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Jia-Ming Liu
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1 Basic Concepts of Optical Fields

1.1 NATURE OF LIGHT

Photonics addresses the control and use of light for various applications. Light is electromagnetic radiation of frequencies in the range from 1 THz to 10 PHz, corresponding to wavelengths between ~300 μm and ~30 nm in free space, which is generally divided into the infrared, visible, and ultraviolet regions. In this spectral region, the electromagnetic radiation exhibits the *dual nature of photon and wave*. The photon nature has to be considered in the generation, amplification, frequency conversion, or detection of light, whereas the wave nature is important in all processes but especially in the propagation, transmission, interference, modulation, or switching of light.

1.1.1 Photon Nature of Light

The energy of a photon is determined by its frequency ν or, equivalently, its angular frequency $\omega = 2\pi\nu$. Associated with its particle nature, a photon has a momentum determined by its wavelength λ or, equivalently, its *wavevector* $\mathbf{k}$. These characteristics are summarized below for a photon in free space:

speed	$c = \lambda\nu$;
energy	$h\nu = \hbar\omega = pc$;
momentum	$p = h\nu/c = h/\lambda, \quad \mathbf{p} = \hbar\mathbf{k}$.

The energy of a photon that has a wavelength of λ in free space can be calculated using the formula:

$$h\nu = \frac{1.2398}{\lambda}\ \mu\text{m eV} = \frac{1239.8}{\lambda}\ \text{nm eV}. \tag{1.1}$$

The photon energy at the optical wavelength of 1 μm is 1.2398 eV, and its frequency is 300 THz.

EXAMPLE 1.1

The visible spectrum ranges from 700 nm wavelength at the red end to 400 nm wavelength at the violet end. What is the frequency range of the visible spectrum? What are the energies of visible photons?

Solution:
The 700 nm optical wavelength at the red end has a frequency of

$$\nu_{\text{red}} = \frac{c}{\lambda_{\text{red}}} = \frac{3 \times 10^8 \text{ m s}^{-1}}{700 \text{ nm}} = 429 \text{ THz}$$

and a photon energy of

$$h\nu_{\text{red}} = \frac{1239.8}{\lambda_{\text{red}}} \text{ nm eV} = \frac{1239.8}{700} \text{ eV} = 1.77 \text{ eV}.$$

The 400 nm optical wavelength at the violet end has a frequency of

$$\nu_{\text{violet}} = \frac{c}{\lambda_{\text{violet}}} = \frac{3 \times 10^8 \text{ m s}^{-1}}{400 \text{ nm}} = 750 \text{ THz}$$

and a photon energy of

$$h\nu_{\text{violet}} = \frac{1239.8}{\lambda_{\text{violet}}} \text{ nm eV} = \frac{1239.8}{400} \text{ eV} = 3.10 \text{ eV}.$$

Therefore, the frequency range of the visible spectrum is from 429 THz to 750 THz. Visible photons have energies in the range from 1.77 eV to 3.10 eV.

The energy of a photon is determined only by its frequency or, equivalently, by its free-space wavelength, but not by the light intensity. The intensity, I, of monochromatic light is related to the *photon flux density*, or the number of photons per unit time per unit area, by

$$\text{photon flux density} = \frac{I}{h\nu} = \frac{I}{\hbar\omega}.$$

The *photon flux*, or the number of photons per unit time, of a monochromatic optical beam is related to the beam power P by

$$\text{photon flux} = \frac{P}{h\nu} = \frac{P}{\hbar\omega}.$$

EXAMPLE 1.2
Find the photon flux of a monochromatic optical beam that has a power of $P = 1$ W by taking its wavelength at either end of the visible spectrum. What are the momentum carried by a red photon and the momentum carried by a violet photon? What is the total momentum carried by the beam in a time duration of $\Delta t = 1$ s?

