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        \[ r_t = \left\{ e^{-\kappa(t-t_0)} \right\} r_0 + \left\{ 1 - e^{-\kappa(t-t_0)} \right\} \theta + \int_{t_0}^t e^{-\kappa(t-s)} \sigma dz_s \] 181

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