

A General Algebraic Semantics for Sentential Logics Second Edition

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In this volume, the 7th publication in the Lecture Notes in Logic series, Font and Jansana develop a very general approach to the algebraization of sentential logics and present their results on a number of particular logics. The authors compare their approach, which uses abstract logics, to the classical approach based on logical matrices and the equational consequence developed by Blok, Czelakowski, Pigozzi and others. This monograph presents a systematized account of some of the work on the algebraic study of sentential logics carried out by the logic group in Barcelona in the 1970s.

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Second Edition

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