CAMBRIDGE
Cambridge University Press & Assessment
978-1-107-17790-1 — Linear Algebra
Elizabeth S. Meckes , Mark W. Meckes
Table of Contents
More Information

Contents

Preface *page* xiii
To the Student xvii

1 Linear Systems and Vector Spaces 1

1.1 Linear Systems of Equations 1
Bread, Beer, and Barley 1
Linear Systems and Solutions 4
1.2 Gaussian Elimination 9
The Augmented Matrix of a Linear System 9
Row Operations 11
Does it Always Work? 14
Pivots and Existence and Uniqueness of Solutions 18
1.3 Vectors and the Geometry of Linear Systems 24
Vectors and Linear Combinations 24
The Vector Form of a Linear System 27
The Geometry of Linear Combinations 29
The Geometry of Solutions 33
1.4 Fields 39
General Fields 39
Arithmetic in Fields 42
Linear Systems over a Field 44
1.5 Vector Spaces 49
General Vector Spaces 49
Examples of Vector Spaces 53
Arithmetic in Vector Spaces 57

2 Linear Maps and Matrices 63

2.1 Linear Maps 63
Recognizing Sameness 63
Linear Maps in Geometry 65
Matrices as Linear Maps 67
Eigenvalues and Eigenvectors 69

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-107-17790-1 — Linear Algebra
Elizabeth S. Meckes , Mark W. Meckes
Table of Contents
More Information

The Matrix–Vector Form of a Linear System 73
2.2 More on Linear Maps 78
Isomorphism 78
Properties of Linear Maps 80
The Matrix of a Linear Map 83
Some Linear Maps on Function and Sequence Spaces 86
2.3 Matrix Multiplication 90
Definition of Matrix Multiplication 90
Other Ways of Looking at Matrix Multiplication 93
The Transpose 96
Matrix Inverses 97
2.4 Row Operations and the LU Decomposition 102
Row Operations and Matrix Multiplication 102
Inverting Matrices via Row Operations 105
The LU Decomposition 107
2.5 Range, Kernel, and Eigenspaces 114
Range 115
Kernel 118
Eigenspaces 120
Solution Spaces 123
2.6 Error-correcting Linear Codes 129
Linear Codes 129
Error-detecting Codes 130
Error-correcting Codes 133
The Hamming Code 134

3 Linear Independence, Bases, and Coordinates 140

3.1 Linear (In)dependence 140
Redundancy 140
Linear Independence 142
The Linear Dependence Lemma 145
Linear Independence of Eigenvectors 146
3.2 Bases 150
Bases of Vector Spaces 150
Properties of Bases 152
Bases and Linear Maps 155
3.3 Dimension 162
The Dimension of a Vector Space 163
Dimension, Bases, and Subspaces 167
3.4 Rank and Nullity 172
The Rank and Nullity of Maps and Matrices 172
The Rank–Nullity Theorem 175

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-107-17790-1 — Linear Algebra
Elizabeth S. Meckes , Mark W. Meckes
Table of Contents
More Information

Consequences of the Rank–Nullity Theorem 178
Linear Constraints 181
3.5 Coordinates 185
Coordinate Representations of Vectors 185
Matrix Representations of Linear Maps 187
Eigenvectors and Diagonalizability 191
Matrix Multiplication and Coordinates 193
3.6 Change of Basis 199
Change of Basis Matrices 199
Similarity and Diagonalizability 203
Invariants 206
3.7 Triangularization 215
Eigenvalues of Upper Triangular Matrices 215
Triangularization 218

4 Inner Products 225

4.1 Inner Products 225
The Dot Product in $\mathbb{R}^n$ 225
Inner Product Spaces 226
Orthogonality 229
More Examples of Inner Product Spaces 233
4.2 Orthonormal Bases 239
Orthonormality 239
Coordinates in Orthonormal Bases 241
The Gram–Schmidt Process 244
4.3 Orthogonal Projections and Optimization 252
Orthogonal Complements and Direct Sums 252
Orthogonal Projections 255
Linear Least Squares 259
Approximation of Functions 260
4.4 Normed Spaces 266
General Norms 267
The Operator Norm 269
4.5 Isometries 276
Preserving Lengths and Angles 276
Orthogonal and Unitary Matrices 281
The QR Decomposition 283

5 Singular Value Decomposition and the Spectral Theorem 289

5.1 Singular Value Decomposition of Linear Maps 289
Singular Value Decomposition 289
Uniqueness of Singular Values 293

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-107-17790-1 — Linear Algebra
Elizabeth S. Meckes , Mark W. Meckes
Table of Contents
More Information

5.2 Singular Value Decomposition of Matrices 297
Matrix Version of SVD 297
SVD and Geometry 301
Low-rank Approximation 303
5.3 Adjoint Maps 311
The Adjoint of a Linear Map 311
Self-adjoint Maps and Matrices 314
The Four Subspaces 315
Computing SVD 316
5.4 The Spectral Theorems 320
Eigenvectors of Self-adjoint Maps and Matrices 321
Normal Maps and Matrices 324
Schur Decomposition 327

6 Determinants 333

6.1 Determinants 333
Multilinear Functions 333
The Determinant 336
Existence and Uniqueness of the Determinant 339
6.2 Computing Determinants 346
Basic Properties 346
Determinants and Row Operations 349
Permutations 351
6.3 Characteristic Polynomials 357
The Characteristic Polynomial of a Matrix 358
Multiplicities of Eigenvalues 360
The Cayley–Hamilton Theorem 362
6.4 Applications of Determinants 366
Volume 366
Cramer's Rule 370
Cofactors and Inverses 371

Appendix 378

A.1 Sets and Functions 378
Basic Definitions 378
Composition and Invertibility 380
A.2 Complex Numbers 382
A.3 Proofs 384
Logical Connectives 384
Quantifiers 385
Contrapositives, Counterexamples, and Proof by Contradiction 386
Proof by Induction 388

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-107-17790-1 — Linear Algebra
Elizabeth S. Meckes , Mark W. Meckes
Table of Contents
More Information

Addendum 390
Hints and Answers to Selected Exercises 391
Index 423

© in this web service Cambridge University Press & Assessment www.cambridge.org