Index

acknowledgments, xiii
agent, 25, 87, 173
allocation curve, 29
area under, 31
allocation rule, 26
  approximately welfare-maximizing, 43
cycle monotonicity, 89, 93
defered, 107, 111–112
greedy, 44, 106, 112
implementable, 27
monotone, 27
non-monotone, 45
scoring function, 108
virtual threshold, 78
virtual welfare-maximizing, 62
welfare-maximizing, 19, 22, 42, 50, 89
approximation algorithm, 43, 44
auction
  approximately optimal, 75, 84
  ascending, 21, 97–98, 112, 115
  clinching, 116–119, 126
  combinatorial, see combinatorial auction
  competition, 81

English, see auction, ascending
first-price, 12, 64, 72–73
ideal, 15–16
Japanese, 98
k-unit, 20, 25, 37, 71
knapsack, 39–40, 43–46
multi-item, see combinatorial auction
multi-unit, 114, 126
online, 22
optimal, 57, 64
prior-free, 83
prior-independent, 79–82, 86
procurement, 21
revenue-maximizing, 57
reverse, 21, 106
sealed-bid, 12
second-price, 13, 32
separate single-item, 98
sequential, 100
simple, 75, 77–79, 84
simultaneous sealed-bid, 101, 201
single-item, 11–16, 25, 31, 88
sniping, 102
spectrum, see spectrum auction
sponsored search, see
sponsored search
third-price, 20
two-step design approach, 18
uniform-price, 114–116
versus negotiation, 81
weird, 75, 83
welfare-maximizing, 15
with budgets, 113
with one bidder, 56, 57

badminton, 1–3
Bayesian analysis, 56, 64, 72–73
Bayesian incentive compatible (BIC), 64
best response, 175
best-response dynamics, 216
as a graph walk, 217
as local search, 274
convergence time, 218
$\epsilon-$, 219, 275
fast convergence, 220–223, 227, 228, 228
in atomic selfish routing, 220–223, 228
in congestion games, 274
in potential games, 217
in scheduling, 227–229
in smooth games, 223–226
in symmetric congestion games, 275
lower bounds, 274, 275
maximum-gain, 219, 224
maximum-relative-gain, 228
maximum-weight, 227
non-convergence, 227, 229
vs. better-response dynamics, 217
bid, 26
critical, 41, 63
false-name, 92, 94
bimatrix game, 7, 279
bounded rationality, 8
Braess’s paradox, 4, 145, 158
Brouwer’s fixed-point theorem, 290, 296
and Nash’s theorem, 291, 293, 296
in higher dimensions, 291
in atomic selfish routing, 220–223, 228
in congestion games, 274
in potential games, 217
in scheduling, 227–229
in smooth games, 223–226
in symmetric congestion games, 275
lower bounds, 274, 275
maximum-gain, 219, 224
maximum-relative-gain, 228
maximum-weight, 227
non-convergence, 227, 229
vs. better-response dynamics, 217
bid, 26
critical, 41, 63
false-name, 92, 94
bimatrix game, 7, 279
bounded rationality, 8
Braess’s paradox, 4, 145, 158
Brouwer’s fixed-point theorem, 290, 296
and Nash’s theorem, 291, 293, 296
in higher dimensions, 291
in atomic selfish routing, 220–223, 228
in congestion games, 274
in potential games, 217
in scheduling, 227–229
in smooth games, 223–226
in symmetric congestion games, 275
lower bounds, 274, 275
maximum-gain, 219, 224
maximum-relative-gain, 228
maximum-weight, 227
non-convergence, 227, 229
interpretation, 241
tractability, 178, 241, 260, 263, 293
coffee, 290
collusion, 23, 38, 92, 94
combinatorial auction, 88, 94–95, 97
applications, 88
approximation, 96
complements, 99
exposure problem, 104
package bidding, 105
price of anarchy, 201
substitutes, 99
common prior, 48, 57, 72
communication network, 159, 169
computational efficiency, 16
congestion game, 181, 271
as a potential game, 186, 271
computing a correlated equilibrium, 277
intractability of pure Nash equilibria, 272–275
lower bound for best-response dynamics, 274, 275
symmetric, 274–275
\(\text{coNP}\), 281
core allocation, 122
correlated equilibrium, 176–177
and no-swap-regret dynamics, 248, 258
as a linear program, 260, 277
in congestion games, 277
interpretation, 176, 241
swapping function, 184, 247
tractability, 177, 249, 260, 263, 277, 293
traffic light example, 177
cost function
in cost-minimization games, 174
in selfish routing, 148
cryptography, 293, 295
CTR, see sponsored search, click-through rate
defered acceptance algorithm, 137–141
applicant-optimality, 140
convergence, 139
incentive properties, 141, 144
defeated allocation rule, see allocation rule, deferred
demand
aggregate, 115
of a bidder, 114
reduction, 103, 115
residual, 116
density function, 57
dependencies, xii
direct-revelation, see mechanism, direct-revelation
discussion forum, xii
distribution function, 57
dominant strategy, 13
dominant-strategy incentive compatible, see DSIC
DSIC, 15, 16, 46
for randomized mechanisms, 86, 126
dynamics, 216
  best-response, see best-response dynamics
  no-regret, see no-regret dynamics
  no-swap-regret, see no-swap-regret dynamics
  vs. algorithms, 263

