

Index

- \Leftarrow , 385
- \Leftrightarrow , 385
- \Rightarrow , 385
- \cap , 378
- \cup , 378
- \exists , 386
- \forall , 386
- \in , 5, 378
- \subseteq , 378
- \subsetneq , 378
- V , 51
- \mathbb{C} , 40, 46
- \mathbb{F} , 39
- \mathbb{F}^∞ , 56
- \mathbb{F}^n , 52, 59
- \mathbb{F}_2 , 41, 46
- \mathbb{Q} , 41, 46
- \mathbb{R} , 5, 46
- \mathbb{R}^n , 25, 35
- \mathbb{Z} , 41
- $C[a, b]$, 50
- $D[a, b]$, 57
- $\mathcal{L}(V, W)$, 64
- $\mathcal{L}(V)$, 64
- $M_{m,n}(\mathbb{F})$, 44
- $M_{m,n}(\mathbb{R})$, 10
- $\mathcal{P}_n(\mathbb{F})$, 57
- c_0 , 56
- $\operatorname{Im} z$, 382
- $\operatorname{Re} z$, 382
- $|z|$, 226, 382
- \bar{z} , 226, 382
- $f(A)$, 380
- $f : X \rightarrow Y$, 379
- $f \circ g$, 381
- f^{-1} , 381
- $x \mapsto f(x)$, 380
- $\langle v, w \rangle$, 226
- $\|v\|$, 267
 - in an inner product space, 229
- $\langle v_1, v_2, \dots, v_k \rangle$, 53
- $\operatorname{diag}(d_1, \dots, d_n)$, 68
- A^* , 97
- A^T , 96
- A^{-1} , 97
- $\|A\|_F$, 234
- $\|A\|_{op}$, 272
- T^* , 311
- $\operatorname{tr} A$, 208
- $\operatorname{tr} T$, 209
- a_{ij} , 10
- $p(T), p(A)$, 217
- $C(A)$, 116
- $\dim V$, 164
- $\operatorname{Eig}_\lambda(T), \operatorname{Eig}_\lambda(A)$, 120
- $\ker T, \ker A$, 118
- null T , null A , 175
- rank T , rank A , 173
- n -dimensional, 164
- $P_{c,i,j}$, 102
- $Q_{c,i}$, 102
- $R_{i,j}$, 102
- $[T]_{\mathcal{B}_V, \mathcal{B}_W}$, 187
- $[T]_{\mathcal{B}_V, \mathcal{B}_W}$, 195
- $[v]_{\mathcal{B}}$, 185, 195
- L^1 norm, 268
- ℓ^1 norm, 267
- ℓ^∞ norm, 267
- S_n , 351
- ι , 351
- $\operatorname{sgn}(\sigma)$, 352
- $\det(A)$, 336
- A_{ij} , 339
- $p_A(x)$, 358

- absolute value (of a complex number), 226, 382
- addition mod p , 49
- adjoint operator, 311, 311–318
- adjugate, 372, 373
- affine subspace, 123, 181
- algebraically closed, 217, 221
- alternating, 335, 344
- argument (of a complex number), 383
- augmented matrix, 11, 9–11, 45

- back-substitution, 14
- base field, 51
- basis, 150, 150–162
- bijjective, 78, 380
- binary code, 129
 - linear, 130, 136

- Cauchy–Schwarz inequality, 232, 235
- Cayley–Hamilton Theorem, 362, 364, 365
- change of basis matrix, 199, 209
- characteristic polynomial, 358, 358–364
- Cholesky decomposition, 330
- circulant matrix, 330
- closed under addition, 55
- closed under scalar multiplication, 55
- codomain, 64, 115, 379
- coefficient matrix, 10, 116, 120
- cofactor matrix, 372
- collinear, 38, 148
- column space, 116, 125
- column vector, 25
- companion matrix, 366
- complex conjugate, 226, 382
- composition, 380
- condition number, 274, 287, 310
- conjugate transpose, 97, 227, 234
- consistent linear system, 7, 7, 18, 21
- contrapositive, 386
- coordinate representation
 - of a linear map, 187
 - of a vector, 185
- coordinates, 185, 185–199
 - in orthonormal bases, 241–244, 247
- counterexample, 386
- Courant–Fischer min–max principle, 332
- Cramer’s rule, 370, 373

