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John T. Baldwin works in the Department of Mathematics, Statistics and Computer Science at the University of Illinois, Chicago.

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PERSPECTIVES IN LOGIC

Fundamentals of Stability Theory

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ASSOCIATION FOR SYMBOLIC LOGIC

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To Sharon, Katie,
and my parents

Preface to the Series

Perspectives in Mathematical Logic

(Edited by the Ω-group for "Mathematische Logik" of the Heidelberger Akademie der Wissenschaften)

On Perspectives. *Mathematical logic arose from a concern with the nature and the limits of rational or mathematical thought, and from a desire to systematise the modes of its expression. The pioneering investigations were diverse and largely autonomous. As time passed, and more particularly in the last two decades, interconnections between different lines of research and links with other branches of mathematics proliferated. The subject is now both rich and varied. It is the aim of the series to provide, as it were, maps of guides to this complex terrain. We shall not aim at encyclopaedic coverage; nor do we wish to prescribe, like Euclid, a definitive version of the elements of the subject. We are not committed to any particular philosophical programme. Nevertheless we have tried by critical discussion to ensure that each book represents a coherent line of thought; and that, by developing certain* themes, *it will be of greater interest than a mere assemblage of results and techniques.*

The books in the series differ in level: some are introductory, some highly specialised. They also differ in scope: some offer a wide view of an area, others present a single line of thought. Each book is, at its own level, reasonably self-contained. Although no book depends on another as prerequisite, we have encouraged authors to fit their book in with other planned volumes, sometimes deliberately seeking coverage of the same material from different points of view. We have tried to attain a reasonable degree of uniformity of notation and arrangement. However, the books in the series are written by individual authors, not by the group. Plans for books are discussed and argued about at length. Later, encouragement is given and revisions suggested. But it is the authors who do the work; if, as we hope, the series proves of value, the credit will be theirs.

History of the Ω-Group. *During 1968 the idea of an integrated series of monographs on mathematical logic was first mooted. Various discussions led to a meeting at Oberwolfach in the spring of 1969. Here the founding members of the group (R.O. Gandy, A. Levy, G.H. Müller, G. Sacks, D.S. Scott)* *discussed the project in earnest and decided to go ahead with it. Professor F.K. Schmidt and Professor Hans Hermes gave us encouragement and support. Later Hans Hermes joined the group. To begin with all was fluid. How ambitious should we be? Should we write the books ourselves? How long would it take? Plans for authorless books were promoted, savaged and scrapped. Gradually there emerged a form and a method. At the end of an infinite discussion we found our name, and that of the series. We established our centre in Heidelberg. We agreed to meet twice a year together with authors, consultants and assistants, generally in Oberwolfach. We soon found the value of collaboration: on the one hand the permanence of the founding group gave coherence to the overall plans; on the other hand the stimulus of new contributors kept, the project alive and flexible. Above all, we found how intensive discussion could modify the authors' ideas and our own. Often the battle ended with a detailed plan for a better book which the author was keen to write and which would indeed contribute a perspective.*

Oberwolfach, September 1975

Acknowledgements. *In starting our enterprise we essentially were relying on the personal confidence and understanding of Professor Martin Barner of the Mathematisches Forschungsinstitut Oberwolfach, Dr. Klaus Peters of Springer-Verlag and Dipl.-Ing. Penschuck of the Stiftung Volkswagenwerk. Through the Stiftung Volkswagenwerk we received a generous grant (1970–1973) as an initial help which made our existence as a working group possible.*

Since 1974 the Heidelberger Akademie der Wissenschaften (Mathematisch-Naturwissenschaftliche Klasse) has incorporated our enterprise into its general scientific program. The initiative for this step was taken by the late Professor F.K. Schmidt, and the former President of the Academy, Professor W. Doerr.

Through all the years, the Academy has supported our research project, especially our meetings and the continuous work on the Logic Bibliography, in an outstandingly generous way. We could always rely on their readiness to provide help wherever it was needed.

Assistance in many various respects was provided by Drs. U. Felgner and K. Gloede (till 1975) and Drs. D. Schmidt and H. Zeitler (till 1979). Last but not least, our indefatigable secretary Elfriede Ihrig was and is essential in running our enterprise.

We thank all those concerned.

Heidelberg, September 1982

R. O. Gandy *H. Hermes*
A. Levy *G.H. Müller*
G. Sacks *D.S. Scott*

Acknowledgements

The subject of this book was created by Michael Morley and Saharon Shelah. As anyone familiar with the area could easily guess, results which are not explicitly credited elsewhere are due to Shelah. I thank, Alistair Lachlan not only for introducing me to the area but for his own seminal contributions. The development of this book has been mightily influenced by the French school, notably the work of Lascar and Poizat. The discussion of the spectrum problem depended not only on the original papers of Shelah but from the thorough reworking already accomplished by Harrington and Makkai.

During the preparation of this book the support and encouragement of G.H. Müller, the Ω group and the Heidelberg Academy have been invaluable. The ability to make legible early versions of the text and to cope with the ensuing multiple revisions arose only from the generous computer facilities of the University of Illinois at Chicago. I must particularly thank George Yanos and Richard Larson for their efforts in making TEX available. Work proceeded in Jerusalem for one year with the support of Institute for Advanced Studies of the Hebrew University. Many of the final preparations were made on a computer at Carnegie Mellon University in Pittsburgh. My thanks to Dana Scott for making these arrangements. The Minio and Manders-Boelema families are thanked for their hospitality. Roberto Minio translated my unformatted TEX into a file which produces acceptable output. I greatly appreciate his generosity and his patience with the interminable revisions.

For four summers I lectured one week to the ten to twenty participants in the 'Forking Festival' on the material written the previous year. The details of this work have benefitted immensely from the supervision of these model theorists. I do not have a list of all the participants. But I must thank first Bruce Rose for his help in beginning the process and some of the most persistent critics. These include Gisela Ahlbrandt, Steve Buechler, Andrew Glass, Bradd Hart, Ward Henson, Matt Kaufmann, Julia Knight, David Kueker, Doug Miller, Anand Pillay, Mike Prest, Charles Steinhorn, Bill Tait, and Carol Wood.

Finally, I want to thank those who at one time or another read over large sections of the text and provided me with detailed comments. The introduction benefitted from the criticism of Joel Berman, Wim Blok, Greg Cherlin, and Sol Feferman. Major commentators on the rest of the book include Steve Buechler, Greg Cherlin, Victor Harnik, Wilfrid Hodges, Matt Kaufmann, Jürgen Saffe, Charles Steinhorn, John Vaughn, and Martin Ziegler. They have not caught all the errors. But if I had a penny for each one they caught I could refuse the royalties.

My wife and daughter have borne up well in my absence. It will be nice to see them again.

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