Solution:

From Example 1.1, the photon energy of the 700 nm wavelength at the red end is $h\nu_{\text{red}} = 1.77$ eV, and that of the 400 nm wavelength at the violet end is $h\nu_{\text{violet}} = 3.10$ eV. Therefore, the photon flux of a beam that has a power of $P = 1$ W at the 700 nm red wavelength is

$$\text{red photon flux} = \frac{P}{h\nu_{\text{red}}} = \frac{1}{1.77 \times 1.6 \times 10^{-19}}\ \text{s}^{-1} = 3.53 \times 10^{18}\ \text{s}^{-1},$$

and the photon flux of a beam that has a power of $P = 1$ W at the 400 nm violet wavelength is

$$\text{violet photon flux} = \frac{P}{h\nu_{\text{violet}}} = \frac{1}{3.10 \times 1.6 \times 10^{-19}}\ \text{s}^{-1} = 2.02 \times 10^{18}\ \text{s}^{-1}.$$

The momentum carried by a red photon is

$$p_{\text{red}} = \frac{h\nu_{\text{red}}}{c} = \frac{1.77 \times 1.6 \times 10^{-19}}{3 \times 10^{8}}\ \text{N s} = 9.44 \times 10^{-28}\ \text{N s},$$

and that carried by a violet photon is

$$p_{\text{violet}} = \frac{h\nu_{\text{violet}}}{c} = \frac{3.10 \times 1.6 \times 10^{-19}}{3 \times 10^{8}}\ \text{N s} = 1.65 \times 10^{-27}\ \text{N s}.$$

The total momentum carried by an optical beam that has a power of P during a time duration of Δt is independent of the optical wavelength:

$$\text{total momentum} = (\text{photon flux})p\Delta t = \frac{P}{h\nu} \cdot \frac{h\nu}{c}\Delta t = \frac{P\Delta t}{c}.$$

Therefore, irrespective of whether the wavelength of the beam is at the red or the violet end, the total momentum carried by the beam in a time duration of $\Delta t = 1$ s is

$$\text{total momentum} = \frac{P\Delta t}{c} = \frac{1 \times 1}{3 \times 10^{8}} = 3.33 \times 10^{-9}\ \text{N}.$$

1.1.2 Wave Nature of Light

An optical wave is characterized by the space and time dependence of the optical field, which is composed of coupled electric and magnetic fields governed by Maxwell's equations. It varies with time at an optical carrier frequency, and it propagates in a spatial direction determined by a wavevector. The behavior of an optical wave is strongly dependent on the optical properties of the medium. An optical field is a vectorial field characterized by five parameters: polarization, magnitude, phase, wavevector, and frequency. Polarization and wavevector are vectorial quantities; magnitude, frequency, and phase are scalar quantities. The general properties of optical fields are described in the following sections.

1.2 OPTICAL FIELDS AND MAXWELL'S EQUATIONS

An electromagnetic field in a medium is characterized by four vectorial fields:

electric field	$\boldsymbol{E}(\mathbf{r}, t)$	$\mathrm{V\,m^{-1}}$,
electric displacement	$\boldsymbol{D}(\mathbf{r}, t)$	$\mathrm{C\,m^{-2}}$,
magnetic field	$\boldsymbol{H}(\mathbf{r}, t)$	$\mathrm{A\,m^{-1}}$,
magnetic induction	$\boldsymbol{B}(\mathbf{r}, t)$	T or $\mathrm{Wb\,m^{-2}}$.

The response of a medium to an electromagnetic field generates the polarization and the magnetization:

polarization (*electric polarization*)	$\boldsymbol{P}(\mathbf{r}, t)$	$\mathrm{C\,m^{-2}}$,
magnetization (*magnetic polarization*)	$\boldsymbol{M}(\mathbf{r}, t)$	$\mathrm{A\,m^{-1}}$.

