

Fundamentals of Massive MIMO

Written by pioneers of the concept, this is the first complete guide to the physical and engineering principles of Massive MIMO. Assuming only a basic background in communications and statistical signal processing, it will guide readers through key topics in multi-cell systems such as propagation modeling, multiplexing and de-multiplexing, channel estimation, power control, and performance evaluation. The authors' unique capacity-bounding approach will enable readers to carry out effective system performance analyses and develop advanced Massive MIMO techniques and algorithms. Numerous case studies, as well as problem sets and solutions accompanying the book online, will help readers put knowledge into practice and acquire the skill set needed to design and analyze complex wireless communication systems. Whether you are a graduate student, researcher, or industry professional working in the field of wireless communications, this will be an indispensable guide for years to come.

Thomas L. Marzetta is the originator of Massive MIMO. He is a Bell Labs Fellow at Nokia and a Fellow of the IEEE, and previously worked at Schlumberger-Doll Research and Nichols Research Corporation. He has received numerous recognitions and awards, including the IEEE W. R. G. Baker Award (2015), the IEEE Communications Society Stephen O. Rice Prize (2015), and the IEEE Guglielmo Marconi Prize Paper Award (2013). He is the recipient of an Honorary Doctorate from Linköping University.

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“Massive MIMO has over the past few years become one of the hottest research topics in wireless, and will be a key component of 5G. This book is written by pioneers of the area in a systematic and lucid way, and works out the fundamentals without getting lost in the details. I highly recommend it to anybody working in this field.”

Andreas Molisch, *University of Southern California*

“Bright and profound, this book provides the fundamentals to understand the unique capabilities of Massive MIMO and illustrates the benefits for specific use cases. The authors are scientific pioneers and masters in explaining and sharing their proficiency in this book: it is an intellectual treat for everyone fascinated by Massive MIMO technology!”

Liesbet Van der Perre, *KU Leuven*

“A very timely text by some of the ‘founding fathers’ of massive MIMO. This is a great book for the beginner, with its simple but enlightening examples, as well as a great reference text for the more experienced engineer. The book is concise and to the point, and the summary and key points at the end of each chapter make it easy to focus your reading. Highly recommended for those that want to get an in-depth understanding of massive MIMO without spending months doing so.”

Lee Swindlehurst, *University of California, Irvine*

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PREFACE

Simplification of modes of proof is not merely an indication of advance in our knowledge of a subject, but is also the surest guarantee of readiness for further progress.

– W. Thomson [1st Baron Kelvin] and P. G. Tait,
Elements of Natural Philosophy, 1873

There are three timeless truths in the field of wireless communications:

- Demand for wireless throughput, both mobile and fixed, will always increase.
- The quantity of available electromagnetic spectrum will never increase, and the most desirable frequency bands that can propagate into buildings and around obstacles and that are unaffected by weather constitute only a small fraction of the entire spectrum.
- Communication theorists and engineers will always be pressured to invent or to discover breakthrough technologies that provide higher spectral efficiency.

Given the history of more than a century of wireless innovation, we must look to the physical layer for breakthrough technologies. The central assertion of this book is that Massive MIMO constitutes a breakthrough technology. It is a scalable technology whereby large numbers of terminals simultaneously communicate through the entire allocated frequency spectrum. What enables this aggressive multiplexing is, first, an excess number of service antennas compared with terminals, and, second, performing the multiplexing and de-multiplexing based on *measured* propagation characteristics rather than on *assumed* propagation characteristics. A disproportionate number of service antennas compared with active terminals makes it likely that the propagation channels are conducive to successful multiplexing, and basing the multiplexing on direct channel measurements makes the antenna array tolerances independent of the number of antennas. The activity of growing the number of antennas relative to the number of active terminals renders the simplest types

of multiplexing and de-multiplexing signal processing exceedingly effective, and it permits the same quality of service with reduced radiated power. Low radiated power is conducive to frequency reuse. The combined action of many antennas eliminates frequency-dependent fading and simplifies power control. By virtue of the time-division duplexing operation, the propagation channel characteristics are measured on the uplink and used both for uplink data detection and downlink beamforming. This facilitates operation in high-mobility scenarios. Also, by performing appropriate power control, Massive MIMO yields uniformly good service to all terminals, as measured in terms of 95%-likely throughputs on both the uplink and downlink.

Ostensibly, analyzing the performance of a Massive MIMO system is a daunting task because of the sheer number of frequency dependent propagation channels at work, and the fact that all terminals transmit and receive information over all frequencies. The central message of this book is that substantially closed-form performance expressions are obtainable for even the most complicated multi-cell Massive MIMO deployments. We achieve analytical tractability in three ways: first, we model small-scale fading – a-priori unknown to everyone – as independent, Rayleigh distributed; second, we assume that large-scale fading is known to everyone who needs to know it; and, third, we restrict attention to the simplest linear multiplexing and de-multiplexing – zero-forcing and maximum-ratio processing, both on the uplink and on the downlink. Collectively, these assumptions admit Bayesian analysis and ergodic capacity lower bounds, whose derivation requires only elementary mathematical techniques. For multi-cell Massive MIMO deployment, we obtain comprehensive but remarkably simple non-asymptotic expressions for the capacity lower bounds. Massive MIMO, in effect, creates a dedicated virtual circuit between the home base station and each of its terminals, comprising a frequency independent channel whose quality depends only on large-scale fading and power control. Our capacity lower bounds account for receiver noise, channel estimation errors, the overhead associated with pilots, power control, the imperfections of the particular multiplexing or de-multiplexing signal processing that is employed, non-coherent inter-cell interference, and coherent inter-cell interference that arises from pilot reuse. These bounds yield considerable intuitive insight into the workings of Massive MIMO systems, the interplay of system parameters, and system scalability. Numerical case studies illustrate the tremendous potential of Massive MIMO as well as the value of the capacity bounds as system design tools.

This book should appeal to three classes of readers. Wireless engineers will find a clear exposition of the principles of Massive MIMO that uses only elementary communication theory and statistical signal processing. While exhaustive end-to-end system simulations may be employed for detailed system performance analyses, and possibly for engineering designs, our capacity-bound approach is likely to see extensive use because of its speed and simplicity, the insight that it gives into the interaction of system parameters, and as

an independent check on simulations. Researchers who are devising advanced Massive MIMO techniques and algorithms can use baseline performance measures, embodied in our capacity bounds, to quantify performance improvements versus implementation complexity. For the student, this book should be an ideal vehicle for learning how to translate basic information theory, and communication and signal processing principles, into the analysis of complicated communication systems.

How to Read This Book

With the aid of the appendices, this book is self-contained and requires only linear algebra and undergraduate-level probability theory as prerequisites.

Practicing systems engineers looking for a quick insight into the potential of Massive MIMO technology may start by reading Chapter 6 on case studies, and then for insights into the performance evaluation methodology, read Chapter 3 for the single-cell analysis, and Chapter 4 for the multi-cell analysis. The other chapters and appendices can be used as references when needed.

Professors looking to cover the topic in depth (a one-semester graduate-level course), or students and researchers looking for a solid background in Massive MIMO performance analysis, may first read Chapters 1–2, next study the background material in Appendices A–C, and then read Chapters 3–8 in sequence, referring to the rest of the appendices whenever needed.

A set of problems to each chapter, and an accompanying solution manual, are available to course instructors and may be obtained by contacting the authors.