contact with Pure Solid</i>	835
<i>Liquid Mixture in Contact with Gas</i>	836
13.3 Transient Diffusion through a Stationary Solid	838
13.4 Diffusion of a Species in a Fluid	842
13.4.1 Diffusive and Advective Mass Transfer	842
13.4.2 Evaporation through a Layer of Gas	843
13.5 Momentum, Energy, and Mass Transfer Analogies	847
13.6 Simultaneous Heat and Mass Transfer	853
13.7 Conclusions and Learning Objectives	858
References	859
Problems	859
Concentration Relationships	859
The Diffusion Coefficient and Boundary Conditions	859
Transient Diffusion through a Solid	860
Diffusion of a Species in a Fluid	861
Heat Mass Transfer Analogy	862
Simultaneous Heat and Mass Transfer	863
Projects	864
14 Radiation	866
14.1 Introduction	866
14.1.1 Electromagnetic Radiation	866
14.1.2 The Electromagnetic Spectrum	866
14.2 Emission of Radiation by a Blackbody	867
14.2.1 Introduction	867
14.2.2 Blackbody Emission	868
<i>Planck’s Law</i>	868
<i>Blackbody Emission in Specified Wavelength Bands</i>	869
14.3 Radiation Exchange between Black Surfaces	872
14.3.1 Introduction	872
14.3.2 View Factors	873
<i>Inspection</i>	873
<i>The View Factor Integral</i>	873
<i>The Enclosure Rule</i>	874
<i>Reciprocity</i>	875
<i>Other View Factor Relationships</i>	875
<i>The Crossed and Uncrossed Strings Method</i>	876
<i>The View Factor Libraries</i>	879
14.3.3 Blackbody Radiation Calculations	879
<i>The Space Resistance</i>	879
<i>N-Surface Solutions</i>	887
14.4 Radiation Characteristics of Real Surfaces	892
14.4.1 Introduction	892

14.4.2 Emission from Real Surfaces	892
<i>Spectral, Directional Emissivity</i>	892
<i>Hemispherical Emissivity</i>	893
<i>Total Hemispherical Emissivity</i>	893
<i>The Diffuse Surface Approximation</i>	895
<i>The Diffuse Gray Surface Approximation</i>	896
14.4.3 Reflectivity, Absorptivity, and Transmissivity	896
<i>Diffuse and Specular Surfaces</i>	897
<i>Hemispherical and Total Hemispherical Reflectivity, Absorptivity, and Transmissivity</i>	897
<i>Kirchhoff's Law</i>	897
<i>The Diffuse Surface Approximation</i>	897
<i>The Diffuse Gray Surface Approximation</i>	898
14.5 Diffuse Gray Surface Radiation Exchange	900
14.5.1 Introduction	900
14.5.2 Radiosity	900
14.5.3 Diffuse Gray Surface Radiation Calculations	901
<i>Resistance Network</i>	902
<i>N-Surface Solutions</i>	905
14.6 Conclusions and Learning Objectives	909
References	910
Problems	910
Blackbody Radiation and the Electromagnetic Spectrum	910
View Factors	913
Blackbody Radiation Exchange	914
Properties of Real Surfaces	918
Diffuse Gray Surface Radiation Exchange	920
Projects	931
Appendix A Thermophysical Properties of Solids	940
Appendix B Thermophysical Properties of Liquids	943
Appendix C Thermophysical Properties of Gases	945
Appendix D Thermophysical Properties of Saturated Liquids	948
Appendix E Engineering Equation Solver (EES) Software	952
Appendix F Finite Element Heat Transfer (FEHT) Software	966
Appendix G MATLAB Software	978
Appendix H Answers to Selected Problems	987
Index	993

Preface

The objective of this book is to provide engineering students with the capability, tools, and confidence to solve real-world heat transfer problems. This objective has resulted in a textbook that differs from existing heat transfer textbooks in an important way. This textbook introduces fundamental heat transfer concepts at an introductory, undergraduate level that is appropriate for a practicing engineer and integrates these concepts with modern computational tools. The text provides extensive examples and problems that utilize these tools. The practicing engineer of today is expected to be proficient with computer tools; engineering education must evolve accordingly. Most real engineering problems cannot be solved using a sequential set of calculations that can be easily carried out with a pencil and a hand calculator. Engineers must have the ability and confidence to utilize the powerful computational tools that are available and essential for design and optimization of real-world systems.