The electric field $\boldsymbol{E}(\mathbf{r}, t)$ and the magnetic induction $\boldsymbol{B}(\mathbf{r}, t)$ are the macroscopic forms of the microscopic fields seen by the charge and current densities in the medium. The polarization $\boldsymbol{P}(\mathbf{r}, t)$ and the magnetization $\boldsymbol{M}(\mathbf{r}, t)$ are the macroscopically averaged densities of microscopic electric dipoles and magnetic dipoles that are induced by the presence of the electromagnetic field in the medium. These macroscopic forms are obtained by averaging over a volume that is small compared to the dimension of the optical wavelength but is large compared to the atomic dimension. The electric displacement $\boldsymbol{D}(\mathbf{r}, t)$ and the magnetic field $\boldsymbol{H}(\mathbf{r}, t)$ are macroscopic fields defined as

$$\boldsymbol{D}(\mathbf{r}, t) = \epsilon_0 \boldsymbol{E}(\mathbf{r}, t) + \boldsymbol{P}(\mathbf{r}, t), \tag{1.2}$$

and

$$\boldsymbol{H}(\mathbf{r}, t) = \frac{1}{\mu_0} \boldsymbol{B}(\mathbf{r}, t) - \boldsymbol{M}(\mathbf{r}, t), \tag{1.3}$$

where $\epsilon_0 \approx 1/36\pi \times 10^{-9}\ \mathrm{F\,m^{-1}} = 8.854 \times 10^{-12}\ \mathrm{F\,m^{-1}}$ is the *electric permittivity* of free space and $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$ is the *magnetic permeability* of free space. In addition to the induced charge density and current density that respectively generate electric dipoles and magnetic dipoles for $\boldsymbol{P}(\mathbf{r}, t)$ and $\boldsymbol{M}(\mathbf{r}, t)$, an independent charge or current density, or both, from external sources may exist:

charge density	$\rho(\mathbf{r}, t)$	$\mathrm{C\,m^{-3}}$,
current density	$\boldsymbol{J}(\mathbf{r}, t)$	$\mathrm{A\,m^{-2}}$.

The behavior of a space- and time-varying electromagnetic field in a medium is governed by space- and time-dependent macroscopic *Maxwell's equations*:

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \quad \text{Faraday's law;} \tag{1.4}$$

$$\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} + \boldsymbol{J}, \quad \text{Ampère's law;} \tag{1.5}$$

$$\nabla \cdot \boldsymbol{D} = \rho, \quad \text{Gauss's law, Coulomb's law;} \tag{1.6}$$

$$\nabla \cdot \boldsymbol{B} = 0, \quad \text{absence of magnetic monopoles.} \tag{1.7}$$

Note that Gauss's law in the form of (1.6) is equivalent to Coulomb's law because one can be derived from the other. The current and charge densities are constrained by the *continuity equation*:

$$\nabla \cdot \boldsymbol{J} + \frac{\partial \rho}{\partial t} = 0, \quad \text{conservation of charge.} \tag{1.8}$$

The total current density in an optical medium has two contributions: the polarization current from the bound charges of the medium and the current from free charge carriers, thus $\boldsymbol{J}_{\text{total}} = \boldsymbol{J}_{\text{bound}} + \boldsymbol{J}_{\text{free}}$. The free-carrier current has two possible origins, one from the response of the conduction electrons and holes of the medium to the optical field and the other from an external current source: $\boldsymbol{J}_{\text{free}} = \boldsymbol{J}_{\text{cond}} + \boldsymbol{J}_{\text{ext}}$. Both $\boldsymbol{J}_{\text{bound}}$ and $\boldsymbol{J}_{\text{cond}}$ are induced by the optical field; thus

$$\boldsymbol{J}_{\text{total}} = \boldsymbol{J}_{\text{bound}} + \boldsymbol{J}_{\text{free}} = \boldsymbol{J}_{\text{bound}} + \boldsymbol{J}_{\text{cond}} + \boldsymbol{J}_{\text{ext}} = \boldsymbol{J}_{\text{ind}} + \boldsymbol{J}_{\text{ext}}, \tag{1.9}$$

where $\boldsymbol{J}_{\text{ind}} = \boldsymbol{J}_{\text{bound}} + \boldsymbol{J}_{\text{cond}}$. Similarly, the total charge density can be decomposed as

$$\rho_{\text{total}} = \rho_{\text{bound}} + \rho_{\text{free}} = \rho_{\text{bound}} + \rho_{\text{cond}} + \rho_{\text{ext}} = \rho_{\text{ind}} + \rho_{\text{ext}}. \tag{1.10}$$

