CLASSICAL KINETIC THEORY OF WEAKLY TURBULENT NONLINEAR PLASMA PROCESSES

Kinetic theory of weakly turbulent nonlinear processes in plasma helped form the foundation of modern plasma physics. This book provides a systematic overview of the kinetic theory of weak plasma turbulence from a modern perspective. It covers the fundamentals of weak turbulence theory, including the foundational concepts and the mathematical and technical details. Applications to some key space plasma problems are also covered, including the origin of nonthermal charged particle population and radio burst phenomena from the sun. Treating both collective and discrete particle effects, it provides a valuable reference for researchers looking to familiarize themselves with plasma weak turbulence theory.

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Preface

Plasma is an ionized gas in which dynamical processes are governed predominantly by a collective behavior rather than that of individual particle interactions. For this reason, the plasma is often regarded as free of inter-particle collisions. However, the discrete nature of the plasma particles or, equivalently, the single-particle behavior of plasma can be important, and often the description of plasma is incomplete if the discrete particle effects are not properly taken into account. This monograph is concerned with the kinetic theory of weakly turbulent plasma processes in which collective and discrete particle effects are systematically treated. The methodology is classical since dilute high-temperature plasmas in laboratory, near-Earth space, and most astrophysical environments do not require quantum mechanical approaches. The book is exclusively concerned with incoherent nonlinear plasma phenomena. Such processes are known as microscopic or kinetic plasma turbulence, as opposed to macroscopic or fluid turbulence.

The purpose of this book is to present essential ideas for treating the weak plasma turbulence on the basis of kinetic theory. In order to simplify the discussion, the plasma of interest is considered to be free of influence from externally applied electric or magnetic fields. The plasma is also considered as uniform on average, without spatial inhomogeneity associated with it. Despite these simplifying assumptions, basic theoretical tools and concepts developed in the present monograph can, in principle, be applied to situations in which the plasma is immersed in external fields, especially, the ambient magnetic field. Realistic laboratory or space/astrophysical plasmas are magnetized. Consequently, applications of the theory developed in this book are limited to high-frequency phenomena for which the approximation of field-free plasma may be valid.

The weak plasma turbulence theory is a perturbative nonlinear theory, and the essential formalism was developed by the pioneers of modern plasma physics beginning in the late 1950s and 1960s, and continued on through the early 1980s.
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It is the purpose of the present author to systematically overview various aspects of this venerable theory, but also certain new developments are included in the discussion.

This monograph comprises four parts. In Part I, the simplest kinetic theory is discussed, where it is assumed that the particles interact primarily through electrostatic force and without collisions. Chapter 1 formulates the perturbative kinetic theory based upon the relatively simple Vlasov–Poisson equation. Chapter 1 establishes conceptual foundations for a statistical description of plasmas, and also includes a detailed discussion of various linear and nonlinear response functions. Chapter 2 takes the formal results developed in Chapter 1 in a further direction, and derives the wave kinetic equation that describes weakly turbulent plasma processes. Chapter 3 derives the corresponding particle kinetic equations for electrons and ions interacting with the waves.

In Part II, the discrete particle effects are discussed within the framework of electrostatic approximation. The conceptual approach is based upon the Klimontovich equation, hence Part II is entitled “Klimontovich Weak Turbulence Theory.” Chapter 4 revisits the perturbative nonlinear theory already discussed in earlier chapters, but additional effects that arise from particle discreteness are incorporated. The result is a weak turbulence theory in which various processes are expressed in the balanced form between the so-called spontaneous and induced processes. Chapter 5 considers applications. The first example is on the spontaneous emission of electrostatic fluctuations in thermal equilibrium plasma. The second application relates to plasmas slightly out of equilibrium, where collisional relaxation processes bring such plasmas back to thermal equilibrium. In Chapter 6, further application is made to a situation where collisional processes are not sufficient to relax the system to equilibrium, but rather the relaxation is achieved via excitation of instability. The sample problem considered involves an electron beam interacting with the background plasma, leading to an excitation of the so-called Langmuir turbulence. Numerical solutions of the basic equations show that the long-time evolution of electron beam-plasma instability involves the formation of a nonthermal population in the electron velocity distribution function. Chapter 6 closes with the discussion of the time asymptotic state of the plasma turbulence. Relevance to the near-Earth space environment is also discussed.

Part III returns to the collision-free problem, but it generalizes Part I to fully electromagnetic situation. Chapter 7 formulates the general framework of Vlasov–Maxwell weak turbulence theory, and also discusses the electromagnetic linear and nonlinear response functions. Chapter 8 derives the specific equations of electromagnetic Vlasov weak turbulence theory for plasma normal modes. In Parts III and IV where theories concerning the electromagnetic weak turbulence formalism
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are discussed, various intermediate steps are deliberately included so that the readers may be able to retrace the derivations of various mathematical formulae.

Part IV generalizes Part III by adding the particle discreteness effects, and it also generalizes Part II by including electromagnetic effects in the Klimontovich weak turbulence formalism, thus bringing all the necessary ingredients together, and completing the generalized formalism of weak turbulence theory. Chapter 9 presents the general formalism, followed by Chapter 10, where we make specific applications, first to the issue of spontaneous emission of magnetic field fluctuations in thermal plasmas, and the radiation emission in thermal plasma. We also discuss the problem of relativistic collisional kinetic equation. Finally, we apply the electromagnetic “Klimontovich” weak turbulence formalism to the problem of plasma emission, which is a physical process that is intimately related to the solar radio bursts phenomena. The backdrop on this problem is also discussed. Various topics and supplemental discussions are included in the appendices.