eBay, 13, 21, 58, 65

economics and computation, xi
  environment, see single-parameter environment
  envy-free, see sponsored search, envy-free

equilibrium
  Bayes-Nash, 48, 57, 64, 72–73
  coarse correlated, see coarse correlated equilibrium
  competitive, see Walrasian equilibrium
  computation, 263
  correlated, see correlated equilibrium
  dominant-strategy, 46
  flow, 152, 164
  hierarchy, 173, 208
  Nash, see Nash equilibrium
  refinement, 208
  sponsored search, 36
  Walrasian, see Walrasian equilibrium
  experts, see online decision making
  externality, 50, 90, 195, 202

negative vs. positive, 202

feasible set (of outcomes), 25

Federal Communications Commission (FCC), 106

first welfare theorem, 111

$\mathcal{FIXP}$, 295

flow network, 152–153, 156

multicommodity, 157

$\mathcal{FNP}$ (functional $\mathcal{NP}$), 280
  and mixed Nash equilibria, 281
  decision vs. search problems, 280
  $\mathcal{FNP}$-completeness, 281

Gale-Shapley algorithm, see deferred acceptance algorithm

game
  compact, 263, 277
  congestion, see congestion game
  constant-sum, 259
  cost-minimization, 173
  description length, 263
  location, 188, 199
  network cost-sharing, see network cost-sharing game
  payoff-maximization, 174, 190
  potential, see potential game
  smooth, see smooth game
  symmetric, 297
  two-player, see bimatrix game
Index

