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Prelude

An Introduction to the Sorites Paradox

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Topic, Structure, Features and Aims of the Book

This book is about what is nowadays regarded as one of the most venerable and difficult philosophical paradoxes (Priest 2003, p. 9): the Sorites Paradox (in better English, 'Heaper Paradox', from ancient Greek *sōritēs* i.e. heaper, in turn from *sōros* i.e. heap). According to the historiographical tradition (e.g. Keefe and Smith 1997a, p. 3), the paradox was first formulated by Eubulides of Miletus, an eminent member of the Megarian school, a group of philosophers well known to have been under Eleatic influence (Chapter 15 supplies further historical background). The paradox does in fact target the concepts that we mostly use in describing the world *as it appears to our senses* – that is, those concepts (such as e.g. the concept of a heap) that categorise objects as falling on one side or the other of a *distinction that seems not to depend on small differences* – and does boldly attempt to show that such concepts are *incoherent*.

This introduction provides the tools necessary for understanding the Sorites Paradox itself as well as a first orientation concerning its solutions and influence. Part I offers a systematic survey of the main types of solutions to the paradox. Part II delves into the main areas where the paradox has exerted a profound influence. A coda recapitulates the pre-analytic history of the paradox vis-à-vis the state of the art exposed in the previous parts.

The book intends, on the one hand, to *take stock of the vertiginous developments* in thought about the Sorites Paradox that have taken place in the last half century (fn. 37) and, on the other hand, to *address some of the new challenges* that have arisen in the context of such developments. While a few recent excellent readers and collected volumes on vagueness exist (Keefe and Smith 1997b; Fara and Williamson 2002; Beall 2003; Dietz and Moruzzi 2010; Égré and Klinedinst 2011; Ronzitti 2011), from a more *structural* point of view the book constitutes an absolute novelty in several respects (which suits its inclusion in the series *Classic Philosophical Arguments*). First, the book focuses on the Sorites Paradox in particular rather than on vagueness in general.[1] Second, the book is so organised as to

[1] In this, the book reflects what has actually been an interesting tendency in the most recent literature on vagueness – witnessed e.g. by many contextualist works – which, in theorising about vagueness, variously prioritises the phenomenon of

provide a systematic treatment of the paradox, by devoting exactly one chapter to each main type of solution to the paradox and to each area where the paradox has exerted a profound influence. Third, each chapter is so conceived as to be accessible to novices in its topic, by gradually leading them from an introductory presentation of its general subject to a discussion of some open problems at the edge of contemporary research.[2] From a more *thematic* point of view, the book also constitutes a substantial novelty, in that it extensively covers fairly recent solutions (i.e. dialetheism and non-transitivism) and somewhat underrepresented areas (i.e. practical philosophy and psychology). Even for those solutions and areas that in general have already received sustained attention in previous publications, the relevant chapter not only provides the state of the art, but also pushes research further on that front, by putting forth new ideas on some selected issues.

Consisting of chapters written by first-rate experts, we hope that the book will significantly advance the debate on the Sorites Paradox, and will thus become essential reading for every researcher on the topic and, more generally, for many philosophers of logic. Moreover, given the book's overall systematic character and each chapter's gradual approach, we believe that it can constitute an excellent source for graduate courses covering vagueness, and that it should also appeal to members of the general educated public curious to find out what the Sorites Paradox is all about. Furthermore, we think that the deep connections, reflected in many of the chapters, that numerous strands of thought on the paradox have with fundamental issues in philosophy of language, epistemology and metaphysics (e.g. context dependence, indiscriminability and spatiotemporality) should make substantial parts of the book stimulating also for scholars specialising in those areas. We intend some chapters to attract an even wider audience, in that they illustrate the ways in which the paradox has acted as a powerful generator of ideas and insights also in the more remote areas of practical philosophy, linguistics and psychology. All in all, we expect that the book will be both a record of the fecund role played by the paradox in all these areas and a spur to further work for those doing research in them.

Characterisation and Extension of Vagueness

The Sorites Paradox arises in connection with the general phenomenon of vagueness, and so this must antecedently be introduced. Most expressions in natural languages are vague,[3] in the sense that, although they apply in some cases and do not apply in others, *they seem*

Sorites-susceptibility over that of borderline cases (e.g. Zardini 2008b, pp. 11–2). Consequently, this introduction itself offers a novel kind of presentation of the lay of the land, which forgoes use of the notion of a borderline case.