The text reinforces good engineering problem solving technique by delineating the formulation and analysis steps from the solution and exploration steps. In the formulation step, the problem itself is defined and, through appropriate approximations, simplified to the point where it can be represented by a set of mathematical equations. These equations are derived from first principles in the analysis step. Many textbooks stop their presentation at this point. However, the solution step where the equations are solved is equally important. In some cases hand calculations are appropriate for solving the equations. More typically, the complexity of the problem dictates that some type of computational software must be used for the solution step. Each of these steps is essential. It is not possible to move to the solution step until the formulation and analysis steps are complete. Separating these steps forces the student to understand that the computational software cannot be used to “think” for them, but rather provide powerful tools for helping them solve the relevant equations. Computational software is essential for the exploration step in which the engineer carries out parametric, optimization, and design studies that allow a deeper understanding of the problem and provide more useful results. Exploration studies are a natural first step to becoming an effective practicing engineer.

This book integrates the computational software Engineering Equation Solver (EES), MATLAB, and Finite Element Heat Transfer (FEHT) directly with the heat transfer material so that students can see the relevance of these tools. The specific commands and output associated with these software packages are used in the solution and exploration steps of numerous examples so that the integration is seamless and does not detract from the presentation of the heat transfer concepts. The computational software tools used in this book are all common in industry and have existed for more than a decade; therefore, while this software will certainly continue to evolve, it is not likely to disappear. Educational versions of these software packages are available and therefore the use of these tools should not represent an economic hardship to any academic institution or student. These tools are easy to learn and use, allowing students to become proficient with all of them in a reasonable amount of time. Therefore, learning the computer tools will not detract from material coverage. In fact, providing the capability to easily solve the equations developed in the analysis is a motivator to many students. To facilitate this learning process, tutorials for each of the software packages are provided as appendices in this book.

Traditionally, tables and charts have been required to solve heat transfer problems in order to, for example, determine properties, view factors, shape factors, convection relations, and related information. Limited versions of these tables and graphs are provided in the textbook; however, much more extensive libraries have been made available as functions and procedures in the EES software so that they can be easily accessed and used to solve problems. The Heat Transfer Library that has been developed and integrated with EES as part of the preparation of this textbook and the more advanced textbook, *Heat Transfer*, enables a profound shift in the focus of the educational process. It is trivial to obtain, for example, the value of a shape factor or a view factor using the Heat Transfer Library. Therefore, it is possible to assign problems involving design and optimization studies that would be computationally impossible without these computer tools.

Integrating the study of heat transfer with computer tools does not diminish the depth of understanding of the underlying physics that students obtain. Conversely, our experience indicates that the innate understanding of the subject matter is enhanced by appropriate use of these tools for several reasons. First, the software allows the student to tackle practical and relevant problems as opposed to the comparatively simple problems that must otherwise be assigned. Real-world engineering problems are more satisfying to the student. Therefore, the marriage of computer tools with theory motivates students to understand the governing physics as well as to learn how to apply the computer tools. When a solution is obtained, students can carry out a more extensive investigation of its behavior and therefore a more intuitive and complete understanding of the subject of heat transfer. Along with the typical homework problems, each chapter includes several project type problems that allow a guided exploration of advanced topics using computer tools. Real-world problems often require a combination of English and SI units. The EES software provides unit checking that should prevent the student (and practicing) engineer from making unit conversion errors. Therefore, the examples and problems in this book use mixed units.

This book is unusual in its linking of classical theory and modern computing tools. It fills an obvious void that we have encountered in teaching undergraduate heat transfer. The text was developed over many years from our experiences teaching Introduction to Heat Transfer (an undergraduate course) at the University of Wisconsin. It is our hope that this text will not only be useful during the heat transfer course, but also a life-long resource for practicing engineers.

