Twenty Lectures on Algorithmic Game Theory

Computer science and economics have engaged in a lively interaction over the past 15 years, resulting in the new field of algorithmic game theory. Many problems central to modern computer science, ranging from resource allocation in large networks to online advertising, involve interactions between multiple self-interested parties. Economics and game theory offer a host of useful models and definitions to reason about such problems. The flow of ideas also travels in the other direction, and concepts from computer science are increasingly important in economics.

This book grew out of the author's Stanford course on algorithmic game theory, and aims to give students and other newcomers a quick and accessible introduction to many of the most important concepts in the field. The book also includes case studies on online advertising, wireless spectrum auctions, kidney exchange, and network management.

Tim Roughgarden is an Associate Professor of Computer Science at Stanford University. For his research in algorithmic game theory, he has been awarded the ACM Grace Murray Hopper Award, the Presidential Early Career Award for Scientists and Engineers (PECASE), the Kalai Prize in Game Theory and Computer Science, the Social Choice and Welfare Prize, the Mathematical Programming Society's Tucker Prize, and the EATCS-SIGACT Gödel Prize. He wrote the book *Selfish Routing and the Price of Anarchy* (2005) and coedited the book *Algorithmic Game Theory* (2007).

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To Emma

Contents

Preface

Computer science and economics have engaged in a lively interaction over the past 15 years, resulting in a new field called *algorithmic game theory* or alternatively *economics and computation*. Many problems central to modern computer science, ranging from resource allocation in large networks to online advertising, fundamentally involve interactions between multiple self-interested parties. Economics and game theory offer a host of useful models and definitions to reason about such problems. The flow of ideas also travels in the other direction, as recent research in computer science complements the traditional economic literature in several ways. For example, computer science offers a focus on and a language to discuss computational complexity; has popularized the widespread use of approximation bounds to reason about models where exact solutions are unrealistic or unknowable; and proposes several alternatives to Bayesian or average-case analysis that encourage robust solutions to economic design problems.

This book grew out of my lecture notes for my course "Algorithmic Game Theory," which I taught at Stanford five times between 2004 and 2013. The course aims to give students a quick and accessible introduction to many of the most important concepts in the field, with representative models and results chosen to illustrate broader themes. This book has the same goal, and I have stayed close to the structure and spirit of my classroom lectures. Brevity necessitates omitting several important topics, including Bayesian mechanism design, compact game representations, computational social choice, contest design, cooperative game theory, incentives in cryptocurrencies and networked systems, market equilibria, prediction markets, privacy, reputation systems, and social computing. Many of these areas are covered in the books by Brandt et al. (2016), Hartline (2016), Karlin and Peres (2017), Nisan et al. (2007), Parkes and Seuken (2016), Shoham and Leyton-Brown (2009), and Vojnović (2016).

Reading the first paragraph of every lecture provides a quick sense of the book's narrative, and the "top 10 list" on pages 299–300 summarizes the key results in the book. In addition, each lecture includes an "Upshot" section that highlights its main points. After the introductory lecture, the book is loosely organized into three parts. Lectures 2–10 cover several aspects of "mechanism design"—the science of rule-making—including case studies in online advertising, wireless spectrum auctions, and kidney exchange. Lectures 11–15 outline the theory of the "price of anarchy"—approximation guarantees for equilibria of games found "in the wild," such as large networks with competing users. Lectures 16–20 describe positive and negative results for the computation of equilibria, both by distributed learning algorithms and by computationally efficient centralized algorithms. The second and third parts can be read independently of the first part. The third part depends only on Lecture 13, with the exceptions that Sections 16.2–16.3 depend on Section 12.4 and Section 16.4 on Lecture 14. The starred sections are the more technical ones, and they can be omitted on a first reading.

I assume that the reader has a certain amount of mathematical maturity, and Lectures 4, 19, and 20 assume familiarity with polynomial-time algorithms and $\mathcal{NP}$-completeness. I assume no background in game theory or economics, nor can this book substitute for a traditional book on these subjects. At Stanford, the course is attended by advanced undergraduates, masters students, and first-year PhD students from many different fields, including computer science, economics, electrical engineering, operations research, and mathematics.

Every lecture concludes with brief bibliographic notes, exercises, and problems. Most of the exercises fill in or reinforce the lecture material. The problems are more difficult, and often take the reader step-by-step through recent research results. Hints to exercises and problems that are marked with an "*(H)*" appear at the end of the book.

Videos of my classroom lectures in the most recent (2013) offering of the course have been uploaded to YouTube and can be accessed through my home page (`www.timroughgarden.org`). Lecture notes and videos on several other topics in theoretical computer science are also available there. Readers can interact with me and each other about the book's content and problems through the book's discussion

forum (`twentylecturesonagt.freeforums.net/`).

I am grateful to all of the Stanford students who took my course, which has benefited from their many excellent questions and comments. I am especially indebted to my teaching assistants: Peerapong Dhangwatnotai, Kostas Kollias, Okke Schrijvers, Mukund Sundararajan, and Sergei Vassilvitskii. Kostas and Okke helped prepare several of the figures in this book. I thank Yannai Gonczarowski, Warut Suksompong, and Inbal Talgam-Cohen for particularly detailed feedback on an earlier draft of this book, and Lauren Cowles, Michal Feldman, Vasilis Gkatzelis, Weiwei Jiang, Yishay Mansour, Michael Ostrovsky, Shay Palachy, and Rakesh Vohra for many helpful comments. The cover art is by Max Greenleaf Miller. The writing of this book was supported in part by NSF awards CCF-1215965 and CCF-1524062.

I always appreciate suggestions and corrections from readers.

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