In an optical medium, charge conservation requires that an increase of charge density induced by an optical field at a location is always accompanied by a reduction at another location, resulting in no net macroscopic induced charge density. Therefore, $\rho_{\text{ind}} = 0$ and $\rho_{\text{total}} = \rho_{\text{ext}}$ for a macroscopic optical field. By contrast, an induced macroscopic current density of $\boldsymbol{J}_{\text{ind}} \neq 0$ can exist in an optical medium.

In an optical medium that is free of external sources, $\boldsymbol{J}_{\text{ext}} = 0$ and $\rho_{\text{total}} = \rho_{\text{ext}} = 0$, but $\boldsymbol{J}_{\text{total}} = \boldsymbol{J}_{\text{bound}} + \boldsymbol{J}_{\text{cond}} = \boldsymbol{J}_{\text{ind}} \neq 0$. Both $\boldsymbol{J}_{\text{bound}}$ and $\boldsymbol{J}_{\text{cond}}$ are induced currents in response to an optical field. The bound-electron polarization current $\boldsymbol{J}_{\text{bound}}$ is a displacement current that is always included in the $\partial \boldsymbol{D}/\partial t$ term but not in the $\boldsymbol{J}$ term in (1.5). The conduction current $\boldsymbol{J}_{\text{cond}}$ is also an induced current, but it is carried by free charge carriers in the medium. In the case when both external current and external charge are absent, the form of Maxwell's equations depends on how the conduction current is treated. There are generally two alternatives.

1. Being an induced current, $\boldsymbol{J}_{\text{cond}}$ can be considered as a displacement current to be included in the $\partial \boldsymbol{D}/\partial t$ term so that $\boldsymbol{J} = 0$ in (1.5). Then, Maxwell's equations are

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \tag{1.11}$$

$$\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t}, \tag{1.12}$$

$$\nabla \cdot \boldsymbol{D} = 0, \tag{1.13}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{1.14}$$

where $\boldsymbol{D}$ is the electric displacement that includes optical-field-induced responses from all bound and conduction charges in the medium.

2. Being a current carried by free charge carriers, $\boldsymbol{J}_{\text{cond}}$ can be separated from the $\partial\boldsymbol{D}/\partial t$ term so that $\boldsymbol{J} = \boldsymbol{J}_{\text{cond}}$ in (1.5). Then, Maxwell's equations have the form:

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \tag{1.15}$$

$$\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}_{\text{bound}}}{\partial t} + \boldsymbol{J}_{\text{cond}}, \tag{1.16}$$

$$\nabla \cdot \boldsymbol{D}_{\text{bound}} = 0, \tag{1.17}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{1.18}$$

with $\nabla \cdot \boldsymbol{J}_{\text{cond}} = 0,$ where $\boldsymbol{D}_{\text{bound}}$ is the electric displacement that includes only the contribution from bound charges and excludes that from the conduction current.

These two alternative forms of Maxwell's equations are equivalent. The form using (1.16) is taken only when a specific effect of the conduction current is considered, as in Section 2.4. Otherwise, the form using (1.12) is generally taken. Therefore, we use the general form given in (1.11)–(1.14) unless the situation calls for specific attention to a conduction current.

1.2.1 Transformation Properties

Maxwell's equations and the continuity equation are the basic physical laws that govern the behavior of electromagnetic fields. They are invariant under the transformation of *space inversion*, in which the spatial vector $\mathbf{r}$ is changed to $\mathbf{r}' = -\mathbf{r},$ i.e., $\mathbf{r} \to -\mathbf{r},$ or $(x, y, z) \to (-x, -y, -z),$ and under the transformation of *time reversal*, in which the time variable t is changed to $t' = -t,$ i.e., $t \to -t$. This means that the form of these equations is not changed when we perform the space-inversion transformation or the time-reversal transformation, or both together.

