

Notation Index

Introduction

$\{\cdots\}$	set of elements 3
$\{\cdots \cdots\}$	set defined by property 3
$\in$	is an element of 3
$\notin$	is not an element of 3
$\subseteq$	is a subset of 3
$\not\subseteq$	is not a subset of 3
$\subset$	is a proper subset of 3
$\not\subset$	is not a proper subset of 3
$\cup$	union 3
$\cap$	intersection 3
$-$	difference 3
Δ	symmetric difference 3
$\max(A)$	maximum of A 3
$\min(A)$	minimum of A 3
$\times$	cartesian product 3
$\langle \cdots \rangle$	tuple 3
A^k	k th power of A 3
$A^{<\omega}$	set of finite sequences from A 3
$\bar{x}^{[k]}$	k th coordinate of x 3
$S^{[k]}$	k th column of S 3
$\emptyset$	empty set 3
$\mathbb{N}$	set of natural numbers 3
$\oplus$	direct sum 3
$ A $	cardinality of A 3
$\aleph_0, \aleph_1$	cardinal numbers 3
$2^{\aleph_0}$	cardinality of continuum 3
$\varphi: A \rightarrow B$	function notation 3
$\downarrow$	converges 3
$\uparrow$	diverges 3
$x \mapsto \varphi(x)$	x is mapped to $\varphi(x)$ 3
$\text{dom}(\varphi)$	domain of φ 3
$\text{rng}(\varphi)$	range of φ 3
χ_S	characteristic function of S 3
$\upharpoonright$	restriction 3
$\lim_s$	limit 4
$\limsup_s$	limit supremum 4
$\liminf_s$	limit infimum 4
$\lambda x f(x, y)$	lambda notation 4
$\subseteq$	extension for functions 4
2^S	power set of S 4
$[\cdots]$	interval notation 4
$(\cdots)$	interval notation 4
$[\cdots)$	interval notation 4

$(\dots]$	interval notation 4
(a, ∞)	interval notation 4
$(-\infty, a]$	interval notation 4
$\&$	and 4
$\vee$	or 4
$\sim, \neg$	not 4
$\rightarrow, \Rightarrow$	implies 4
$\leftrightarrow, \Leftrightarrow$	if and only if 4
$\exists$	existential quantifier 4
$\forall$	universal quantifier 4
$\bigwedge_{i=0}^n \sigma_i$	finite conjunction 4
$\bigvee_{i=0}^n \sigma_i$	finite disjunction 4

Chapter I

μ	least number operator 7
$\mathcal{R}$	class of recursive functions 7
$\mathcal{P}$	class of partial recursive functions 9
$\mathcal{S}$	space of strings 10
$\mathcal{S}_f$	space of f -valued strings 10
$\mathcal{S}_c$	space of strings with values $\leq c$ 10
$\mathcal{S}_2$	space of binary strings 10
$\subseteq$	extension for strings 10
$\text{lh}(\sigma)$	length of string 10
$\sigma * \tau$	concatenation of strings 10
$\mathcal{R}_f$	functions recursive in f 11
$\leq_T$	Turing reducibility 11
χ_R	characteristic function of relation 11
φ	enumeration function 12
$\Phi^\theta, \Phi_e^\theta, \Phi^A, \Phi_e^A$	enumeration functionals 13

Chapter II

$\mathbf{f}, \mathbf{A}$	degree of f, A 15
$\equiv_T$	Turing equivalence 15
$\mathbf{D}$	degrees of unsolvability 15
$\leq$	partial ordering on $\mathbf{D}$ 15
$\cup$	join operation on $\mathbf{D}$ 15
$\mathcal{D}$	poset of degrees 15
$\mathcal{D}\mathcal{U}$	usl of degrees 15
$\mathbf{0}$	degree of recursive functions 15
$\cup \{\mathbf{a}_i\}$	lub of finite set of degrees 15
$\cap \{\mathbf{a}_i\}$	glb of finite set of degrees 15
$\mathbf{a} \mid \mathbf{b}$	incomparability for degrees 17
$p < q$	p refines q 19
$p \mid q$	incompatibility for forcing conditions 19
$\models$	satisfies 21
$\Vdash$	forces 21
$\hookrightarrow$	embedding 23
$\simeq$	isomorphism 23
$\theta^{[F]}$	union of columns of θ 24
$\theta^{[j]}$	union of all but one column of θ 24

