Synchrotron Radiation and Free-Electron Lasers

Principles of Coherent X-Ray Generation

Learn about the latest advances in high-brightness X-ray physics and technology with this authoritative text. Drawing upon the most recent theoretical developments, pre-eminent leaders in the field guide you through the fundamental principles and techniques of high-brightness X-ray generation from both synchrotron and free-electron laser sources. A wide range of topics is covered, including high-brightness synchrotron radiation from undulators, self-amplified spontaneous emission, seeded high-gain amplifiers with harmonic generation, ultra-short pulses, tapering for higher power, free-electron laser oscillators, and X-ray oscillator and amplifier configuration. Novel mathematical approaches and numerous figures accompanied by intuitive explanations enable easy understanding of key concepts, while practical considerations of performance improving techniques and discussion of recent experimental results provide the tools and knowledge needed to address current research problems in the field.

This is a comprehensive resource for graduate students, researchers, and practitioners who design, manage, or use X-ray facilities.

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Preface

X-rays produced when highly relativistic electrons are accelerated along a curved trajectory, generally referred to as synchrotron radiation, have served as an important tool for studying the structure and dynamics of various atomic and molecular systems. The first dedicated synchrotron radiation facility was built in the 1970s using an electron storage ring, and since that time the demand for synchrotron radiation has steadily increased due to its high intensity, narrow angular opening, and broad spectral coverage.[1] Over the past few decades the effectiveness of synchrotron radiation has been further advanced by improvements in storage ring design that led to an increase in the electron beam phase space density, and by the use of magnetic devices such as undulators that dramatically increase the X-ray brightness over traditional bending magnets. These developments have widened and deepened the reach of "photon sciences" around the globe.

Another revolutionary advance in X-ray generation was made with the development of X-ray free-electron lasers (FELs). The radiation produced in an FEL acts back on the electron beam in a positive feedback loop, resulting in X-rays with dramatically improved intensity and coherence over those produced with storage-ring based sources. The X-ray FEL became feasible thanks to improvements in linear accelerator technology in general, and in particular to advances in the injector (electron source).

High-brightness, high-energy electron beams from a linear accelerator can now drive a high-gain X-ray FEL amplifier in a long undulator. The gain can be so high that the initially incoherent undulator radiation evolves to an intense, quasi-coherent field known as self-amplified spontaneous emission (SASE). The SASE pulse can be made ultrashort by using an ultrashort electron bunch. With X-ray FELs, experimental techniques developed for traditional synchrotron light sources can be made much more efficient, and new areas of material, chemistry, and biology research, such as ultrafast dynamics, have become accessible to study.

The advent of X-ray FELs, however, has by no means rendered other synchrotron radiation sources obsolete. For certain applications, including those that require high levels of stability or high levels of average flux, synchrotron radiation from storage rings can be more attractive than SASE. Future synchrotron radiation facilities can provide even brighter X-rays than those that are feasible with the current, "third"-generation

[1] The name "synchrotron radiation" came about because it was first observed in a synchrotron in 1947. However, the oft-used practice of referring to the storage ring as a "synchrotron source" should be avoided, since the source is a storage ring in which the electron beam is in a steady state, rather than a synchrotron in which electrons go through an acceleration cycle.

sources by improving the storage ring design; for example, "multi-bend achromats" can significantly improve the electron beam brightness and hence the radiation brightness. Also, advances in X-ray FEL capabilities are underway in several directions, including the production of fully-coherent soft X-rays via harmonic generation, improvements in hard X-ray coherence via the self-seeding technique, and the tailoring of X-ray pulse characteristics for each user including multi-color and/or multi-pulse X-ray production, among others. An X-ray FEL oscillator (XFELO) that provides full coherence and high spectral purity also appears to be feasible in the hard X-ray region by using an X-ray cavity in which Bragg crystals serve as the main mirrors. A grand X-ray facility could be envisaged in which the XFELO output serves as input for a high-gain amplifier, providing ultimate capabilities for future photon sciences.

The physics of X-ray production in synchrotron radiation sources and free-electron lasers is therefore of significant contemporary interest. We attempt to provide a unified, coherent account of X-ray generation in this book, even though there are several excellent references already devoted to these topics, some of which we list at the end of this book. While overlaps in exposition are unavoidable, we hope that the perspective and approach offered here can help readers develop a more complete understanding of coherent X-ray generation, and lay the foundation for potentially new innovations. We outline and highlight our philosophy and approach as follows.