2 The overall book has been designed to be accessible to new graduate students, and only basic notions of philosophy and logic (as well as familiar terminological and notational conventions) are systematically presupposed, which can easily be recovered from any good introduction to philosophy and logic; given the prominence, in the topic, of philosophy of logic and of alternative, philosophically motivated logical systems, Priest (2008) is a particularly helpful source.

3 Arguably, vagueness attaches not only to *linguistic* representations, but also to *mental* ones, in particular to concepts (contrast with e.g. ambiguity, which would seem to attach only to expressions: plausibly, while a vague expression such as 'heap' corresponds to a single vague concept of a heap, an ambiguous expression such as 'bank' corresponds to the two non-ambiguous concepts of a river bank and of a money bank). Throughout, to fix ideas, we focus on expressions, sometimes switching to concepts when more natural.

to lack a sharp boundary between the former cases and the latter ones. For example, the noun 'heap' is vague: it applies to collections of many enough, say, grains, it does not apply to collections of too few grains, but it seems to lack a sharp boundary between those two kinds of collections – there seems to be no i[4] such that 'heap' applies to collections of i grains but does not apply to collections of $i - 1$ grains.[5]

Although nouns and adjectives are probably the two most paradigmatic linguistic categories of vague expressions, *virtually every linguistic category is affected by vagueness* (Chapter 13). Verbs can be vague, as witnessed e.g. by 'rain': it applies to precipitations of many enough, say, droplets, it does not apply to precipitations of too few droplets, but it seems to lack a sharp boundary between those two kinds of precipitations – there seems to be no i such that 'rain' applies to precipitations of i droplets but does not apply to precipitations of $i - 1$ droplets. Determiners can be vague, as witnessed e.g. by 'many': it applies to groups of many enough, say, people, it does not apply to groups of too few people, but it seems to lack a sharp boundary between those two kinds of groups – there seems to be no i such that 'many' applies to groups of i people but does not apply to groups of $i - 1$ people. Prepositions can be vague, as witnessed e.g. by 'by': it applies to pairs of, say, buildings that are close enough to each other, it does not apply to pairs of buildings that are too far from each other, but it seems to lack a sharp boundary between those two kinds of pairs – there seems to be no i such that 'by' applies to pairs of buildings that are at i yards from each other but does not apply to pairs of buildings that are at $i + 1$ (i' for short) yards from each other. Having noted all this, for concreteness we'll henceforth take as our leading example of vagueness 'bald', focusing on the construction 'A man with i hairs is bald' (which we'll often formalise as Bi). 'Bald' is also vague: it applies to men with 1 hair, it does not apply to men with 100,000 hairs,[6] but it seems to lack a sharp boundary between those two kinds of men – there seems to be no i such that 'bald' applies to men with i hairs but does not apply to men with i' hairs. The sense of 'vagueness' just introduced (i.e. *seeming lack of sharp boundaries between positive and negative cases*) is the one intended in the contemporary philosophical debate on vagueness. The reader is advised to keep it sharply distinct from other senses of 'vagueness' commonly found in ordinary discourse: 'vagueness' in the sense of ambiguity (i.e. the fact that the same syntactic string is associated with *different meanings*, as when one says that 'bank' is 'vague'), 'vagueness' in the sense of underspecificity (i.e. the fact that an expression as used in a certain context is *too general* for the informative purposes of the context, as when one says that 'Something' is a vague answer to the question 'What are you thinking?'), 'vagueness' in the sense of indeterminacy (i.e. the fact that an expression has *unsettled*

[4] Throughout, 'i' and its relatives range over the relevant set of natural numbers.

[5] 'Heap' as well as most expressions in natural languages is also multi-dimensional, in the sense that *there is more than one factor* ('dimension of comparison') relevant for its application: to wit, whether a collection of grains is a heap depends not only on their number, but also on e.g. the shape of their arrangement (a row of 100,000 grains on the floor is not a heap). Throughout, we exclusively focus on one single dimension of comparison presupposing that all the other ones are kept fixed from one case to the other (see Zardini 2018c for some discussion of the relation between seeming lack of sharp boundaries and multi-dimensionality).

[6] Background information: apparently, men with around 100,000 hairs are paradigmatically not bald.

application to certain cases, as when one says that it is vague whether 'Newtonian mass' applies to rest mass or relativistic mass).