- determinant, 336, 333–377
 - and inverse matrix, 372
 - and solution of linear systems, 370–373
 - and volume, 366, 373
 - computation by row operations, 349, 354
 - existence and uniqueness, 339–344
 - expansion along a column, 349, 354
 - expansion along a row, 346, 354
 - product of eigenvalues, 345
 - sum over permutations, 353, 354
- determinant function, 339
- diagonal matrix, 68, 191
- diagonalizable, 192, 195, 209
 - map, 192
 - matrix, 204
 - unitarily, 320
- differentiation operator, 86, 88
- dimension, 164, 162–172
- direct sum, *see* orthogonal direct sum
- discrete Fourier transform, 285
- division, 40
- domain, 64, 379

- eigenspace, 120, 120–122, 125
- eigenvalue, 69, 69–73, 75, 122
 - and determinant, 362
 - and trace, 362
 - geometric multiplicity, 214, 365
 - multiplicity, 361, 360–362, 364
 - of a diagonal matrix, 71, 122
 - of a self-adjoint map linear map, 314
 - of a unitary matrix, 286
 - of an upper triangular matrix, 215, 220, 361
 - of similar matrices, 207
 - of transpose matrix, 179
 - root of characteristic polynomial, 358
- eigenvector, 69, 69–73, 75
 - linear independence of eigenvectors, 146
 - of commuting linear maps, 325
 - orthogonality of, for self-adjoint linear maps, 314
 - orthogonality of, for normal matrices, 331
- element (of a set), 378
- elementary matrices, 103
- encoding function, 129
 - linear, 130
- encoding matrix, 130
- entry (of a matrix), 10, 44
- error propagation, 273, 273
- error-correcting code, 133, 136

- error-detecting code, 131, 136
 extending by linearity, 156, 159
- feasible production plan, 74
 field, 39, 39–49
 finite-dimensional, 150
 four fundamental subspaces, 315
 free variable, 16, 21
 Frobenius inner product, 234, 235
 Frobenius norm, 234
 function, 379
 function space, 57
 functional calculus, 324
- Gaussian elimination, 12, 21
 Gram–Schmidt process, 244, 244–247
- Hadamard’s inequality, 355
 Hamming code, 134, 134–136
 Hermitian matrix, 314, 318
 homogeneous linear system, 6, 54
 Householder matrix, 285
- identity matrix, 68
 identity operator, 64
 image, 115, 380
 imaginary part, 382
 inclusion map, 319
 inconsistent linear system, 7
 infinite-dimensional, 150
 injective, 380
 linear map, 120, 125
 inner product, 226, 225–239
 inner product space, 227, 225–239
 integral kernel, 87
 integral operator, 87, 88
 intersection, 378
 invariant of a matrix, 207, 206–209
 invariant subspace, 69
 inverse matrix, 97, 97–107
 computing via determinants, 372
 computing via row operations, 105, 110
 invertible, 98
 isometry, 276, 276–288
 isomorphism, 78, 78–80, 88
 isoscopic, 333
- Jordan measurable, 367
- kernel, 118, 118–120, 125
- Laplace expansion, 354
 along a column, 349
 along a row, 346
- LDU decomposition, 114
 least squares, 259, 259–260, 262
 length
 in a normed space, 273
 in an inner product space, 235
 linear combination, 26, 35, 53
 linear constraints, 181, 181–182
 Linear Dependence Lemma, 145, 145–150
 linear map, 64, 63–90
 diagonalizable, 192
 linear operator, *see* linear map
 linear regression, *see* least squares
 linear system of equations, 44, 44–49
 matrix–vector form, 73, 73–75
 over \mathbb{R} , 5, 2–7
 vector form, 27–28
 linear transformation, *see* linear map
 linearly dependent, 141, 140–150
 linearly independent, 142, 140–150
 logical connectives, 384
 low-rank approximation, 303, 303–308
 lower triangular matrix, 101, 107, 221
 LU decomposition, 107, 107–110
 LUP decomposition, 110
- magnitude, 29
 matrix, 21, 44
 as a linear map, 67–69
 diagonalizable, 204
 over \mathbb{R} , 10
 matrix exponential, 324
 matrix decompositions
 Cholesky, 330
 LDU, 114
 LU, 107, 107–110
 LUP, 110
 QR, 283, 283–284
 Schur, 327, 327–329
 singular value, *see also* singular value
 decomposition, 297
 spectral, *see also* Spectral Theorem, 322
 matrix invariant, *see* invariant of a matrix
 matrix multiplication, 91, 90–100, 139
 in coordinates, 193