zero-sum, see zero-sum game
Google, 17
greedy, see allocation rule, greedy
group-strategyproof, 38
weakly, 112
GSP, see sponsored search, generalized second-price auction
hints, xii, 301–308
H_k, 205
house allocation, 119–122, 125
and kidney exchange, 129
implementable, see allocation rule, implementable
incentive guarantee, 16
individual rationality, 15, 26, 46
information rent, 60
intended audience, xii
intractability
of approximate mixed Nash equilibria, 295
of local search, 267–271
of mixed Nash equilibria, 8, 284, 292–293
of pure Nash equilibria, 272–275
of welfare maximization, 41, 92
Iran, 129
item, 11, 88
kidney exchange, 128–136
altruistic donor, 131
chain, 131, 143
for money, 129
incentives for hospitals, 134
incompatibility, 128
pairwise, 131–134
patient-donor pair, 128
priorities, 132
simultaneous vs. sequential, 130
knapsack problem, 40
auction, see auction, knapsack
fully polynomial-time approximation scheme (FPTAS), 45, 52
greedy heuristic, 43
learning, see dynamics
online, see online decision making
lecture videos, xii
Lemke-Howson algorithm, 292, 295
linear programming, 260, 277, 292
local optimum, 265
local search, 265
abstract, 267
and best-response dynamics, 274
as a walk in a graph, 265, 269
generic procedure, 268
improving move, 265
intractability, 267–271
polynomial, see PLS
unconditional lower bounds, 271
Markov chain, 252, 258
matching
  in kidney exchange, 131
  stable, see stable matching
maximum cut problem, 264
  and congestion games, 273
  is \( PLS \)-complete, 270, 277
  unconditional lower bounds, 271
  with unit edge weights, 266
mechanism, 25
  anonymous, 127
  approximately optimal, 75, 84, 96
  direct-revelation, 26, 46
  ideal, 51, 89
  indirect, 21, 26, 97–98
  onto, 127
  optimal, 57
  prior-free, 83
  prior-independent, 79–82, 86
  priority, 125
  randomized, 86, 126
  serial dictatorship, 125
  simple, 84
  single-sample, 86
  VCG, see VCG mechanism
mechanism design, 3
  algorithmic, 42–43, 45
  applications, 3, 16, 65, 106
  Bayesian, xi, 48
  direct-revelation, 91
  multi-parameter, 87
  preference elicitation, 91
  single-parameter, 24
  two-step design approach, 18, 89
with budgets, 113
Minimax theorem, 254–257
  and linear programming duality, 258
  and no-regret dynamics, 255, 259
  equivalent to equilibrium existence, 258
  for constant-sum games, 259
history, 258
  interpretation, 255
  mix and match, 259
mixed Nash equilibrium, 7, 175
  brute-force search algorithm, 296
  existence, 175, 290
  in bimatrix games, 279
  intractability, 8, 176, 284, 292–293
  intractability of approximate equilibria, 295
  intractability with three or more players, 295
  quasitractability of approximate equilibria, 295–297
mixed strategy, 7, 175
MNE, see mixed Nash equilibrium
monopoly price, 58, 85
monotone, see allocation rule, monotone
monotone hazard rate (MHR), 70, 85
multiplicative weights (MW) algorithm, see no-regret algorithm, multiplicative weights
Index

Myerson’s lemma, 28–31
  in multi-parameter environments, 89
Myerson’s payment formula, 30
Nash equilibrium, 7
  approximate, 197, 219, 295–297
  as a predictor of behavior, 8, 293
  best-case, see price of stability
  existence (mixed), 290
  existence (pure), 179–182
  existence (strong), 210, 214
  in zero-sum games, 255, 258, 260
  \( \ell \)-strong, 215
  mixed, see mixed Nash equilibrium
  non-uniqueness, 293
  pure, see pure Nash equilibrium
  strong, 209
  worst-case, see price of anarchy
Nash’s theorem, 7, 290
  network cost-sharing game, 203
  examples, 203–205
  opting out, 205
  price of anarchy, 204, 214
  price of stability, 205–208
  strong, Nash equilibria, 208–212
  undirected networks, 214
  VHS or Betamax, 204
  with concave cost functions, 214
  network formation, 202, 213
  network over-provisioning, 159, 169
no-regret algorithm, 232
  design principles, 234
  deterministic, 233
  existence, 234
  follow-the-leader (FTL), 233
  follow-the-perturbed-leader (FTPL), 245–246
  learning rate, 235, 239
  multiplicative weights, 234–239
  with unknown time horizon, 239, 243
no-regret dynamics, 239
  converges to coarse correlated equilibria, 240–241, 245
  in payoff-maximization games, 239
  in smooth games, 241, 243
  in zero-sum games, 255, 259
  non-convergence to mixed Nash equilibria, 259
no-swap-regret algorithm, 248
  existence, 249
  reduction to a no-regret algorithm, 249–252
no-swap-regret dynamics, 248
  converges to correlated equilibria, 248, 258
\( \text{NP} \), 42
\( \text{NP} \) search problem, see \( \text{FNP} \)
NP-completeness, 264, 276
unsuitability for equilibrium computation, 281–283
NP-hard, 8, 42, 107

Olympic scandal, 1–3
online decision making, 230
adversary (adaptive), 231, 241
adversary (oblivious), 236
algorithm, 231
bandit model, 231, 243
examples, 231–234
history, 243
in games, see no-regret dynamics
mistake bound, 244
regret-minimization, see no-regret algorithm
with an omniscient expert, 244
with large costs, 243
with payoffs, 231, 255
organization of book, xii
outcome, 87, 174