Sample Program of Study

A sample program of study is laid out below for a one-semester undergraduate course. The format assumes that there are 45 lectures within a 15-week semester.

Lecture	Sections in book	Topics
1	Chapter 1	Introduction
2	2.1–2.2.2	Fourier’s Law, 1-D steady-state conduction
3	2.2.3–2.2.5	Resistance concepts and circuits
4	2.3	1-D steady-state with generation
5	2.4	Numerical solutions
6	3.1–3.2	Extended surface approximation and analytical solution
7	3.3	Fin behavior, fin efficiency, and finned surfaces
8	3.4	Numerical solution to extended surface problems
9	4.1–4.2	2-D steady-state conduction, shape factors
10	4.3.1–4.3.3	Finite difference solutions with EES
11	4.3.4–4.3.5	Finite difference solutions using matrix decomposition and Gauss–Seidel iteration
12	4.4	Finite element solutions
13	5.1–5.2	Lumped capacitance approximation and analytical solution
14	5.3	Numerical solution to lumped capacitance problems
15	6.1	1-D transient conduction concepts
16	6.2	Analytical solutions to 1-D transient problems
17	6.3	Numerical solutions to 1-D transient problems
18	6.4.3	Finite element solution to 2-D transient problems
19	7.1–7.2	Laminar and turbulent boundary layer concepts
20	7.3–7.4	The boundary layer equations and dimensional analysis
21	8.1–8.2	External flow correlations and flow over a flat plate
22	8.3–8.5	Flow over extrusions and spheres
23	9.1.1	Internal flow hydrodynamic concepts
24	9.1.2	Internal flow thermal concepts
25	9.2	Internal flow correlations
26	9.3	The energy balance for an internal flow
27	10.1–10.2	Free convection concepts and dimensionless parameters
28	10.3–10.4	Free convection correlations
29	10.5	Combined free and forced convection
30	11.1–11.2	Pool boiling
31	11.3–11.5	Boiling and condensation correlations
32	12.1–12.2	Heat exchanger configurations & concepts
33	12.3	Log-mean temperature difference method
34	12.4.1–12.4.4	Effectiveness– <i>NTU</i> method
35	12.4.5	Behavior of ϵ - <i>NTU</i> solutions and heat exchanger design
36	13.1–13.2	Introduction to mass transfer and mass diffusion
37	13.3	Diffusion in a stationary solid
38	13.4	Diffusion in a fluid
39	13.5–13.6	Mass transfer analogies and simultaneous heat and mass transfer

xviii

40	14.1–14.2	Introduction to radiation and blackbodies
41	14.3.1–14.3.2	View factors
42	14.3.3	Blackbody radiation exchange
43	14.4	Radiation characteristics of real surfaces
44	14.5	Diffuse, gray surface radiation exchange
45		Multi-mode heat transfer problems

