

Contents

Introduction	1
Prerequisites from Logic and Analysis	6
Part I. Computability in Classical Analysis	9
<i>Chapter 0. An Introduction to Computable Analysis</i>	<i>11</i>
Introduction	11
1. Computable Real Numbers	13
2. Computable Sequences of Real Numbers	17
3. Computable Functions of One or Several Real Variables	24
4. Preliminary Constructs in Analysis	28
5. Basic Constructs of Analysis	33
6. The Max-Min Theorem and the Intermediate Value Theorem	40
7. Proof of the Effective Weierstrass Theorem	44
<i>Chapter 1. Further Topics in Computable Analysis</i>	<i>50</i>
Introduction	50
1. C^n Functions, $1 \leq n \leq \infty$	51
2. Analytic Functions	59
3. The Effective Modulus Lemma and Some of Its Consequences	64
4. Translation Invariant Operators	69
Part II. The Computability Theory of Banach Spaces	75
<i>Chapter 2. Computability Structures on a Banach Space</i>	<i>77</i>
Introduction	77
1. The Axioms for a Computability Structure	80
2. The Classical Case: Computability in the Sense of Chapter 0	82
3. Intrinsic L^p -computability	83

X	Contents
4. Intrinsic I^p -computability	85
5. The Effective Density Lemma and the Stability Lemma	85
6. Two Counterexamples: Separability Versus Effective Separability and Computability on $L^\infty[0, 1]$	88
7. Ad Hoc Computability Structures	90
<i>Chapter 3. The First Main Theorem and Its Applications</i>	93
Introduction	93
1. Bounded Operators, Closed Unbounded Operators	96
2. The First Main Theorem	101
3. Simple Applications to Real Analysis	104
4. Further Applications to Real Analysis	108
5. Applications to Physical Theory	115
Part III. The Computability Theory of Eigenvalues and Eigenvectors	121
<i>Chapter 4. The Second Main Theorem, the Eigenvector Theorem, and Related Results</i>	123
Introduction	123
1. Basic Notions for Unbounded Operators, Effectively Determined Operators	125
2. The Second Main Theorem and Some of Its Corollaries	128
3. Creation and Destruction of Eigenvalues	130
4. A Non-normal Operator with a Noncomputable Eigenvalue	132
5. The Eigenvector Theorem	133
6. The Eigenvector Theorem, Completed	139
7. Some Results for Banach Spaces	142
<i>Chapter 5. Proof of the Second Main Theorem</i>	149
Introduction	149
1. Review of the Spectral Theorem	151
2. Preliminaries	158
3. Heuristics	166
4. The Algorithm	175
5. Proof That the Algorithm Works	177
6. Normal Operators	184
7. Unbounded Self-Adjoint Operators	187
8. Converses	188
Addendum: Open Problems	192
Bibliography	195
Subject Index	201