The Sorites Paradox

The informal characterisation of vagueness given in 'Characterisation and Extension of Vagueness', in this introduction, is fairly minimal and neutral, thereby constituting the common starting point of theories of vagueness, which basically focus on providing a comprehensive account of the seeming lack of sharp boundaries of vague expressions. Now, a very natural such account consists in maintaining that what underlies the *seeming* lack of sharp boundaries is a *real* lack of sharp boundaries: vague expressions seem to lack sharp boundaries because they do indeed lack sharp boundaries! In turn, as already implicitly done in 'Characterisation and Extension of Vagueness', in this introduction, the informal notion of lack of sharp boundaries is very naturally understood as the *non-existence of a positive case immediately followed by a negative cases*: for example, on this understanding, B lacks sharp boundaries iff, for every i, it is not the case that Bi holds and Bi' does not.

It is this very natural account of the seeming lack of sharp boundaries of vague expressions that is apparently shattered by the Sorites Paradox. Focusing on 'bald', the account, which may be called 'the naive theory of vagueness' (Zardini 2008a, pp. 337–40), has it in effect that:

(N_1^{bald}) A man with 1 hair is bald;
(N_2^{bald}) A man with 100,000 hairs is not bald;
(N_{3LSB}^{bald}) For every i, it is not the case that a man with i hairs is bald and man with i' hairs is not bald

(these and similar principles obviously have analogues for other vague expressions, which we assume to be labelled by replacing superscript 'bald' with the relevant vague expression, so that e.g. (N_1^{heap}) is something like 'A collection of 100,000 grains is a heap'; further, we drop a superscript or subscript to refer to the totality of the corresponding principles got by adding a possible value for that superscript or subscript, so that e.g. (N_1) is the totality of the principles ((N_1^{bald}), (N_1^{heap}), (N_1^{rain}), etc.) that identify a paradigmatically positive case of a vague expression, and, at the limit, (N) is the totality of the principles of the naive theory). A prominent version of the Sorites Paradox consists then in apparently showing that (N_1^{bald}) and (N_{3LSB}^{bald}) entail the contradictory of (N_2^{bald}), so that (N^{bald}) would be inconsistent.[7] In detail, by universal instantiation,[8] (N_{3LSB}^{bald}) entails $\neg(B1 \,\&\, \neg B2)$, which, together with (N_1^{bald}), by *modus ponendo tollens*,[9] entails $\neg\neg B2$, which in turn, by double-negation elimination,[10] entails $B2$. We can now repeat this argumentative routine to get to $B3$:

[7] Throughout, by 'entailment' and its relatives we mean the converse of the relation of logical consequence (so that a sequence of sentences Γ entails a sentence φ iff φ logically follows from Γ), and by 'equivalence' and its relatives we mean two-way entailment. By 'imply' and its relatives we mean instead an operation expressed by an indicative conditional (so that φ implies ψ iff, if φ holds, so does ψ).
[8] Formally, letting as usual $\vdash$ be the relation of logical consequence and φ_{τ_0/τ_1} the result of replacing in φ all free occurrences of τ_1 with free occurrences of τ_0, the principle that $\forall\xi\varphi \vdash \varphi_{\tau/\xi}$ holds.
[9] Formally, the principle that $\varphi, \neg(\varphi \,\&\, \psi) \vdash \neg\psi$ holds.
[10] Formally, the principle that $\neg\neg\varphi \vdash \varphi$ holds.

by universal instantiation, $(\mathrm{N}_{3\mathrm{LSB}}^{bald})$ entails $\neg(B2 \,\&\, \neg B3)$, which, together with the just established conclusion $B2$, by *modus ponendo tollens* entails $\neg\neg B3$, which in turn, by double-negation elimination, entails $B3$. By repeating the same argumentative routine another 99,997 times, we eventually get to $B100{,}000$, i.e. the contradictory of (N_2^{bald}).