The field quantities that appear in Maxwell's equations, however, do not have to be invariant under space inversion or time reversal. Their transformation properties are summarized as follows.

1. **Electrical fields**: The electric field vectors $\boldsymbol{E},$ $\boldsymbol{D}$, and $\boldsymbol{P}$ are *polar vectors* associated with the charge-density distribution. They change sign under space inversion but not under time reversal.
2. **Magnetic fields**: The magnetic field vectors $\boldsymbol{B}, \boldsymbol{H}$, and $\boldsymbol{M}$ are *axial vectors* associated with the current-density distribution. They change sign under time reversal but not under space inversion.
3. **Charge density**: The charge density ρ is a scalar. It does not change sign under either space inversion or time reversal.
4. **Current density**: The current density $\boldsymbol{J}$ is a polar vector that is the product of charge density and velocity: $\boldsymbol{J} = \rho\mathbf{v}$. It changes sign under either space inversion or time reversal following the sign change of the velocity vector under either transformation.

1.2.2 Optical Response of a Medium

Polarization and magnetization are generated in a medium by the response of the medium to the electric and magnetic fields, respectively: $\boldsymbol{P}(\mathbf{r},t)$ depends on $\boldsymbol{E}(\mathbf{r},t)$, and $\boldsymbol{M}(\mathbf{r},t)$ depends on $\boldsymbol{B}(\mathbf{r},t)$. *At an optical frequency, the magnetization vanishes*: $\boldsymbol{M}=0$. Therefore, it is always true for an optical field that

$$\boldsymbol{B}(\mathbf{r},t)=\mu_0\boldsymbol{H}(\mathbf{r},t). \tag{1.19}$$

Because μ_0 is a constant that is independent of the medium, the magnetic induction $\boldsymbol{B}(\mathbf{r},t)$ can be replaced by $\mu_0\boldsymbol{H}(\mathbf{r},t)$ for any equations that describe optical fields, including Maxwell's equations, thus effectively eliminating one field variable. Note that this is not true at DC or low frequencies, however, because a nonzero DC or low-frequency magnetization, $\boldsymbol{M}\neq 0$, can exist in any material. Indeed, it is possible to change the optical properties of a medium through a magnetization induced by a DC or low-frequency magnetic field, leading to the functioning of magneto-optics. It should be noted that even for magneto-optics, the magnetization is induced by a DC or low-frequency magnetic field that is separate from the optical field. No magnetization is induced by the magnetic component of the optical field.

The optical properties of a material are completely determined by the relation between $\boldsymbol{P}(\mathbf{r},t)$ and $\boldsymbol{E}(\mathbf{r},t)$. This relation is generally characterized by an *electric susceptibility tensor*, $\boldsymbol{\chi}$, through the following definition for electric polarization,

$$\boldsymbol{P}(\mathbf{r},t)=\epsilon_0\int_{-\infty}^{t}\iiint_{\text{all }\mathbf{r}'}\boldsymbol{\chi}(\mathbf{r}-\mathbf{r}',t-t')\cdot\boldsymbol{E}(\mathbf{r}',t')\mathrm{d}\mathbf{r}'\mathrm{d}t'. \tag{1.20}$$

The relation between $\boldsymbol{D}(\mathbf{r},t)$ and $\boldsymbol{E}(\mathbf{r},t)$ is characterized by the *electric permittivity tensor*, $\boldsymbol{\epsilon}$, of the medium:

$$\boldsymbol{D}(\mathbf{r},t)=\epsilon_0\boldsymbol{E}(\mathbf{r},t)+\boldsymbol{P}(\mathbf{r},t)=\int_{-\infty}^{t}\iiint_{\text{all }\mathbf{r}'}\boldsymbol{\epsilon}(\mathbf{r}-\mathbf{r}',t-t')\cdot\boldsymbol{E}(\mathbf{r}',t')\mathrm{d}\mathbf{r}'\mathrm{d}t'. \tag{1.21}$$