- matrix of a linear map
 in $\mathcal{L}(\mathbb{R}^n, \mathbb{F}^m)$, 83, 83–86, 88
 with respect to a basis, 187, 195
- matrix–vector multiplication, 67
- modulus, 226, 382
- multilinear, 334, 344
- multiplication mod p , 49
- multiplication operator, 86, 88
- negation, 387
- norm, 29, 101, 267
 L^1 , 268
 ℓ^1 , 267
 ℓ^∞ , 267
 Frobenius, 234
 in an inner product space, 229
 operator, 271, 272, 269–273
 spectral, 271
 strictly convex, 276
 supremum, 268
- normal matrix, 325, 329
- normal operator, 325, 329
- normed space, 267, 266–274
- null space, *see* kernel
- nullity, 175, 175, 182
- one-to-one, 380
- one-to-one correspondence, 380
- onto, 380
- operator, *see* linear map
- operator norm, 271, 269–273
 of a matrix, 272
- orthogonal, 229, 235
- orthogonal complement, 252, 261
- orthogonal direct sum, 254
- orthogonal matrix, 281, 284
- orthogonal projection, 75, 255, 255–262
 algebraic properties, 255
 geometric properties, 258
- orthonormal, 239
- orthonormal basis, 239, 239–252
- overdetermined linear system, 20
- parallelogram identity, 268, 273
- parity bit code, 131
- parity-check matrix, 131, 136
- permanent, 356
- permutation, 280, 351, 354
- permutation matrix, 109, 351
- perpendicular, 226, 229, 235
- Perspectives
 bases, 223
 determinants, 376
 eigenvalues, 223, 376
 isometries, 288
 isomorphisms, 224, 377
 matrix multiplication, 139
- pivot, 15, 18–20
- pivot variable, 16, 21
- polarization identities, 238, 277
- positive definite matrix, 323
- positive homogeneity, 229
- positive semidefinite matrix, 330
- proof by induction, 388
- pseudoinverse, 310
- QR decomposition, 283, 283
- quantifier, 386
- range, 115, 115–118, 125, 379
- rank, 173, 172–175, 182
- Rank–Nullity Theorem, 175, 175–182
- Rat Poison Principle, 6, 7
- real part, 382
- recipe, 390
- reduced row-echelon form (RREF), 15, 21
- row operations, 12, 11–14, 21
- row rank, 174, 182
- row space, 174
- row vector, 94
- row-echelon form (REF), 15, 21
- scalar, 25, 51
- scalar multiplication, 51
 in \mathbb{R}^n , 25
- Schur decomposition, 327, 327–329
- self-adjoint, 314
- set, 378
- sign of a permutation, 352
- similar matrices, 203, 209
- singular matrix, 98
- singular value decomposition
 computing, 316, 318
 geometric interpretation, 301–303
 of a map, 289, 289–295
 of a matrix, 297, 297–309
- singular values
 computing, 299

- of a map, 289, 295
- of a matrix, 299, 308
- uniqueness, 293–295
- singular vectors
 - of a map, 289
 - of a matrix, 299
- solution (of a linear system), 44
 - over \mathbb{R} , 3, 5
 - via determinants, 370–373
- solution space, 123–125
- span, 26, 35, 53, 150
- spectral decomposition, 322
- spectral norm, see operator norm
- Spectral Theorem, 320–329
 - for Hermitian matrices, 321
 - for normal maps and matrices, 326
 - for self-adjoint maps, 321
- spectrum, 321
- stable rank, 310
- standard basis, 68, 150
- standard basis vectors
 - of \mathbb{R}^n , 26
- strictly upper triangular matrix, 222
- subfield, 60
- subset, 378
 - proper, 378
- subspace, 55, 59
- subtraction, 40
- supremum norm, 268
- surjective, 380
- SVD, see singular value decomposition
- symmetric group, 351
- symmetric matrix, 234, 314, 318
- trace, 60, 208, 209
- transpose, 96, 100
- triangle inequality, 267
 - in an inner product space, 232, 235
- triangularization, 219, 219–221
- underdetermined linear system, 20
- union, 378
- unique solution, 7, 7, 21
- unit vector, 229
- unitarily invariant norm, 287
- unitary matrix, 281, 284
- upper triangular linear system, 19, 48
- upper triangular matrix, 101, 107, 215, 215–216
- Vandermonde determinant, 357
- vector, 51
 - over \mathbb{R} , 25
- vector addition, 51
- vector space, 51, 49–62
 - complex, 51
 - real, 51
- vector sum
 - in \mathbb{R}^n , 26
- volume, 366