A closely related version of the Sorites Paradox comes into view by reflecting on a feature of vagueness closely related to seeming lack of sharp boundaries: namely, on the fact that vague expressions seem tolerant, in the sense that it seems that, *assuming that a vague expression applies to a certain case, it also applies to every case similar to it*. For example, it seems that, assuming that 'heap' applies to a collection of i grains, it also applies to collections of $i-1$, $i+1$, $i-2$, etc. grains. Theories of vagueness typically also include an account of the seeming tolerance of vague expressions. Now, a very natural such account consists in maintaining that what underlies *seeming* tolerance is *real* tolerance: vague expressions seem tolerant because they are indeed tolerant! In turn, the informal notion of tolerance is very naturally understood as *implication from a case's being positive to the immediately following case's being positive*: for example, on this understanding, B is tolerant iff, for every i, if Bi holds, so does Bi'.[11]

Let's thus understand (N) as containing not only $(\mathrm{N}_{3\mathrm{LSB}}^{bald})$, but also:

$(\mathrm{N}_{3\mathrm{T}}^{bald})$ For every i, if a man with i hairs is bald, so is a man with i' hairs.

Another prominent version of the Sorites Paradox consists then in apparently showing that (N_1^{bald}) and $(\mathrm{N}_{3\mathrm{T}}^{bald})$ entail the contradictory of (N_2^{bald}), so that (N^{bald}) would again be inconsistent. In detail (and mimicking closely the $(\mathrm{N}_{3\mathrm{LSB}}^{bald})$-based version of the paradox), by universal instantiation, $(\mathrm{N}_{3\mathrm{T}}^{bald})$ entails $B1 \rightarrow B2$, which, together with (N_1^{bald}), by *modus ponens*,[12] entails $B2$. We can now repeat this argumentative routine to get to $B3$: by universal instantiation, $(\mathrm{N}_{3\mathrm{T}}^{bald})$ entails $B2 \rightarrow B3$, which, together with the just established conclusion $B2$, by *modus ponens*, entails $B3$. By repeating the same argumentative routine another 99,997 times, we eventually get to $B100{,}000$, i.e. the contradictory of (N_2^{bald}).

Solutions to the Sorites Paradox

While there are other natural and interesting versions of the Sorites Paradox, the two ones reviewed in 'The Sorites Paradox', in this introduction, stand out for their theoretical centrality and historical salience. Now, according to philosophical tradition (e.g. Sainsbury 2009, p. 1), a paradox is a situation where *apparently true premises apparently entail an apparently false conclusion*,[13] and both the $(\mathrm{N}_{3\mathrm{LSB}})$-based and the $(\mathrm{N}_{3\mathrm{T}})$-based version of the paradox would seem paradigmatic instances of such understanding: (N_1) and $(\mathrm{N}_{3\mathrm{LSB}})$

[11] Notice that, to get to the essentials, in our understanding of tolerance we're focusing on the implication *from Bi to Bi′* rather than its converse (which is also involved in the informal notion of tolerance), since the latter is totally compelling and unproblematic. Notice also that our understanding of tolerance is equivalent with our understanding of lack of sharp boundaries only under the assumption that the implication at work in tolerance is material implication (i.e. such that $\varphi \rightarrow \psi$ is tantamount to $\neg(\varphi \,\&\, \neg\psi)$). But that assumption would seem rather perverse, as there would seem to be a world of difference between saying that *positive cases spread over small differences* and saying that *the difference between a positive case and a negative case is not small*.

[12] Formally, the principle that $\varphi, \varphi \rightarrow \psi \vdash \psi$ holds.

[13] Throughout, we understand falsity as truth of the negation (i.e. φ is false iff $\neg\varphi$ is true).

are apparently true and apparently entail (as per the apparently valid argument in 'The Sorites Paradox', in this introduction) the apparently false contradictory of (N_2); (N_1) and (N_{3T}) are apparently true and they too apparently entail (as per the apparently valid argument in 'The Sorites Paradox', in this introduction) the apparently false contradictory of (N_2).[14]