Nomenclature

A	area (m ²)	C_N	correction factor for number of tubes in a tube bank (-)
\underline{A}	the coefficient matrix in a system of linear equations	C_{nb}	nucleate boiling constant (-)
A_c	cross-sectional area (m ²)	C_R	capacitance ratio (-)
A_p	projected area (m ²)	Co	convection number (-)
A_s	surface area (m ²)	COP	coefficient of performance (-)
$A_{s,fin}$	surface area of a single fin exposed to fluid (m ²)	D	diameter (m)
$A_{s,fins}$	surface area of all of the fins on a finned surface (m ²)	D_h	hydraulic diameter (m)
$A_{s,prime}$	surface area of the base of a finned surface that is exposed to fluid (m ²)	dx	differential distance in the x -direction (m)
$A_{s,total}$	total surface area of fins and base exposed to fluid (m ²)	e	specific energy (J/kg)
AR	aspect ratio of a rectangular duct, defined as the ratio of the minimum to the maximum dimensions of the cross-section		surface roughness (m)
AR_{tip}	tip to perimeter surface area ratio for a fin (-)	E_b	blackbody emissive power (W/m ²)
\underline{b}	the constant vector in a system of linear equations	$E_{b,0-\lambda_1}$	blackbody emissive power for $\lambda < \lambda_1$ (W/m ²)
Bi	Biot number (-)	$E_{b,\lambda}$	blackbody spectral emissive power (W/m ² -μm)
Bo	boiling number (-)	ed	energy density (J/kg)
c	specific heat capacity (J/kg-K)	err	iteration error (varies)
	speed of light (299,792,000 m/s)	f	Moody (or Darcy) friction factor (-)
c_v	specific heat capacity at constant volume (J/kg-K)	$F_{0-\lambda_1}$	fraction of blackbody radiation emitted at $\lambda < \lambda_1$ (-)
c_p	specific heat capacity at constant pressure (J/kg-K)	$F_{i,j}$	view factor from surface i to surface j (-)
C	thermal capacitance (J/K)	$F_{\lambda_1-\lambda_2}$	fraction of blackbody radiation emitted at $\lambda_1 < \lambda < \lambda_2$ (-)
\dot{C}	capacitance rate (W/K)	f_l	friction factor associated with the flow of liquid alone (-)
C_1, C_2	undetermined constant of integration (varies)	\bar{f}	average Moody friction factor (-)
C_{crit}	critical heat flux constant (-)	$f_{Fanning}$	Fanning friction factor (-)
C_D	drag coefficient (-)	fpl	number of fins per length (1/m)
C_f	local friction coefficient (-)	Ec	Eckert number (-)
\bar{C}_f	average friction coefficient (-)	F_D	drag force (N)
C_i	the i th constant in a separation of variables solution (-)	Fo	Fourier number (-)
C_{ms}	heat capacity of microscale energy carrier (J/K)	Fr	Froude number (-)
		Fr_{mod}	modified Froude number (-)
		g	gravitational acceleration (m/s ²)
		G	mass velocity, also known as mass flux (kg/m ² -s)
		\dot{g}	rate of thermal energy generation (W)

xx

\dot{g}'''	rate of thermal energy generation per unit volume (W/m ³)	mL	fin constant (-)
Ga	Galileo number (-)	MW	molecular weight (kg/kmol)
Gr	Grashof number (-)	N	total number of time steps used (in numerical problems)
Gz	Graetz number (-)		intermediate dimensionless parameter for flow boiling correlation (-)
h	local heat transfer coefficient (W/m ² -K)		
\bar{h}	average heat transfer coefficient (W/m ² -K)	n_{ms}	number density of microscale energy carriers (#/m ³)
\tilde{h}	dimensionless heat transfer coefficient for flow boiling (-)	N_L	number of rows of tubes in the longitudinal direction in a tube bank
h_l	superficial heat transfer coefficient of the liquid phase (W/m ² -K)	Nu	local Nusselt number (-)
\bar{h}_{eff}	effective heat transfer coefficient (W/m ² -K)	Nu_x	local Nusselt number based on the characteristic length x (-)
\bar{h}_{rad}	radiation heat transfer coefficient (W/m ² -K)	\overline{Nu}	average Nusselt number (-)
i	specific enthalpy (J/kg) integer index for spatial location (in numerical problems)	NTU	number of transfer units (-)
j	integer index for time (in numerical problems)	OUT	amount or rate of some arbitrary quantity leaving a system
j_H	Colburn j_H factor (-)	p	pressure (Pa)
I_c	current (ampere)	P	$LMTD$ effectiveness (-)
IN	amount or rate of some arbitrary quantity entering a system	\tilde{p}	dimensionless pressure (-)
k	thermal conductivity (W/m-K)	p_{atm}	atmospheric pressure (Pa)
k_c	contraction loss coefficient (-)	p_∞	free stream pressure (Pa)
k_e	expansion loss coefficient (-)	per	wetted perimeter (m)
Kn	Knudsen number (-)	per_h	perimeter exposed to heating (m)
L	length (m)	Pr	Prandtl number (-)
L_c	corrected length for fin calculation (m)	\dot{q}	heat transfer rate (W)
L_{char}	characteristic length (m)	\dot{q}_{cond}	heat transfer rate due to conduction (W)
L_{cond}	conduction length (m)	\dot{q}_{conv}	heat transfer rate due to convection (W)
L_{flow}	length in the flow direction (m)	\dot{q}_{fin}	heat transfer rate to a fin (W)
L_{ms}	average distance between energy carrier interactions (m)	$\dot{q}_{fin, k \rightarrow \infty}$	heat transfer rate to a fin with $k \rightarrow \infty$ (W)
L_{nb}	nucleate boiling length scale (m)	$\dot{q}_{no fin}$	heat transfer rate that would occur from a surface if fin was removed (W)
\dot{m}	mass flow rate (kg/s)	\dot{q}_{rad}	heat transfer rate due to radiation (W)
m	mass (kg)	\dot{q}_r	heat transfer rate in the r -direction (W)
M	total number of nodes used (in numerical problems)	$\dot{q}_x, \dot{q}_y, \dot{q}_z$	heat transfer rate in the x -, y -, and z -directions (W)
		\dot{q}''	heat transfer rate per unit area, heat flux (W/m ²)
		\dot{q}_{conv}''	heat flux due to convection (W/m ²)