From (1.20) and (1.21), the relationship between $\boldsymbol{\chi}$ and $\boldsymbol{\epsilon}$ in the real space and time domain is

$$\boldsymbol{\epsilon}(\mathbf{r},t)=\epsilon_0[\delta(\mathbf{r})\delta(t)\mathbf{I}+\boldsymbol{\chi}(\mathbf{r},t)], \tag{1.22}$$

where $\mathbf{I}$ is the identity tensor that has the form of a 3×3 unit matrix and the delta functions are Dirac delta functions: $\iiint_{\text{all }\mathbf{r}}\delta(\mathbf{r})\mathrm{d}\mathbf{r}$ and $\int_{-\infty}^{\infty}\delta(t)\mathrm{d}t=1$. The relation in (1.22) indicates that $\boldsymbol{\chi}$ and $\boldsymbol{\epsilon}$ contain exactly the same information about the medium: one is known when the other is known.

Because $\boldsymbol{\chi}$ and, equivalently, $\boldsymbol{\epsilon}$ represent the response of a medium to an optical field and thus completely characterize the macroscopic electromagnetic properties of the medium, (1.20) and (1.21) can be regarded as the definitions of $\boldsymbol{P}(\mathbf{r},t)$ and $\boldsymbol{D}(\mathbf{r},t)$, respectively.

1.2.3 Boundary Conditions

At the interface of two media of different optical properties, as shown in Fig. 1.1, the optical field components must satisfy certain boundary conditions. These boundary conditions can be

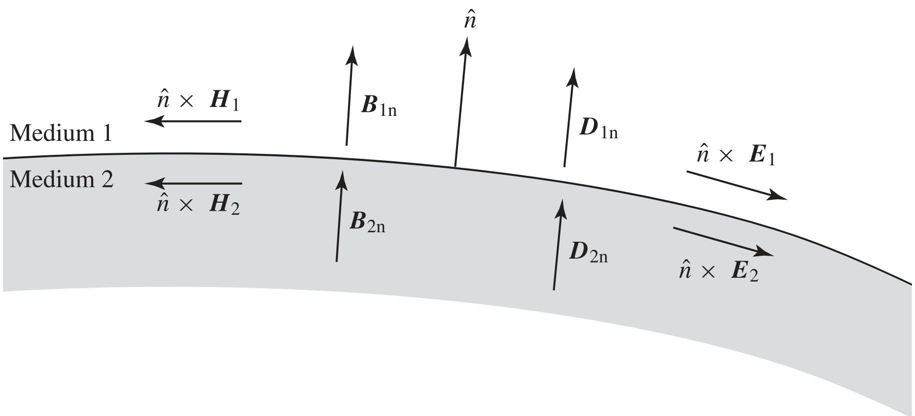

Figure 1.1 Boundary between two media of different optical properties.

derived from Maxwell's equations given in (1.11)–(1.14). From (1.11) and (1.12), the tangential components of the fields at the boundary satisfy

$$\hat{n} \times \boldsymbol{E}_1 = \hat{n} \times \boldsymbol{E}_2, \tag{1.23}$$

$$\hat{n} \times \boldsymbol{H}_1 = \hat{n} \times \boldsymbol{H}_2, \tag{1.24}$$

where $\hat{n}$ is the unit vector normal to the interface as shown in Fig. 1.1. From (1.13) and (1.14), the normal components of the fields at the boundary satisfy

$$\hat{n} \cdot \boldsymbol{D}_1 = \hat{n} \cdot \boldsymbol{D}_2, \tag{1.25}$$

$$\hat{n} \cdot \boldsymbol{B}_1 = \hat{n} \cdot \boldsymbol{B}_2. \tag{1.26}$$

The tangential components of $\boldsymbol{E}$ and $\boldsymbol{H}$ are continuous across an interface, while the normal components of $\boldsymbol{D}$ and $\boldsymbol{B}$ are continuous. Because $\boldsymbol{B} = \mu_0 \boldsymbol{H}$ at an optical frequency, as discussed above, (1.24) and (1.26) also imply that the tangential component of $\boldsymbol{B}$ and the normal component of $\boldsymbol{H}$ are also continuous. Consequently, *all of the magnetic field components in an optical field are continuous across a boundary. Possible discontinuities in an optical field exist only in the normal component of* $\boldsymbol{E}$ *or in the tangential component of* $\boldsymbol{D}$.

