

A First Course in Computational Fluid Dynamics

Fluid mechanics is a branch of classical physics that has a rich tradition in applied mathematics and numerical methods. It is at work virtually everywhere you look, from nature to technology.

This broad and fundamental coverage of computational fluid dynamics (CFD) begins with a presentation of basic numerical methods, and proceeds with a rigorous introduction to the subject. A heavy emphasis is placed on the exploration of fluid mechanical physics through CFD, making this book an ideal text for any new course that simultaneously covers intermediate fluid mechanics and computation. Ample examples, problems and computer exercises are provided to allow students to test their understanding of a variety of numerical methods for solving flow physics problems, including the point-vortex method, numerical methods for hydrodynamic stability analysis, spectral methods and traditional CFD topics.

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Preface

This book has been in preparation for over a decade. Hassan Aref and I had been making substantial additions and revisions each year, in our desire to reach the perfect book for a first course in Computational Fluid Dynamics (CFD). I sincerely wish that we had completed the book a few years ago, so that Hassan was there when the book was published. Unfortunately this was not the case. September 9th 2011 was a tragic day for fluid mechanics. We lost an intellectual leader, a fearless pioneer and for me an inspirational mentor. It is quite amazing how my academic career intersected with Hassan's over the years. He was my Masters Thesis advisor at Brown University. But when Hassan left for the University of California at San Diego, I decided to stay and finish my PhD at Brown. A few years later, when I was an assistant professor at the Theoretical and Applied Mechanics Department at the University of Illinois at Urbana-Champaign, he was appointed as the head of the department. This is when he asked me to join him in this project of writing a non-traditional introductory book on CFD. The project was delayed when Hassan moved to Virginia Tech as the Dean and I moved to the University of Florida as the Chair of the Mechanical and Aerospace Engineering Department. His passing away a few years ago made me all the more determined to finish the book as a way to honor his memory. I am very glad that the book is now finished and can be a monument to his far-sighted vision. Decades ago when CFD was in its infancy he foresaw how powerful computers and numerical methods would dominate the field of fluid mechanics.

Hassan and I shared a vision for this book. Our objective was to write something that would introduce CFD from the perspective of exploring and understanding the fascinating aspects of fluid flows. We wanted to target senior undergraduate students and beginning graduate students. We envisioned the student to have already taken a first level course in fluid mechanics and to be familiar with the mathematical foundations of differential equations. For such a student this book would serve as an ideal source for learning intermediate fluid mechanics, numerical methods and basic CFD – all three topics in an integrated manner. There are many excellent books on numerical methods that cover different techniques of temporal and spatial discretization of ordinary and partial differential equations. They are often written from the perspective of applied mathematics and their focus is on establishing accuracy, stability and conver-

gence properties. In these books CFD is viewed as numerical methodology for a system of nonlinear partial differential equations. On the other hand, there are outstanding books that are dedicated to the topic of CFD. These generally dive straight in the deep-end, with descriptions of complex numerical implementations and associated code listings for solving Navier–Stokes equations in complex three-dimensional settings. From this engineering perspective CFD is viewed as a design and development tool and therefore a considerable fraction of these books is spent on training students to be developers and expert users of large-scale CFD codes.

Our book is different. We start at a considerably lower level by presenting a sequence of basic numerical methods for time and space discretization and build our way towards CFD. Admittedly, as a result, this book will only be a first course in CFD, but the philosophy of CFD as a computational approach for exploring the science of fluid mechanics will be firmly established at an early stage. We take the view that CFD is not limited to time-dependent numerical solutions of the three-dimensional Navier–Stokes equations. On the way to the Navier–Stokes equations we will address numerical techniques for solving point-vortex equations, Kelvin–Kirchhoff equations of motion of a solid in ideal fluid, and Orr–Sommerfeld equations of hydrodynamic stability, which are simpler mathematical models derived from application of appropriate fluid mechanical principles. It is not always necessary to solve the full-blown Navier–Stokes equations in 3D – it may be far more insightful to first obtain a simpler mathematical model and to seek its numerical solution. Furthermore, even if the final answer being sought means solving the Navier–Stokes equations in 3D, it may be advantageous to first start with a simpler model problem. For example, by starting with the Euler equations and progressively increasing viscosity one could pin-point the role of viscosity. Or, before solving the full-blown Navier–Stokes equations in 3D, one could (or must) validate the code by testing it on a simpler limiting problem, whose solution is either known analytically or obtained to very high accuracy with a simpler code.

In essence, the philosophy followed in the book is best articulated by Hamming (1973): the purpose of CFD is physical insight and not just numerical data. As a corollary, the following guiding principle is adopted in this book: always first reduce the problem to its simplest possible mathematical form before seeking its numerical solution. Or in the language of this book, first formulate your (mathematical) *model* before starting to work on your (numerical) *method*. Thus, this book strives to establish a solid foundation where CFD is viewed as a tool for exploring and explaining the varying facets of fluid mechanics. With the foundation laid in this book you will be ready to tackle any in-depth treatment of CFD found in more advanced textbooks.

Pedagogically we feel it is important to learn numerical methods and CFD techniques in context. We will present numerical algorithms and methods in conjunction with a rich collection of fluid mechanical examples, so that the book will also serve as a second course in fluid mechanics. For example, what better

way to learn time-integration techniques for a system of initial-value ODEs than by solving point-vortex equations. Regular and chaotic motion, roll-up of a vortex sheet, conservation constraint in the form of a Hamiltonian, and the extreme sensitivity of the vortex sheet to round-off error are prime examples of characteristics exhibited by numerical solutions of many physical systems that can be best studied in the context of point-vortex systems. Similarly, numerical techniques for two-point eigenvalue problems are best illustrated in the context of Orr–Sommerfeld or Rayleigh stability equations. This way the reader gets to learn not only appropriate numerical techniques for two-point eigenvalue equations, but also gets excellent exposure to hydrodynamic stability theory and important related fluid-mechanical concepts such as inviscid and viscous instability, neutral curve, Squires theorem and inflection-point theorem.

Another motivation for this book stems from our desire to impact the typical undergraduate engineering curriculum in the United States and perhaps in many other countries as well. With decreasing credit hours needed for graduation and increasing amount of information needed to be at the cutting-edge, a typical mechanical, aero, chemical, civil or environmental engineering undergraduate curriculum does not include a CFD course or a second course in fluid mechanics, even as a technical elective. This book will enable a single three-credit course that combines three topics that must otherwise be taught separately: (i) introduction to numerical methods; (ii) elementary CFD; and (iii) intermediate fluid mechanics. The content of this book can also be used as the material for an introductory graduate level course on CFD. Such undergraduate and graduate courses based on versions of this book have been offered both at the University of Illinois, Urbana–Champaign campus and at the University of Florida.

I want to thank Susanne Aref for her support and encouragement in seeing this book to completion as one of the lasting contributions of Hassan to the field of fluid mechanics. I want to thank David Tranah and the Cambridge University Press for their immense help with the preparation and publication of the book. I want to thank B.J. Balakumar for the MATLAB codes that produced some of the figures in the book. I also want to thank the students who took the Computational Mechanics course at the University of Illinois and the CFD course at the University of Florida for spotting numerous typos and for allowing me to reproduce some of their homework figures. Finally, I want to thank my family for their support and understanding, as I spent countless hours working on the book.

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