

Index

- algebraic (element of a field), 158
- algebraic closure
 - model-theoretic, 172
 - of a field, 160
 - relative, 159
- algebraically closed field, 38, 160
- algebraically independent, 163
- Archimedean ordered field, 39, 74, 75
- arithmetic, 34
- arity, 3
- ascending chain condition, 172
- atom (of a Boolean algebra), 106, 116
- atomic Boolean algebra, 107
- atomic formula, 13
- atomic structure, 142
- atomless Boolean algebra, 108
- automorphism, 6, 96, 146, 151
- axiom of extension, 106
- axiomatisable class, 47, 76
- axioms, 32
 - for abelian groups, 33
 - for algebraically closed fields, 38
 - for divisible torsion-free abelian groups, 51
 - for fields, 33
 - for groups, 32
 - for linear orders, 83
 - for ordered fields, 38
 - for real-closed fields, 40
 - for rings, 33
 - for the successor function, 88
 - for vector spaces, 77
- back-and-forth method, 85, 96, 108, 139, 142–144, 146, 147, 162
- basis of a vector space, 78
- binary, 4
- Boolean algebra, 106, 109
- Boolean ring, 108
- canonical model, 59, 134
- Cantor's theorem, 54
- cardinal, 53
 - cardinal arithmetic, 54
 - strongly inaccessible, 149
- cardinality, 53
 - of a language, 55
 - of a vector space, 80
- categorical, 81
 - κ -categorical, 81
- categoricity
 - countable categoricity, 81, 138, 143
 - of ACF_p , 164
 - of atomless Boolean algebras, 108
 - of DLO, 84
 - of DTFAG, 82
 - of the successor function, 89
 - of vector spaces, 81
- cell decomposition theorem, 125
- cells, 124
- characteristic of a field, 38, 51, 156
- Chevalley's theorem, 170
- closed term, 10, 58
- cofinite subset, 90
- compactness theorem, 42, 71, 76, 90, 130, 160, 162
 - proof of, 43, 57–61
 - strong version, 61
- complete Boolean algebra, 108
- complete diagram, 70
- complete ordered field, 39
- complete theory, 35, 38, 40, 57, 81, 101, 147–150

complete type, *see* type
 completeness
 of ACF_p , 165
 of DLO, 85
 of the theory of discrete linear orders, 150
 of the theory of real-closed fields, 102
 of the theory of the successor function, 89
 complex field, 37, 68, 160, 176
 conjunctive normal form, 113, 167
 connected graph, 52
 constant symbols, 3
 constructible set, 169, 170
 countable, 54
 deductively closed, 32, 57
 definable function, 19
 definable sets, 19, 45, 90, 109–126, 133, 150
 \emptyset -definable, 117
 definable with parameters, 117
 in \mathbb{N}_{succ} , 90
 in \mathbb{R} -ring, 119
 in algebraically closed fields, 167
 in dense linear orders, 110
 in real vector spaces, 115
 in vector spaces over finite fields, 111
 dense linear order, 84, 95–98, 110, 150
 descending chain condition, 172
 diagram, 71, 100, 160, 164
 complete diagram, 70
 dimension of a vector space, 79
 discrete linear order, 84, 149
 disjunctive normal form, 113, 169
 distributive lattices, 105
 distributive laws, 105
 divisible (group), 51
 DLO, *see* dense linear order
 domain (integral domain), 155
 domain (of a structure), 3
 Downward Löwenheim–Skolem theorem, 67
 DTFAG, 51, 82
 elementary embedding, 66, 70
 elementary equivalence, 35
 elementary extension, 65, 70, 130
 elementary substructure, 65, 70
 embedding, 6, 15, 65
 endpoints, 84
 entails, 31
 expansion
 of a language, 4
 of a structure, 6
 extension, 6, 24, 65, 100

field of fractions, 155
 finite
 non-standard real number, 75
 set, 54
 finitely axiomatisable, 47, 49
 finitely satisfiable, 42, 57, 133
 set of formulas, 129
 first-order logic (comparison with
 second-order), 40
 formula, 13
 atomic, 13
 existential, 26, 170
 positive, 28
 positive quantifier-free, 167, 169,
 170
 quantifier-free, 17, 25
 universal, 26
 free variables, 14
 function symbols, 3
 fundamental theorem of algebra, 160
 Gödel's first incompleteness theorem, 35
 graph, 51
 Henkin theory, 58, 134
 Hilbert's basis theorem, 158, 169, 172, 175,
 176
 Hilbert's Nullstellensatz, 174
 homogeneous, 146, 151
 homomorphism, 8
 of rings, 156
 ideal of a Boolean algebra, 108
 ideal of a ring, 156, 168
 maximal, 156
 prime, 156
 proper, 156, 175
 radical, 176
 infinite
 non-standard real number, 75
 set, 54
 infinitesimal, 45, 75
 infix notation, 10
 integral domain, 155
 interpretation
 of formulas in a structure, 14
 of symbols in a structure, 3
 of terms in a structure, 10
 irreducible, 157
 irreducible variety, 172
 isomorphism, 6
 kernel, 156