Keeping fixed such platitudes as (N_1) and (N_2),[15] and focusing on the (N_{3LSB})-based version of the Sorites Paradox, we are thus faced with the dilemma of either *rejecting (N_{3LSB})*[16] or *denying the validity of some of the logical principles employed by soritical reasoning*.[17] It's fair to say that the vast majority of solutions to the paradox take the former option (which is not surprising given how minimal the logical resources employed by soritical reasoning are!). On this option, since the soritical argument is valid, it is accepted that (N) is inconsistent, given which the argument can be extended by inferring, by a version of *reductio ad absurdum*,[18] that the negation of (N^{bald}_{3LSB}) holds, and so, by one of the quantified De Morgan laws,[19] that, for some i, both Bi and $\neg Bi'$ hold – i.e. that there is a number of hairs such that adding just one more hair to the scalp of a man with that number of hairs turns him from bald to non-bald! Thricologists will be thrilled. (Let's call the soritical argument as so extended 'Extended Sorites Paradox'.) Solutions to the Sorites Paradox taking the option of rejecting (N_{3LSB}) are thus faced with the dilemma of either *accepting that there is a sharp boundary* or *denying the validity of some of the further logical principles employed by extended soritical reasoning*. Since all those logical principles are valid in classical logic, solutions to the paradox in general are usefully classified as either *preserving classical logic and accepting that vague expressions have sharp boundaries* or *revising classical logic and [accepting (N_{3LSB}) or rejecting that vague expressions have sharp boundaries]* (see fn. 22 for some subtleties concerning the operative understanding of 'classical logic').

14 López de Sa and Zardini (2007, p. 246) criticise the traditional philosophical understanding of paradox as *too narrow*. Their broad line of attack can be applied also in the case of the Sorites Paradox. For example, consider two soritical arguments diverging from the (N_{3LSB})-based version of the Sorites Paradox only in that they stop at B10,000 and B50,000 respectively (rather than going up to B100,000): in both cases, despite the apparent validity of the argument, its premises apparently *do not support its conclusion* (it is apparently wrong, assuming the premises, to regard the argument as establishing the conclusion), thus engendering a real paradox – yet, B10,000 is *apparently true* (rather than apparently false) and B50,000 is *apparently uncertain* (rather than apparently false), thereby indicating that the traditional philosophical understanding of paradox is too narrow.

15 This being philosophy, actually there is also the nihilist option of giving up (N_1) and the trivialist option of giving up (N_2) (Chapters 4, 9 and 11). Given the extreme unattractiveness of such options, we'll henceforth set them aside (see Chapter 4, Section 4.6 and Chapter 11, Section 11.3 for some discussion of nihilism).

16 Throughout, by 'reject' and its relatives we mean *non-acceptance for principled reasons*. By 'deny' and its relatives we mean instead *acceptance of the negation* (in particular, we understand denial of the validity of a logical principle to be acceptance that the principle is not valid). The distinction between rejection and denial is crucial in many solutions to the Sorites Paradox, since, for some φ, some such solutions (supervaluationism, intuitionism, rejection of excluded middle, some versions of degree theory) reject both φ and $\neg\varphi$ (and so, while they reject φ, they do not deny it) whereas the other such solutions (subvaluationism, dialetheism, some other versions of degree theory) accept both φ and $\neg\varphi$ (and so, while they deny φ, they do not reject it). Notice that, for essentially the same reason, all such solutions rely on an analogous distinction between *a sentence's failing to hold* and *its negation's holding*.

17 Without going into details here, we should flag that there are actually more logical principles at work in soritical reasoning than those explicitly mentioned in our presentation in 'The Sorites Paradox', in this introduction (see eleven paragraphs further on for some indications and Chapter 9 for an extended presentation and discussion of such principles).

18 Formally, letting as usual $\Gamma \vdash \varnothing$ mean that Γ is inconsistent, the principle that, if $\Gamma, \varphi \vdash \varnothing$ holds, $\Gamma \vdash \neg\varphi$ holds.

19 Formally, the principle that $\neg\forall\xi\neg\varphi \vdash \exists\xi\varphi$ holds.

Let's start then our overview of solutions to the Sorites Paradox with the solutions that preserve classical logic and therefore accept that vague expressions have sharp boundaries. In general, because of their acceptance of classical logic, such solutions have a quick and easy answer to the paradox itself: they simply concede the soundness of the extended soritical argument. But, precisely because conceding such soundness requires conceding that vague expressions have sharp boundaries, such solutions are then faced with the formidable task of *providing a reasonable account of vagueness that coheres with such a concession*.

Epistemicism (Chapter 1)[20] accepts that vague expressions have sharp boundaries *in the most natural sense*: there really is a *particular* i – it could be, say, 50,000 – such that both Bi and $\neg Bi'$ hold, similarly to how, say, there really is a particular i – it could be, say, 50,000 – such that i straws (of a certain weight) do not break a (certain) camel's back (at a certain time) and i' straws do. But, if vague expressions have sharp boundaries, what does their vagueness consist in? According to epistemicism, their vagueness consists in certain distinctive epistemic features, the most prominent of which is *our distinctive inability to identify their sharp boundaries*, and much of epistemicist theorising is devoted to accounting for the nature and source of such an inability.

