# Energy Transfers in Fluid Flows

Turbulence remains an unsolved problem due to the complex nonlinear interactions among a large number of multiscale structures. For hydrodynamic turbulence, Kolmogorov's theory provides quantitative measures of energy contents of the fluid structures and energy flux. However, this theory based on real space description does not quantify various scale-by-scale energy transfers. In addition, generalisations of Kolmogorov's theory to more complex system—magnetohydrodynamic and buoyancy-driven turbulence, anisotropic flows, etc.—are quite involved. Fortunately, spectral or Fourier space description, which is the theme of this monograph, overcomes some of these deficiencies.

To quantify energy transfers in turbulence, Verma and his collaborators developed a set of important spectral tools: mode-to-mode energy transfers, various energy fluxes, shell-to-shell and ring-to-ring energy transfers, variable energy flux, etc. These diagnostics are quite general, and they do not require the flows to be homogeneous or isotropic, as is assumed in Kolmogorov's theory. Researchers have used the above tools to compute important quantities for various turbulent systems. This analysis provides many valuable insights, e.g., energy transfers responsible for the magnetic energy growth in astrophysical bodies, dynamics of turbulent thermal convection.

In this monograph, Verma systematically describes various techniques of energy transfers in turbulence. These tools include mode-to-mode transfers, fluxes, shell-to-shell and ring-to-ring transfers of energy, as well as enstrophy, kinetic helicity, and magnetic helicity. After developing the framework, the author employs them to turbulence in hydrodynamics, magnetohydrodynamics, passive scalar, buoyancy-driven flows, rotating flows, active scalar and vector, compressible flows, etc. The book describes energy transfers in both real and Fourier space, but the focus is on the latter. The energy transfer diagnostics provide many valuable insights, which have been described throughout the book.

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*Multiscale and Spectral Perspectives*

Mahendra K. Verma

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*To the seekers of knowledge, who guide the world*
*To the workers of the world, who run the world*
*and*
*To the commoners of the world, who keep humanity alive*

# Contents

# Preface

The Navier–Stokes equations, first formulated in 1822, still remain unsolved in the turbulent regime. Over time, scientists and engineers have been studying turbulence in more complex systems, such as thermal convection, scalar flows, flows in magnetofluids, boundary layers, flow across bluff bodies, etc. Turbulent flows have many complex issues—boundary layers, inhomogeneity and anisotropy, multiscale complex structures, spatiotemporal correlations among various fields, etc. In this book we address several fundamental issues of turbulence such as energy transfers in hydrodynamic, magnetohydrodynamic, scalar, compressible, anisotropic, and other forms of turbulence; variable energy flux; effects of enstrophy and kinetic helicity on turbulence.

Kolmogorov (1941) formulated a theory of hydrodynamic turbulence according to which the energy flux that flows from large scales to small scale is related to the third order structure function. This real-space formulation is applicable to isotropic and homogeneous turbulence, and its extension to anisotropic turbulence is difficult. On the other hand, such computations are relatively easier in spectral space. In 1959, Kraichnan derived a formula for the combined energy transfer for a wavenumber triad of hydrodynamic turbulence. This formalism is useful, but its scope is limited. For example, the combined energy transfer formula does not yield the energy transfers that are responsible for the generation of the large-scale magnetic field in dynamos. Curiously, most research works on turbulence report energy spectrum, but energy transfers such as energy flux have not been discussed often.

We (Gaurav Dar, Vinayak Eswaran, and I) started to work on quantifying energy transfers in magnetohydrodynamic (MHD) turbulence way back in 1999–2000. During the investigation, we discovered a very nice formalism called *mode-to-mode energy transfer* that helped us quantify all the energy transfers, energy fluxes, and shell-to-shell transfers in hydrodynamic and MHD turbulence, as well as in dynamos. After publication of this work (with some difficulties) in 2001, we extended this formalism to the following systems and obtained many interesting insights (also see Acknowledgments):

- Passive scalar turbulence
- Helical turbulence
- Buoyancy-driven turbulence—stably stratified turbulence and turbulent thermal convection
- Large-scale and small-scale dynamos
- Anisotropic turbulence
- Rotating turbulence
- Quasi-static MHD turbulence
- Field-theoretic computation of energy transfers, in particular, energy fluxes
- Role of energy transfers in pattern formation

In addition, Daniele Carati and Olivier Debliquy performed energy transfer computations of MHD turbulence using large-resolution data; Bogdan Teaca, Franck Jenko, and coworkers extended this formalism to gyrokinetic plasma turbulence; and Rodion Stepanov and Franck Plunian employed similar techniques to shell models of turbulence.

Several research groups have computed the energy fluxes in various turbulent systems using the numerical procedure of Dar et al. and Frisch. Yet, the underlying formalism of mode-to-mode energy transfer has remained somewhat unnoticed. This book is an attempt to present this powerful formalism and its applications to a variety of turbulent flows in a coherent and general framework so as to reach the turbulence and fluid community. In the monograph I present the fluxes, shell-to-shell and ring-to-ring transfers of kinetic energy, kinematic helicity, enstrophy, etc., for hydrodynamic, MHD, and scalar turbulence. I also describe popular turbulence phenomenologies, and their verification using numerical simulations and experiments. The earlier derivation of mode-to-mode formalism by Dar et al. had an uncomfortable issue of *circulating energy transfer*. In the present book, using tensor analysis and the structure of the nonlinear terms, I show that the circulating energy transfer is zero, thus resolving the ambiguity of the earlier derivation. The present monograph illustrates the usefulness of the energy transfers for understanding turbulence.