First, our view is that synchrotron radiation and FELs should be treated as a unified subject, particularly in the X-ray spectral region. This is because the phenomenon of FEL feedback is always present in an undulator, although in many cases it is too weak to have an appreciable effect on the X-ray radiation properties.

Second, we have emphasized the importance of the phase space distribution for both particle and radiation beams. The phase space distribution of particle beams is familiar from accelerator physics, while that associated with an electromagnetic field can be defined following Wigner's construction. We are then able to identify the phase space density, or the brightness distribution, of synchrotron radiation in a logical manner. This places the electron and radiation beams on more equal footing, and can be used to answer some practically important questions including when are we allowed to neglect the effect of the electron beam phase space on the generated X-rays.

Third, we retain the discrete nature of the electrons by representing the electrons' phase space distribution as a sum of delta functions that indicates the position and momentum of each constituent electron. Such a distribution is known as the Klimontovich distribution function, and in general it contains all the classical information of interest. We then write the Klimontovich distribution as a sum of two parts: the first part describes the smooth, ensemble-averaged distribution, while the second contains both the fast oscillations due to the electromagnetic interaction as well as the fluctuations encapsulated in the discrete sum over delta functions. To render the problem soluble, we regard the second part as a small perturbation about the smooth equilibrium. If we neglect the transverse dimensions, a complete solution to the initial value problem can be found by employing the Laplace transform technique. To account for radiation diffraction and the transverse betatron oscillations of electrons requires a more sophisticated approach. In the low-gain case, the solution can be obtained by the method of integrating along the characteristics, while a formal solution to the high-gain case can be written in terms of the Van Kampen modes, and the resulting equations can be solved numerically once the smooth initial distribution is known.

The practical implementation of coherent X-ray production via FELs is a fast moving frontier. We have therefore not attempted to give an exhaustive account of the wide variety of proposed techniques. Nevertheless, we have provided discussions of several practical topics, including the effects of undulator errors and wakefields, along with several methods to produce coherent, soft X-ray output via nonlinear harmonic generation of an initially longer wavelength laser. We also present a short overview of currently existing and planned high-gain amplifier facilities. An FEL oscillator for hard X-rays making use of Bragg reflectors is discussed in some detail as a possible way to produce fully coherent, hard X-ray beams of high spectral purity.

This book began as a set of lecture notes for a course at the U.S. Particle Accelerator School, and has been gradually maturing as the course was repeated every two to three years over the last fifteen years. Feedback from students has played an important role in improving the notes. Suggestions and encouragements from our colleagues, too many to list here, were also essential for this book to become a reality. The construction of an X-ray FEL theory, and the application of that theory to the design and interpretation of subsequent FEL experiments and X-ray facilities, have been one of the most exciting and successful beam dynamics activities in recent years. It is our great pleasure to share some of these developments with students and with our colleagues, and to acknowledge and thank their contributions.

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Conventions and Notation

Throughout this book we use SI units, with the notable exception that we usually quote energies in eV. We use standard boldface to denote vectors (e.g., $\boldsymbol{x}$) and sans-serif fonts for matrices (M). Our definition of the Fourier transform follows the standard conventions in classical physics, so that the Fourier transform in space and time of the function $f(x, t)$ is defined as

$$f(\omega, x) = f_\omega(x) = \int_{-\infty}^{\infty} dt\ e^{i\omega t} f(x, t)$$

$$f(k, t) = \int_{-\infty}^{\infty} dx\ e^{-ikx} f(x, t).$$

Note that we usually write the frequency argument of the Fourier transform as a subscript. In addition, we typically do not include the limits of integration if they are over the entire line, so that the inverse transforms would appear as

$$f(x, t) = \frac{1}{2\pi} \int d\omega\ e^{-i\omega t} f_\omega(x)$$

$$f(x, t) = \frac{1}{2\pi} \int dk\ e^{ikx} f(k, t).$$

Finally, as is inevitable in a book like this, we introduce a lot of mathematical symbols to represent physical quantities. Rather than list every symbol introduced, here we include a table of only those symbols that appear across different sections and chapters of the book. We note that we also at times employ scaled versions of these variables that we denote with hats; for example, the scaled propagation distance along the undulator would be $\hat{z}$.