Supervaluationism (Chapter 2) accepts that vague expressions have sharp boundaries *only in a nominal sense*: while there is a sharp boundary, there really isn't any *particular* i such that both Bi and $\neg Bi'$ hold – it is absurd, say, that both $B50{,}000$ and $\neg B50{,}001$ hold – similarly to how, on a natural interpretation of the situation, while there is a 20-day period next year in which you'll be on holidays (so much is settled in your work contract), there really isn't any particular 20-day period next year in which you'll be on holidays (which one it is depends on your choice, which you still have to make). When it comes to baldness, no number is The Special One. But how can the existence of such-and-such float free of particular objects' being such-and-such? According to supervaluationism, the *structure* of baldness is settled: it is settled that the basic valuation scheme is classical,[21] that paradigmatic cases such as $B1$ and $\neg B100{,}000$ are true, that totally compelling and unproblematic principles such as $\forall i(Bi' \rightarrow Bi)$ are true, etc.[22] Therefore, enough is settled to make the existential claim $\exists i(Bi \,\&\, \neg Bi')$ true. However, for *every* i, it is *not settled* that the witness $Bi \,\&\, \neg Bi'$ to that claim is true, even though, because of what is settled, these

[20] Notice that, while the admittedly rather sloganised systematic chapter titles fit well with the taxonomical purposes of a companion, for every chapter the authors' personal take on and development of its topic will naturally sometimes transcend what might narrowly be understood under the chapter's title.

[21] Essentially (and assuming that every object has a name): every sentence is either true or false but not both; $\neg\varphi$ is true iff φ is false; $\varphi \,\&\, \psi$ is true iff both φ and ψ are true; $\varphi \vee \psi$ is true iff either φ or ψ are true; $\forall\xi\varphi$ is true iff, for every τ, $\varphi_{\tau/\xi}$ is true; $\exists\xi\varphi$ is true iff, for some τ, $\varphi_{\tau/\xi}$ is true.

[22] Solutions to the Sorites Paradox are possible where supervaluationist ideas and tools are employed but the classicality assumption is abandoned; still, since the key idea of supervaluationism is quite congenial to preservation of classical logic, we focus in our presentation on classical versions of supervaluationism. Similar comments apply to our presentation of contextualism and incoherentism in the next and second next paragraph respectively. The reader is warned that there is an issue (discussed e.g. by McGee and McLaughlin 2004, pp. 132–6) as to whether supervaluationism preserves all the principles that would fairly naturally be listed under the label 'classical logic' (and that it is indeed a merit of the rise of that issue to have underscored *just how fuzzy the boundaries of 'classical logic' are*). However, for our purposes, a natural understanding of 'classical logic' is one as of a logic that basically ensures, as per the spirit of the Sorites Paradox, that the conjunction of (N_1), (N_2) and (N_{3LSB}) is inconsistent and, as per the spirit of the Extended Sorites Paradox, that from such inconsistency it follows that the conjunction of (N_1) and (N_2) entails that there is a sharp boundary.

cases of unsettledness somehow *fail to combine* (which would prevent the *existence* of a sharp boundary). The upshot is then that there is *underdetermination* as to which object plays the role of being the sharp boundary in the structure of baldness, and that is why the existence of the sharp boundary comes decoupled from any particular object's being the sharp boundary.[23]

Contextualism (Chapter 3) starts with the idea that vague expressions are essentially context dependent, at least in the sense that which cases they apply to varies with context. For example, when judging whether one could benefit from a hair treatment, 'bald' might apply to a man with 50,000 hairs, but, when judging whether one could need a sunscreen, it might not. Contextualism then postulates a pragmatic mechanism determining that, for every pair of cases *whose similarity is salient in a context*, one case is positive *in the context* iff the other case is, so that the sharp boundary of a vague expression in a context is determined to lie outside the cases whose similarity is salient in the context. Much of contextualist theorising is devoted to accounting for the nature and source of the context dependence of vague expressions and in particular of the hiding mechanism. Contextualism attaches great theoretical significance to the existence of such a mechanism: epistemologically, it is claimed that the fact that the sharp boundary as such (i.e. as including the similarity between the last positive case and the first negative case) is never salient explains our inability to identify it;[24] psychologically, it is claimed that the fact that the sharp boundary as such is never salient explains the empirical fact that we're biased to believe ($\mathrm{N}_{3\mathrm{LSB}}$). Thus, contextualism accepts that vague expressions have sharp boundaries *only in a relative and negligible sense*: while, in every context, there is a sharp boundary, in different contexts there are different sharp boundaries; moreover, because of the hiding mechanism, in every context the sharp boundary as such is not where we're looking.