In addition to the earlier works (listed above) on the energy transfers, the book contains several new works on energy transfers, which are as follows:

- Derivation of mode-to-mode transfer formulas for kinetic helicity, magnetic helicity, and enstrophy
- Energy transfers in compressible turbulence
- Energy transfers in electron MHD
- Energy transfers in Craya–Herring and helical basis
- Energy transfers in tensor flows, and in spherical flows
- Field-theoretic computation of energy transfers in Craya–Herring basis

The presentations on compressible turbulence, electron MHD, and flows on a sphere are quite elementary and nascent. I believe that a lot more can be done in these areas.

For a thematic presentation, the book is divided into four parts. The first part deals with hydrodynamic flows—two-dimensional and three-dimensional turbulence, helical turbulence, enstrophy transfers, Craya–Herring and helical basis, and Kolmogorov theory of turbulence. In the second part, flows with a scalar that includes passive scalar, stably stratified flows, thermal convection, and binary fluid are covered. The third part is dedicated to flows with vectors, namely MHD and electron MHD. In the last part, compressible and Burgers turbulence, shell model, flows in spherical geometry, etc., have been discussed. The monograph includes works of many researchers, including those of my collaborators, who are listed in Acknowledgments. Yet, many important works could not be included due to lack of space.

Though the book focuses on energy transfers in fluid flows, one can observe that energy transfers would be useful for studying the nature of interactions in other nonequilibrium systems. For example, we expect the detailed balance to be broken in a generic nonequilibrium system, which would lead to directional energy transfers in time, space, and across scales. These transfers also yield direction to time in terms of evolution of the system. We believe that such analysis would be very useful for studying many nonequilibrium systems—quantum turbulence, financial market, coarsening in material science, etc. I do hope that the ideas of energy transfers would be employed to these systems.

Lastly, I hope that the book will be useful to students and researchers. I would greatly welcome comments, criticisms, and ideas on the contents of the book at my email mkv@iitk.ac.in.

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# Acknowledgments

The present monograph includes research works of many of my collaborators. It is great pleasure to acknowledge their contributions.

The *mode-to-mode energy transfer* formalism was discovered in collaboration with Gaurav Dar and Vinayak Eswaran around 1999. This work became part of Garav's PhD thesis in which he analyzed energy transfers in 2D MHD turbulence. I am very grateful to Gaurav and Eswaran for this collaboration.

Daniele Carati and Olivier Debliquy performed numerical computation of energy fluxes for 3D MHD turbulence. Later, Bogdan Teaca, Bernard Knaepen, and Thomas Lessinnes joined this collaboration, and we worked out energy transfers in anisotropic MHD turbulence, shell model, dynamo, etc. We also developed ring spectrum and ring-to-ring energy transfers for anisotropic turbulence. Arvind Ayyer, Amar Chandra, and V. Avinash collaborated with me to employ field-theoretic techniques to analyze helical turbulence and locality in turbulence.

Abhishek Kumar and I worked out energy transfers in stably stratified turbulence and turbulent thermal convection. Using variable energy flux we could show that stably stratified turbulence with moderate stratification shows Bolgiano–Obukhov scaling, while thermal convection is closer to Kolmogorov's scaling for hydrodynamic turbulence. I thank Abhishek Kumar for this collaboration. For thermal convection, I also collaborated with Ambrish Pandey, Pankaj Mishra, Mani Chandra, Anando Chatterjee, Dinesh Nath, Shashwat Bhattacharjee, Sumit Vashishtha, Krishna Kumar, Supriyo Paul, Pinaki Pal, and Roshan Samuel. I also benefited by my collaboration with Jai Sukhatme, Anirban Guha, and Shadab Alam on stably stratified turbulence.

The energy transfer studies on quasi-static MHD turbulence was performed in collaboration with Sandeep Reddy and Raghwendra Kumar, while those on dynamo and dynamo transition was with Rohit Kumar, Ravi Samtaney, and Rakesh Yadav. In recent times, I collaborated with Manohar Sharma, Sagar Chakraborty, and Abhishek Kumar on rotating turbulence, and with Rohith Jayaraman and Akanksha Gupta on two-dimensional turbulence. In addition, I worked with Alex Alexakis and Sita Sundar on anisotropic MHD turbulence.

Recently, I collaborated with Rodion Stepanov, Peter Frick, Valeriy Titov, Andrei Teimurazov, Andrei Sukhanovskii, Franck Plunian, Shubhadeep Sadhukhan, Abhishek Kumar, Satyajit Barman, and Ravi Samtaney under an Indo-Russian project. During this project we developed the framework of kinetic helicity, magnetic helicity, cross helicity, and enstrophy transfers, and performed numerical computations of these quantities in hydrodynamic, MHD, and convective turbulence. We also performed a very high resolution hydrodynamic turbulence simulation, and analyzed energy spectra and fluxes in the inertial–dissipation range.

I started my research career under the mentorship of Melvyn Goldstein and Aaron Roberts. Many thanks to them. I am grateful to Daniele Carati, Ravi Samtaney, Franck Plunian, and Rodion Stepanov for long-term and fruitful collaborations, and camaraderie. I also thank Stephan Fauve, Katepalli Sreenivasan (Sreeni), and Jayant Bhattacharya for many interesting suggestions, encouragement, and constant help throughout my career.

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All the energy transfer algorithms are implemented in our spectral software TARANG. Anando Chatterjee and other group members of our turbulence group contributed to the development and testing of this software, for which I am greatly indebted to them. I am thankful to King Abdullah University of Science and Technology (KAUST) for providing computational access on SHAHEEN I and SHAHEEN II through project K1052. Some of the numerical simulations were also performed on HPC2010, HPC2013, CHAOS clusters of IIT Kanpur, and PARAM YUVA of CDAC.

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