\dot{q}''_{rad}	heat flux due to radiation (W/m ²)	R_{SF}	shape factor thermal resistance (K/W)
$\dot{q}''_x, \dot{q}''_y, \dot{q}''_z$	heat flux in the x -, y -, and z -directions (W/m ²)	R_{sph}	thermal resistance associated with radial conduction through a spherical shell (K/W)
\dot{q}''_s	surface heat flux (W/m ²)	$R_{surface-to-surroundings}$	thermal resistance between the surface of an object and its surroundings (K/W)
$\dot{q}''_{s,crit}$	critical heat flux (W/m ²)	R_{total}	total resistance of a finned surface (K/W)
$\dot{q}''_{s,nb}$	nucleate boiling heat flux (W/m ²)	R_{univ}	universal gas constant (8314 J/kmol-K)
Q	total amount of heat transfer (J)	R''_c	area-specific contact resistance (K-m ² /W)
\tilde{Q}	dimensionless heat transfer (-)	Ra	Rayleigh number (-)
r	radial coordinate, radius (m)	Re	Reynolds number (-)
\tilde{r}	dimensionless radial coordinate (-)	Re_{δ_m}	Reynolds number based on the momentum boundary layer thickness (-)
R	thermal resistance (K/W)	Re_x	Reynolds number based on the characteristic length x (-)
	gas constant (J/kg-K)	RR	radius ratio, ratio of inner to outer radius of an annular duct (-)
	$LMTD$ capacitance ratio (-)	s	a coordinate direction (m)
R_c	contact thermal resistance (K/W)	S	shape factor (m)
R_{cond}	thermal resistance to conduction (K/W)	S_L	spacing between plates (m)
$R_{cond,int}$	thermal resistance to internal conduction within an object (K/W)		tube pitch in the longitudinal direction in a bank of tubes (m)
$R_{cond,x}$	thermal resistance to conduction in the x -direction (K/W)	S_T	tube pitch in the transverse direction in a bank of tubes (m)
$R_{cond,y}$	thermal resistance to conduction in the y -direction (K/W)	St	Stanton number (-)
R_{conv}	thermal resistance to convection (K/W)	$STORED$	amount or rate of some arbitrary quantity being stored in a system
R_{cyl}	thermal resistance associated with radial conduction through a cylindrical shell (K/W)	t	time (s)
R_e	electrical resistance (ohm)	t_j	time at the j th time in a numerical solution (s)
R_f	fouling resistance (K/W)	t_{sim}	simulation time (s)
R_{fin}	thermal resistance of a single fin (K/W)	T	temperature (K)
R_{fins}	thermal resistance of all of the fins on a finned surface (K/W)	\bar{T}	average temperature (K)
R''_f	fouling factor (K-m ² /W)	T_f	film temperature (K)
$R_{i,j}$	space resistance between surfaces i and j in a radiation problem (1/m ²)	T_h	solution to a homogeneous differential equation (K)
R_{pw}	thermal resistance to conduction through a plane wall (K/W)	T_i	temperature of the i th node in a numerical solution (K)
R_{rad}	thermal resistance associated with radiation (K/W)	T_j	temperature at the j th time in a numerical solution (K)
R_s	surface resistance in a radiation problem (1/m ²)		