[23] Subvaluationism (Chapter 2) agrees with supervaluationism about what the latter says is settled, but disagrees about what the latter says is not settled: according to subvaluationism, for *many* i, it is indeed *settled* that the witness $Bi \mathbin{\&} \neg Bi'$ to the existential claim $\exists i(Bi \mathbin{\&} \neg Bi')$ is true, even though, because of what is settled, these cases of settledness somehow *fail to combine* (which would prevent the *uniqueness* of a sharp boundary). The upshot is then that there is *overdetermination* as to which object plays the role of being the sharp boundary in the structure of baldness, and that is why the uniqueness of the sharp boundary comes decoupled from any *singular* object's being the sharp boundary. Thus, subvaluationism too accepts that vague expressions have sharp boundaries *only in a nominal sense*: while there is a sharp boundary, there really isn't this time a *singular* i such that both Bi and $\neg Bi'$ hold – it is valid, say, that both $B50{,}000$ and $\neg B50{,}001$ hold, but it is also valid, say, that both $B50{,}001$ and $\neg B50{,}002$ hold. Again, but for reasons opposite to those submitted by supervaluationism, when it comes to baldness, no number is The Special One. Supervaluationism sweetens the pill of the existence of a (unique) sharp boundary by denying that any particular object is such (and so in effect *accepts a disjunction* – say, $(B45{,}000 \mathbin{\&} \neg B45{,}001) \vee (B45{,}001 \mathbin{\&} \neg B45{,}002) \vee (B45{,}002 \mathbin{\&} \neg B45{,}003) \ldots \vee (B55{,}000 \mathbin{\&} \neg B55{,}001)$ – *while regarding each of its disjuncts as absurd*); subvaluationism sweetens the pill of (the existence of) a unique sharp boundary by denying that any singular object is such (and so in effect *rejects a conjunction* – say, $(B45{,}000 \mathbin{\&} \neg B45{,}001) \mathbin{\&} (B45{,}001 \mathbin{\&} \neg B45{,}002) \mathbin{\&} (B45{,}002 \mathbin{\&} \neg B45{,}003) \ldots \mathbin{\&} (B55{,}000 \mathbin{\&} \neg B55{,}001)$ – *while regarding each of its conjuncts as valid*). Notice that, since, for every i, i's being the sharp boundary entails that, for every $j \neq i$, j is not the sharp boundary, according to subvaluationism, for every i, it is also valid that $\neg(Bi \mathbin{\&} \neg Bi')$ holds, and so subvaluationism is the first solution we meet that accepts *each instance* of ($\mathrm{N}_{3\mathrm{LSB}}$) (although it rejects ($\mathrm{N}_{3\mathrm{LSB}}$) itself – as we've just seen, subvaluationism gives up the traditional idea that *exceptionless universal quantifications and conjunctions must be true*). But, if subvaluationism accepts each instance of ($\mathrm{N}_{3\mathrm{LSB}}$), how does it block soritical reasoning? It does by rejecting the conjunction of the premises of one application of *modus ponendo tollens* in the reasoning: taking the last i such that it is settled that $Bi \mathbin{\&} \neg Bi'$ holds, it is settled that Bi holds only in that it is settled that $Bi \mathbin{\&} \neg Bi'$ holds, whereas it is settled that $\neg(Bi \mathbin{\&} \neg Bi')$ holds only in that, for some $j \neq i$, it is settled that $Bj \mathbin{\&} \neg Bj'$ holds, and so the settledness of Bi fails to combine with the settledness of $\neg(Bi \mathbin{\&} \neg Bi')$.

[24] Thus, we cannot identify the sharp boundary as such neither for *epistemic* nor for *semantic* reasons (as epistemicism and supervaluationism would have it respectively), but for *pragmatic* ones, similarly to how, in every context where there is a closest object that is not salient in the context, we cannot identify that object.