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$\exists_n, \forall_n$	classes of formulas of bounded complexity 25
$\text{Th}(\mathcal{D})$	elementary theory of $\mathcal{D}$ 25
$\subseteq$	substructure 27

Chapter III

$\Sigma_n^0, \Pi_n^0, \Delta_n^0$	levels of arithmetical hierarchy 35
f'	completion of f 39
$\mathbf{f}'$	jump of f 39
f^*	set recursively isomorphic to f' 39
$f^{(n)}$	n th completion of f 39
$\mathbf{f}^{(n)}$	n th jump of f 39
$\mathbf{D}[a, b]$	interval notation for $\mathbf{D}$ 41
$\mathcal{D}[a, b]$	interval notation for $\mathcal{D}$ 41
$\mathbf{J}[0, 0']$	range of jump operator on $\mathbf{D}[0, 0']$ 46
$\mathcal{D}\mathcal{U}'$	usl of degrees with jump 50
$\mathcal{D}'$	poset of degrees with jump 50
α^{-1}	inverse of α 61
$\mathbf{L}_n, \mathbf{H}_n, \mathbf{I}$	levels of high/low hierarchy 62, 63

Chapter IV

$\text{high}_n, \text{low}_n$	classification in high/low hierarchy 75
$\mathbf{a}\text{-high}_n, \mathbf{a}\text{-low}_n$	classification in relativized high/low hierarchy 75
$\mathbf{L}_n(\mathbf{a}), \mathbf{H}_n(\mathbf{a}), \mathbf{I}(\mathbf{a})$	levels of relativized high/low hierarchy 75
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$\mathbf{GL}_n(\mathbf{a}), \mathbf{GH}_n(\mathbf{a}), \mathbf{GI}(\mathbf{a})$	levels of relativized generalized high/low hierarchy 77
Tot	index set of total recursive functions 84
$\text{Tot}(f)$	index set of total functions computable from f 84

Chapter V – XII

$\sigma \upharpoonright \tau$	incompatibility for strings 101
Id	identity tree 102, 126, 150, 199
Ext	extension subtree 102, 126, 150, 230
Sp	splitting subtree 106, 135, 155, 235
Tot	e -total subtree 109, 128, 151
Nar	narrow subtree 111
Pt	pointed subtree 112, 151
$\mathbf{D}_{\text{arith}}$	arithmetical degrees 113
$\equiv$	elementary equivalence 116
$\mathbf{0}^{(\omega)}$	ω -jump of $\mathbf{0}$ 117
$\equiv_j$	congruence for n -tuples 122, 124
$\subseteq_u$	uniform subtree 126
$\sigma^{(i)}$	i th coordinate of string of tuples 127
$\text{tr}(\sigma \rightarrow \tau; \rho)$	transfer strings 127
Diff	differentiating subtree 127, 151, 231
Tr	transfer tree 140
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D_I^*	integers in model of second order arithmetic 170
$\mathbf{0}^{(\omega + \omega)}$	iterate of jump operation 172
PExt	partial extension tree 184, 204

PDiff	partial differentiating tree 184, 204
PSp	partial splitting tree 185, 204, 213
PTot	partial e -total tree 185
$E(\lambda)$	e -state function 215
0_s	string of s consecutive zeroes 219
$I_{\sigma, \tau}$	interval notation 222
$ht(T)$	height of T 223
$s(\xi), p(\xi)$	successor, predecessor of ξ 229
Init	initial tree 229

Appendices

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