$T_{i,j}$	temperature of the i th node and j th time in a numerical solution (K)
\hat{T}_i	an intermediate estimate of the temperature of the i th node in a numerical solution (K)
\hat{T}_j	an intermediate estimate of the temperature at the j th time in a numerical solution (K)
T_{ini}	initial temperature (K)
T_p	solution to a particular differential equation (K)
T_{ref}	reference temperature (K)
T_s	surface temperature (K)
T_{sur}	surrounding temperature (K)
T_∞	free stream temperature (K)
th	thickness (m)
$time$	time duration (s)
tol	tolerance (K)
u	velocity in the x -direction (m/s) specific internal energy (J/kg)
UA	conductance (W/K)
u_{char}	characteristic velocity (m/s)
u_f	fluid approach velocity for an external flow (m/s)
u_m	mean velocity (m/s)
u_{max}	maximum velocity (m/s)
u_∞	free stream velocity (m/s)
\tilde{u}	dimensionless velocity in the x -direction (-)
U	total internal energy (J)
v	velocity in the y -direction (m/s) velocity in the r -direction (m/s)
v_{ms}	average velocity of microscale energy carriers (m/s)
\tilde{v}	dimensionless velocity in the y -direction (-)
V	volume (m ³)
\dot{V}	volumetric flow rate (m ³ /s)
\dot{V}_{oc}	open circuit flow rate produced by a pump with no resistance (m ³ /s)
\dot{w}	work transfer rate, power (W)
W	total amount of work (J) width (m)

x	x -coordinate (m) direction parallel to a surface and in the flow direction for convection problems (m) thermodynamic quality (-) hydrodynamic entry length (m)
$x_{fd,h}$	x -location of the i th node in a numerical solution (m)
x_i	dimensionless x -coordinate (-)
\tilde{x}	the vector of unknown temperatures in a system of linear equations (K)
\underline{X}	Lockhart Martinelli parameter (-)
X_{tt}	y -coordinate (m) direction perpendicular to a surface for convection problems (m)
y	dimensionless y -coordinate (-)
\tilde{y}	z -coordinate (m)
z	

Greek Symbols

α	absorption coefficient (1/m) thermal diffusivity (m ² /s) ratio of gas side surface area to volume (1/m)
β	volumetric thermal expansion coefficient (1/K)
χ	correction factor for pressure drop in tube bank (-)
δ	boundary layer thickness (m)
δ_m	momentum boundary layer thickness (m)
δ_t	thermal penetration depth (m) thermal boundary layer thickness (m)
δ_{vs}	viscous sublayer thickness (m)
Δi_{vap}	latent heat of vaporization (J/kg)
Δp	pressure drop (Pa)
Δp_{dh}	dead head pressure rise produced by a pump with no flow (Pa)
Δp_{pump}	pressure rise generated by a pump (Pa)
Δt	duration of time step (s)
Δt_{crit}	duration of critical time step (s)
ΔT	temperature difference (K)