1.3 OPTICAL POWER AND ENERGY

Taking the dot product of $\boldsymbol{H}$ and (1.4) and that of $\boldsymbol{E}$ and (1.5) yields

$$\boldsymbol{H} \cdot (\nabla \times \boldsymbol{E}) = -\boldsymbol{H} \cdot \frac{\partial \boldsymbol{B}}{\partial t}, \tag{1.27}$$

$$\boldsymbol{E} \cdot (\nabla \times \boldsymbol{H}) = \boldsymbol{E} \cdot \frac{\partial \boldsymbol{D}}{\partial t} + \boldsymbol{E} \cdot \boldsymbol{J}. \tag{1.28}$$

Using the vector identity $\mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B}) = \nabla \cdot (\mathbf{A} \times \mathbf{B})$, (1.27) and (1.28) can be combined to give

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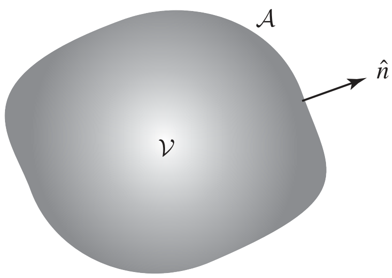

Figure 1.2 Boundary surface enclosing a volume element.

$$-\nabla\cdot(\boldsymbol{E}\times\boldsymbol{H})=\boldsymbol{E}\cdot\boldsymbol{J}+\boldsymbol{E}\cdot\frac{\partial\boldsymbol{D}}{\partial t}+\boldsymbol{H}\cdot\frac{\partial\boldsymbol{B}}{\partial t}. \tag{1.29}$$

Using (1.2) and (1.3) and rearranging (1.29), we obtain

$$\boldsymbol{E}\cdot\boldsymbol{J}=-\nabla\cdot(\boldsymbol{E}\times\boldsymbol{H})-\frac{\partial}{\partial t}\left(\frac{\epsilon_0}{2}|\boldsymbol{E}|^2+\frac{\mu_0}{2}|\boldsymbol{H}|^2\right)-\left(\boldsymbol{E}\cdot\frac{\partial\boldsymbol{P}}{\partial t}+\mu_0\boldsymbol{H}\cdot\frac{\partial\boldsymbol{M}}{\partial t}\right). \tag{1.30}$$

Recall that power in an electric circuit is given by voltage times current and has the unit of W = V A (watts = volts × amperes). Similarly, in an electromagnetic field $\boldsymbol{E}\cdot\boldsymbol{J}$ is the power density and has the unit of V A m^{-3}, or W m^{-3}. From (1.30), the total power dissipated by an electromagnetic field in a volume of $\mathcal{V}$ is simply the integral of $\boldsymbol{E}\cdot\boldsymbol{J}$ over the volume:

$$\int_{\mathcal{V}}\boldsymbol{E}\cdot\boldsymbol{J}\mathrm{d}v=-\oint_{\mathcal{A}}\boldsymbol{E}\times\boldsymbol{H}\cdot\hat{n}\mathrm{d}a-\frac{\partial}{\partial t}\int_{\mathcal{V}}\left(\frac{\epsilon_0}{2}|\boldsymbol{E}|^2+\frac{\mu_0}{2}|\boldsymbol{H}|^2\right)\mathrm{d}v-\int_{\mathcal{V}}\left(\boldsymbol{E}\cdot\frac{\partial\boldsymbol{P}}{\partial t}+\mu_0\boldsymbol{H}\cdot\frac{\partial\boldsymbol{M}}{\partial t}\right)\mathrm{d}v, \tag{1.31}$$

where the first term on the right-hand side is a surface integral over the closed surface $\mathcal{A}$ of the volume $\mathcal{V}$ and $\hat{n}$ is the outward-pointing unit normal vector of the surface, as shown in Fig. 1.2.