P, 42
Pareto optimal, 124
payment rule, 26
computation, 42, 51
explicit formula, 30, 33, 41
nonnegative, 26
uniqueness, 28
payoff, 7
performance guarantee, 16
ex post, 56
Pigou’s example, see selfish routing, Pigou’s exam-
ple
player, 253
PLS, 269, 276
as a subset of \(\mathcal{FNP}\), 281
as a subset of \(\mathcal{T\mathcal{FN\ell}}\), 283
as a syntactic class, 286
as a walk in a graph, 269
PLS-completeness, 270
analogy with \(NP\)-completeness, 267
justification, 283
of computing a locally maximum cut, 270
of computing a pure Nash equilibrium of a congestion game, 272–275
vs. unconditional results, 266
PNE, see pure Nash equilibrium
POA, see price of anarchy
polynomial time, 41, 263
posted price, 56, 85
potential function, 181, 206
generalized ordinal, 227
minimizer, 207, 214, 215
potential game, 181, 185–186, 199
as a congestion game, 186
convergence of best-response dynamics, 217
equilibrium existence, 182, 217
generalized ordinal, 227
smooth, 223
PPAD, 286, 294
and mixed Nash equilibria, 292–293
Index

and Sperner’s lemma, 289
as a subset of $\mathcal{TFNP}$, 288
as a syntactic class, 287
as a walk in a graph, 286
evidence of intractability, 293
$\mathcal{PPAD}$-completeness
of computing a mixed Nash equilibrium of a bimatrix game, 284
of computing a trichromatic triangle, 289
prerequisites, xii
price of anarchy, 5
four-step recipe, 187
in location games, 188–194
in network cost-sharing games, 204, 214
in scheduling, 171–172, 185, 200
in selfish routing, 146–155
in selfish routing (atomic), 165–168
in smooth games, 195–198, 200
in sponsored search, 200
of correlated equilibria, 177
of mixed Nash equilibria, 176
of no-regret dynamics, 241, 243
of strong Nash equilibria, 208–212
optimistic, see price of stability
price of stability, 205
in atomic selfish routing networks, 215
in network cost-sharing games, 205–208
interpretation, 208
private, 12
prophet inequality, 75–77, 83, 84
public project, 25
pure Nash equilibrium, 174
existence, 179–182, 184–185
intractability, 272–275, 277
tractability, 278
pure strategy, 174
quasilinear utility, 12, 23, 26
quasipolynomial time, 297
reduction (between search problems), 269, 280, 297
regret, 232
-minimization algorithm, see no-regret algorithm
external, 232
external vs. swap, 258
internal, 248
lower bound, 233, 244
swap, 248
vs. stronger benchmarks, 231
regular distribution, see valuation distribution, regular
report, 25
reserve price, 58, 65–66
anonymous, 79, 83
bidder-specific, 79, 84, 85
| resource augmentation | 161 |
| revelation principle | 46–48, 64, 89 |
| revenue | 59 |
| curve | 60, 72 |
| equals virtual welfare | 60 |
| equivalence | 70, 72 |
| monotonicity | 94 |
| of a mechanism | 59, 61 |
| target | 37 |
| revenue maximization | see auction, revenue-maximizing |
| Rock-Paper-Scissors | 7, 254 |
| routing | see selfish routing |
| SAA | see spectrum auction, simultaneous ascending |
| satisfiability (SAT) | 107 |
| functional version | 281 |
| scheduling | 171–172, 185, 200, 227–229 |
| science of rule-making | see mechanism design |
| secretary problem | 22 |
| selfish routing | 4, 148 |
| affine cost functions | 151, 156, 171 |
| affine cost functions (atomic) | 165 |
| α-bounded jump condition | 220 |
| atomic | 163–168, 170, 184, 198 |
| atomic splittable | 169 |
| best-response dynamics (atomic) | 220–223, 228, 277 |
| Braess’s paradox | 145, 158 |
| computing an equilibrium flow (atomic) | 277, 278 |
| concave cost functions | 151, 156 |
| cost function | 148 |
| cost of a flow | 153 |
| equilibrium existence | 146, 165, 179 |
| equilibrium flow | 152 |
| equilibrium flow (atomic) | 164 |
| equilibrium uniqueness | 146, 165, 180 |
| examples | 145–147 |
| flow | 152 |
| history | 156 |
| M/M/1 cost function | 162 |
| maximum travel time | 157 |
| multicommodity | 157, 170 |
| nonlinear Pigou’s example | 147 |
| over-provisioned network | 160–161, 170 |
| Pigou bound | 151, 157 |
| Pigou’s example | 146–147 |
| Pigou’s example (atomic) | 164 |
| Pigou-like network | 150 |
| polynomial cost functions | 151, 156 |
| potential function | 179, 180, 221 |
| price of anarchy | 146, 148–152 |
| price of anarchy (atomic) | 165, 171 |
| resource augmentation bound | 161–163, 170 |
### Index