Symbol	Physical meaning/description
a_ν	Scaled electric field at dimensionless frequency ν
α	Fine structure constant $\alpha \equiv e^2/(4\pi\epsilon_0\hbar c) \approx 1/137$
$\alpha_{x,y}$	Twiss parameter for the correlation $\alpha_x = -\langle xx'\rangle/\varepsilon_x$
$\mathcal{B}$	Radiation brightness (or Wigner) function
B_0	Undulator peak magnetic field on axis
$\beta_{x,y}$	Transverse beta function $\beta_x = \langle x^2\rangle/\varepsilon_x$

$\bar{\beta}_x$	Average transverse beta function
β_n	Natural beta function determined by the undulator focusing
c	Speed of light in vacuum
$\Delta\nu$	Relative frequency detuning $\Delta\nu = \nu - 1 = (\omega - \omega_1)/\omega_1$
e	Magnitude of the electron charge
E_x	Transverse electric field
E	Slowly-varying transverse electric field amplitude
E_ν	Fourier component of the transverse electric field
$\mathcal{E}(\phi)$	Angular representation of the transverse electric field
η	Relative energy deviation from resonance $(\gamma - \gamma_r)/\gamma_r$
ϵ_0	Vacuum permittivity
ε or $\varepsilon_{x,y}$	Electron beam transverse emittance $\varepsilon_x = \langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2$
$\varepsilon_{x,n}$	Normalized transverse emittance of the electron beam ε/γ
ε_r	Radiation emittance $\varepsilon_r \geq \lambda/4\pi$
f or F	Electron phase space distribution function
f_ν	Fourier component of the distribution function
γ	Electron energy (in units of mc^2)
γ_r	Resonant electron energy (in units of mc^2)
γ_0	Initial/reference electron energy (in units of mc^2)
h	Odd harmonic order $h = 1, 3, 5, \ldots$
$\hbar$	Planck's constant over 2π
I	Peak electron bunch current
I_A	Alfvén current $\approx$ 17 kA
J_x	Transverse current
$\mathcal{J}_{x,y}$	Transverse particle action
J_n	Bessel function of order n $(n = 0, 1, 2, \ldots)$
[JJ]	Undulator Bessel function factor at the fundamental $[\mathrm{JJ}]_1 = [\mathrm{JJ}]$
$[\mathrm{JJ}]_h$	Undulator Bessel function factor at harmonic h
k_1	Fundamental radiation wavenumber ω_1/c
k_β	Average betatron focusing wavenumber $= 1/\bar{\beta}_x$
k_u	Undulator wavenumber
K	Undulator deflection parameter $K = eB_0/mck_u$
L_{G0}	1D FEL power gain length of a monoenergetic beam
L_G	3D FEL power gain length
L_u	Undulator length
λ_1	Fundamental FEL wavelength
λ_h	FEL wavelength at harmonic h
λ_u	Undulator period
m	Electron rest mass
M	Total number of independent modes in a radiation pulse
$M_{T,L}$	Number of transverse or longitudinal modes, respectively
μ	Scaled complex growth rate of the linear FEL
μ_3	Growth rate of the exponentially growing mode in 1D
$\mu_{\ell m}$	Growth rate of transverse mode with radial order ℓ and azimuthal order m

Symbol	Physical meaning/description
n_e	Electron volume density
N_e	Total number of electrons in a bunch
ω_1	Fundamental undulator radiation frequency
ν	Ratio of the radiation frequency ω to the fundamental ω_1
$\boldsymbol{p}$	Electron angle from the axis $\boldsymbol{p} = (x', y')$
P	Radiation power
P_{beam}	Electron beam power $(I/e)\gamma mc^2$
P_{sat}	FEL saturation power
$\boldsymbol{\phi}$	Radiation angle with respect to the optical axis
ρ	FEL Pierce parameter
σ_η	RMS relative energy spread of the electron beam
σ_r, $\sigma_{r'}$	RMS transverse size, divergence of the radiation
σ_ω	RMS radiation bandwidth
σ_x, $\sigma_{x'}$	RMS transverse size, divergence of the electron beam
$t_j(z)$	Electron's arrival time at the undulator location z
t_j	Shorthand for the electron's arrival time at $z = 0$, $t_j = t_j(0)$
$\bar{t}_j(z)$	Electron's arrival time averaged over an undulator period
T	Flat-top electron bunch duration
t_{coh}	Radiation temporal coherence time
θ	Electron's phase relative to the radiation wave
U	Total radiation energy
$\boldsymbol{v}$	Electron's transverse velocity
v_z	Electron's longitudinal velocity
$\bar{v}_z$	Electron's average longitudinal velocity in a planar undulator
$\boldsymbol{x}$	Electron's horizontal and vertical position (x, y)
$\boldsymbol{x}'$	Electron's horizontal and vertical angle (x', y')
z	Propagation distance from the undulator beginning
z_{sat}	FEL saturation distance
Z_R	Rayleigh length of the radiation