- L -structure, 3
- language, 3
- lattice, 105
- least upper bound, 105
- lexicographic product, 87, 149
- Lindenbaum algebra, 110, 116, 139
- Lindenbaum's lemma, 58
- linear order, 83, 148
- linearly independent, 78, 133, 136
- logical consequence, 31
- Łoś–Vaught test, 81, 85, 101, 147, 149, 165

- Mersenne prime, 75
- method of diagrams, 70, 72, 76, 160
- method of new constants, 43–45, 71, 130, 134
- method of preservation under automorphisms, 21, 90, 97, 130, 146, 150
- minimal polynomial, 157, 161
- model, 15
- model completeness, 103

- n -type, 129
- Nash function, 122
 - piecewise Nash function, 123
- non-standard
 - analysis, 75
 - model, 72
 - natural number, 72, 131
 - real number, 75

- o -minimal, 120
- oligomorphic permutation group, 141
- omitting types, 130, 139
- omitting types theorem, 134
 - of Vaught, 136
- Order Property, 148

- parameters, 117, 144
- parametrically definable, *see* definable with parameters
- partial orders, 104
- partial type, 134
- Peano arithmetic, 35
- polynomials, 155, 167
- power set, 104
- prefix notation, 10
- pregeometry, 166
- preservation
 - of atomic formulas, 15
 - of definable sets, 21
 - of existential formulas, 26
 - of formulas, 15
 - of quantifier-free formulas, 18, 26
 - of terms, 11
 - of types, 130
 - of universal formulas, 27
- prime model, 142
- prime subfield, 156
- principal DLO formula, 95, 110
- principal formula, 110, 115
- principal ideal domain, 157
- principal type, 142

- quantifier elimination, 97, 100, 117, 150
 - for \mathbb{N}_{succ} , 103
 - for $\mathbb{Q}_{<}$, 97
 - for $T_{=}^{\infty}$, 98
 - for ACF, 165, 169, 172, 174
 - for atomless Boolean algebras, 108
 - for DLO, 98, 150
 - for real-closed fields, 102, 132, 136
 - for structures, 97
 - for theories, 97
 - for vector spaces, 101
- quotient ring, 156

- Rabinowitsch trick, 169, 176
- rational functions, 156
- real algebraic number, 136
- real field, 38, 45, 74, 176
- real-closed field, 40, 102, 132
- realise a type, 130
- realising types, 139
- recursive definition
 - of existential formulas, 26
 - of formulas, 13
 - of quantifier-free formulas, 25
 - of terms, 9
 - of universal formulas, 26
- reduct, 6
- relation symbols, 3
- relative algebraic closure, 159
- ring, 155
- Ryll–Nardzewski theorem, 139, 143, 144

- satisfiable, 32
 - formula with respect to a theory, 110
- saturated, 147
 - \aleph_0 -saturated, 144
 - κ -saturated, 147
- Schröder–Bernstein theorem, 53
- second-order logic, 37
- second-order logic (comparison with first-order), 40
- semantics, 57
- semi-algebraic sets, 21, 119

sentence, 14
 signature, 3
 simple extension of a field, 157
 Skolem functions, 68
 Skolem paradox, 67
 small theory, 144
 spanning set, 78
 stability theory, 134, 148
 stable, 148
 κ -stable, 148
 0-stable, 144
 Stone space, 138
 strongly \aleph_0 -homogeneous, 146
 strongly κ -homogeneous, 151
 strongly minimal, 173
 structure, 3
 sub-language, 4
 substructure, 6, 24, 65, 99
 substructure completeness, 100, 165
 syntax, 57
 Tarski–Vaught test, 66, 67
 term, 9
 closed, 10
 theory, 32
 first-order, 32
 of a class of structures, 31
 torsion (in groups), 48
 transcendence base, 163
 transcendence degree, 164
 transcendental, 136, 157, 158

transfinite induction, 161, 162
 twin prime conjecture, 73
 type, 129, 133, 150
 complete type, 129, 134, 148
 in \mathbb{N}_S -ring, 131
 in \mathbb{R}_O -ring, 136
 in DLO, 132
 in vector spaces, 136, 141
 non-principal type, 134
 of a tuple in a model, 129
 partial type, 134
 principal type, 134, 138
 type over parameters, 144
 type-definable sets, 134
 unary, 4
 uncountable, 54
 universal axiomatisation, 76
 universal model, 143
 universal sentences, 155
 unstable, 148
 Upward Löwenheim–Skolem theorem, 72, 89,
 143
 variety, 168, 169
 irreducible, 172
 vector space, 77, 101, 111, 133
 vocabulary, 3
 witness property, 58
 Zariski-closed subset, 168