ΔT_{cond}	temperature difference due to conduction (K)	τ_{yx}	viscous stress on the y-face of a control volume in the x-direction (Pa)
$\Delta T_{cond,x}$	temperature difference due to conduction in x-direction (K)	τ_{yy}	viscous stress on the y-face of a control volume in the y-direction (Pa)
$\Delta T_{cond,y}$	temperature difference due to conduction in the y-direction (K)	τ_{diff}	diffusive time constant (s)
ΔT_{conv}	temperature difference due to convection (K)	τ_{lumped}	lumped capacitance time constant (s)
ΔT_e	excess temperature, surface minus saturation temperature (K)	ν	kinematic viscosity (m^2/s)
ΔT_{lm}	log mean temperature difference (K)	ζ	angle relative to horizontal (radian)
Δx	distance between nodes in the x-direction (m)	ζ_1	the 1st eigenvalue in a separation of variables solution (-)
Δy	distance between nodes in the y-direction (m)	ζ_i	the i-th eigenvalue in a separation of variables solution (-)
ε	emissivity (-) effectiveness (-)		
ε_{fin}	fin effectiveness (-)		
ϕ	viscous dissipation function (W/m^3)	Subscripts	
η	efficiency (-)	b	base
η_{fin}	fin efficiency (-)	c	contact, corrected
η_o	overall efficiency of a finned surface (-)	C	cold
κ	Von Kármán constant, 0.41 (-)	cond	conduction
λ	wavelength (μm)	conv	convection
μ	dynamic viscosity ($N \cdot s / m^2$)	crit	critical time step where simulation becomes unstable
θ	temperature difference (K)		critical Reynolds number for laminar-to-turbulent transition
$\tilde{\theta}$	dimensionless temperature difference (-)	cyl	cylinder
ρ	density (kg / m^3)	diff	diffusive
ρ_e	electrical resistivity ($\Omega \cdot m$)	fc	forced convection
σ	Stefan–Boltzmann constant ($5.67 \times 10^{-8} W / m^2 \cdot K^4$)	fd	fully developed
	surface tension (N/m)	fin	fin
	ratio of free flow to frontal area (-)	h	homogeneous
τ	shear stress (Pa)	H	hot
τ_s	shear stress at a surface (Pa)		solution for constant heat flux boundary condition
$\overline{\tau_s}$	average shear stress on surface (Pa)	in	entering a system, inner (e.g., diameter or radius)
τ_{xx}	viscous stress on the x-face of a control volume in the x-direction (Pa)	ini	initial, at time t = 0
τ_{xy}	viscous stress on the x-face of a control volume in the y-direction (Pa)	int	internal, within an object
		is	inner surface
		l,sat	saturated liquid
		lam	laminar
		max	maximum possible amount
		nc	natural convection
		p	particular
		pw	plane wall

<i>o</i>	overall	$a_{i,j}$	the value of a at node i and time j
<i>os</i>	outer surface		
<i>out</i>	leaving a system, outer (e.g., diameter or radius)	a^k	the value of a for iteration k
<i>rad</i>	radiation	$a_{x=x_1}$	the value of a evaluated at the x -location x_1
<i>s</i>	at the surface	$a(x)$	a is a function only of x
<i>semi-∞</i>	related to the semi-infinite body solution	$\frac{da}{dx}$	the ordinary derivative of a with respect to x (a is only a function of x)
<i>SF</i>	shape factor		
<i>sph</i>	sphere	$\left.\frac{da}{dx}\right _{x=x_1}$	the ordinary derivative of a with respect to x evaluated at x -location x_1
<i>surface-to-surroundings</i>	from the surface of an object to the surroundings	$\frac{\partial a}{\partial x}$	the partial derivative of a with respect to x (a is a function of variables other than x)
<i>T</i>	solution for constant temperature boundary condition	\underline{a}	a one-dimensional vector of values
<i>total</i>	total resistance	$\underline{\underline{a}}$	a two-dimensional matrix of values
<i>turb</i>	turbulent		
<i>uh</i>	unheated	$\underline{\underline{a}}^{-1}$	the inverse of $\underline{\underline{a}}$, a two-dimensional matrix of values
<i>v,sat</i>	saturated vapor		
<i>vs</i>	viscous sublayer		
<i>x</i>	in the x -direction		
<i>y</i>	in the y -direction		
Superscripts and Abbreviations (Where a and b are Arbitrary Quantities)		$\text{Max}(a_i) \ i = 1 \dots M$	the maximum value of the elements of vector a with indices $i = 1$ to M
a'	per unit length	$\text{Min}(a_i) \ i = 1 \dots M$	the minimum value of the elements of vector a with indices $i = 1$ to M
a''	per unit area		
a'''	per unit volume	$\sum_{i=1}^M a_i$	the sum of the elements in vector a with indices $i = 1$ to M
\bar{a}	average value of a	$a \parallel b$	quantity a in parallel with quantity b , shorthand for $(\frac{1}{a} + \frac{1}{b})^{-1}$
\hat{a}	prediction of a obtained during a predictor step		
\tilde{a}	dimensionless form of the variable a	$O(a)$	order of magnitude of the quantity a
a_i	the value of a at node i		
a_j	the value of a at time j		