Each term in (1.31) has the unit of power, and each has an important physical meaning.

1. The vectorial quantity

$$\boldsymbol{S}=\boldsymbol{E}\times\boldsymbol{H} \tag{1.32}$$

is called the *Poynting vector* of the electromagnetic field. It represents the *instantaneous magnitude and direction of the power flow* of the field.

2. The scalar quantity

$$u_0=\frac{\epsilon_0}{2}|\boldsymbol{E}|^2+\frac{\mu_0}{2}|\boldsymbol{H}|^2 \tag{1.33}$$

has the unit of energy per unit volume and is the *energy density stored in the propagating field*. It consists of two components, thus accounting for energies stored in both electric and magnetic fields at any instant of time.

3. The last term in (1.31) also has two components associated with electric and magnetic fields, respectively. The quantity

$$W_{\mathrm{p}}=\boldsymbol{E}\cdot\frac{\partial\boldsymbol{P}}{\partial t} \tag{1.34}$$

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is the *power density expended by the electromagnetic field on the polarization*. It is the rate of energy transfer from the electromagnetic field to the medium on inducing the electric polarization in the medium. Similarly, the quantity

$$W_{\mathrm{m}} = \mu_0 \boldsymbol{H} \cdot \frac{\partial \boldsymbol{M}}{\partial t} \tag{1.35}$$

is the *power density expended by the electromagnetic field on the magnetization.*

With these physical meanings attached to the terms in (1.31), it can be seen that (1.31) simply states the law of conservation of energy in any arbitrary volume element $\mathcal{V}$ in the medium. The total electromagnetic energy in the medium equals that contained in the propagating field plus that stored in the electric and magnetic polarizations.

For an optical field, $\boldsymbol{E} \cdot \boldsymbol{J} = 0$ and $W_{\mathrm{m}} = 0$ because $\boldsymbol{J} = 0$ and $\boldsymbol{M} = 0$, as discussed above. Then, (1.31) becomes

$$-\oint_{\mathcal{A}} \boldsymbol{S} \cdot \hat{n} \mathrm{d}a = \frac{\partial}{\partial t} \int_{\mathcal{V}} u_0 \mathrm{d}v + \int_{\mathcal{V}} W_{\mathrm{p}} \mathrm{d}v, \tag{1.36}$$

which states that the total optical power flowing into volume $\mathcal{V}$ through its boundary surface $\mathcal{A}$ is equal to the rate of increase with time of the energy stored in the propagating fields in $\mathcal{V}$ plus the power transferred to the polarization of the medium in this volume.

1.4 WAVE EQUATION

By applying $\nabla \times$ to (1.11) and using (1.19) and (1.12), we obtain the *wave equation*:

$$\nabla \times \nabla \times \boldsymbol{E} + \mu_0 \frac{\partial^2 \boldsymbol{D}}{\partial t^2} = 0. \tag{1.37}$$

By using (1.2), the wave equation can be expressed as

$$\nabla \times \nabla \times \boldsymbol{E} + \frac{1}{c^2} \frac{\partial^2 \boldsymbol{E}}{\partial t^2} = -\mu_0 \frac{\partial^2 \boldsymbol{P}}{\partial t^2}, \tag{1.38}$$

where

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \approx 3 \times 10^8 \text{ m s}^{-1} \tag{1.39}$$

is the speed of light in free space.

The *wave equation* in (1.38) describes the space-and-time evolution of the electric field of the optical wave. Its right-hand side can be regarded as the driving source for the optical wave; that is, the polarization in a medium drives the evolution of an optical field. This wave equation can take on various forms depending on the characteristics of the medium, as will be seen on various occasions later. Here we leave it in this general form.