171  
road traffic, 149, 156  
total travel time, 153  
weighted atomic, 169, 184–185  
with a common origin and destination (atomic), 220, 277–278  
semantic complexity class, 284, 295  
single-item auction, see auction, single-item  
single-parameter environment, 24  
downward-closed, 85  
single-peaked preferences, 126  
sink vertex, 265  
smooth game, 194–198  
best-response dynamics, 223–226  
examples, 195, 200–201  
interpretation, 195  
potential, 223  
price of anarchy of approximate equilibria, 198, 200  
price of anarchy of coarse correlated equilibria, 196  
price of anarchy of pure Nash equilibria, 196  
with respect to a strategy profile, 195  
social welfare, 15, 55, 87  
approximation, 43, 92  
with budgets, 124, 126  
Sotheby’s, 98  
source vertex, 286  
spectrum auction, 97–109  
activity rule, 102, 105  
bid signaling, 104  
deferred allocation rule, 107  
demand reduction, 103  
descending implementation, 109  
exposure problem, 104  
hierarchical packages, 105  
in New Zealand, 101  
in Switzerland, 100  
opening bids, 109  
package bidding, 105  
price discovery, 102  
repacking, 106  
rookie mistake, 100, 101  
sanity checks, 102  
scoring function, 108  
simultaneous ascending, 102  
substitutes vs. complements, 99, 104  
Sperner’s lemma, 288  
and Brouwer’s fixed-point theorem, 290  
and Nash’s theorem, 290–292  
as a \(PPAD\) problem, 289  
as a walk in a graph, 289  
legal coloring, 288  
trichromatic triangle, 288  
click-through rate, 17  
DSIC payment formula, 33  
envy-free, 36  
equilibrium, 36  
equivalence of DSIC and GSP auctions, 35
Index

generalized second-price auction, 33, 35
locally envy-free, 37
price of anarchy, 200
revenue maximization, 71
slot, 17
welfare maximization, 22
stable matching, 136–141
applicant-optimality, 140
blocking pair, 137
defered acceptance, 137–141
DSIC mechanism, 141
existence, 139
in practice, 143
starred sections, xii
strategy profile, 174
strategy set, 7, 174
strings and springs, 5
submodular function, 192, 199
syntactic complexity class, 283

$\mathcal{TFN} \mathcal{P}$ (total functional $\mathcal{NP}$), 283, 294
lack of complete problems, 283
reasons for membership, 284
Top Trading Cycle algorithm, 119–122
in kidney exchange, 129–131
truthful mechanism, see mechanism, direct-revelation
TTC, see Top Trading Cycle algorithm
unit-demand, see valuation,
unit-demand utility, see quasilinear utility
valuation, 12, 25, 87
downward-sloping, 126
marginal, 126
single-minded, 53
subadditive, 96
unit-demand, 94, 112, 201
valuation distribution, 57
correlated, 69
irregular, 63, 69
regular, 62, 70
strictly regular, 64
unknown, 80
VCG mechanism, 90, 103
allocation rule, 89
and affine maximizers, 96
ascending implementation, 123
flaws, 91–92, 97, 105
non-monotonicity, 92, 94
payment rule, 90, 93
pivot term, 93, 95
revenue, 92
with unit-demand valuations, 94
Vickrey auction, see auction, second-price
Vickrey-Clarke-Groves, see VCG mechanism
virtual valuation, 59
ironing, 69
nondecreasing, 62
virtual welfare, 61
Walrasian equilibrium, 110–112
welfare, see social welfare
Index 341

wireless spectrum, *see* spectrum auction

Yahoo, 65–66

zero-sum game, 7, 253–254
  convergence of no-regret dynamics, 255, 259
  minimax pair, 258
Minimax theorem, *see* Minimax theorem
mixed Nash equilibrium, 254, 263
value, 255