Stochastic Geometry Analysis of Cellular Networks

Achieve faster and more efficient network design and optimization with this comprehensive guide. Some of the most prominent researchers in the field explain the very latest analytic techniques and results from stochastic geometry for modeling the signal-to-interference-plus-noise ratio (SINR) distribution in heterogeneous cellular networks. This book will help you understand the effects of combining different system deployment parameters on such key performance indicators as coverage and capacity, enabling efficient allocation of simulation resources. In addition to covering results for network models based on the Poisson point process, this book presents recent results for when non-Poisson base station configurations appear Poisson due to random propagation effects such as fading and shadowing, as well as non-Poisson models for base station configurations, with a focus on determinantal point processes and tractable approximation methods. Theoretical results are illustrated with practical long-term evolution (LTE) applications and compared with real-world deployment results.

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"These four renowned experts deliver a comprehensive yet curated treatment on the modeling and analysis of modern cellular networks using stochastic geometry, which has been one of the most important recent lines of wireless research. Highly recommended for interested researchers and engineers. Can serve as a useful companion to Haenggi's landmark stochastic geometry textbook, which had fairly minimal treatment of cellular networks."

Jeff Andrews,
University of Texas at Austin

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Bartłomiej Błaszczyszyn , Martin Haenggi , Paul Keeler , Sayandev Mukherjee
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Preface

Since 2010, our knowledge of coverage and capacity in heterogeneous cellular networks (HetNets) has expanded rapidly, primarily through analytical results using stochastic geometry. Most of these results assume that the locations of the base stations in a given tier of the HetNet are the points of a homogeneous Poisson point process (PPP). This modeling assumption was made for mathematical tractability, and the coverage results for a single tier were determined to be about as pessimistic (relative to the real coverage in real-world deployments) as the regular hexagonal lattice location model was optimistic (Andrews, Baccelli, & Ganti 2011).

However, since 2013 we have obtained results showing the fundamental importance of the PPP model to the analysis of real-world deployments in two ways: (a) the set of propagation losses to the typical location in an arbitrary network deployment converge asymptotically to that from a PPP deployment of base stations; (b) long before this convergence is reached, the coverage results for a PPP deployment can be employed to obtain very accurate approximations to the coverage results for various regular deployments.

This book provides the first detailed expository treatment of these results, and includes additional exact analytical results on coverage for certain special non-PPP deployment models. We expect the book to be of interest to researchers in academia and industry, and anyone interested in the application of stochastic geometry to problems in communication. HetNets are an important component of future cellular network standards (LTE Release 12 and later),[1] and the theoretical results in the book are illustrated with examples of their application to transmission scenarios specified in the LTE standard.

Although we have made every effort to make the book self-contained, the reader will benefit from having had some prior exposure both to the theory and techniques of stochastic geometry, and to their application to the derivation of coverage results for PPP base station deployments. For an introduction to stochastic geometry with emphasis on wireless communications, we recommend Haenggi's *Stochastic Geometry for Wireless Networks* (Haenggi 2012) at the introductory level (a condensed version of which is available in Chapter 3) and the two volumes of Baccelli and Błaszczyszyn's *Stochastic Geometry and Wireless Networks* (Baccelli & Błaszczyszyn 2009a, 2009b) for a comprehensive treatment at the advanced level. Note that both references rely on

[1] See Appendix B.

mathematical results from a measure-theoretic treatment of stochastic geometry, such as that found in Chiu et al. 2013. For a simplified treatment of some of the results on coverage and capacity in PPP network deployments in Chapter 5, the interested reader is referred to Mukherjee (2014), which does not require knowledge of measure theory.

Knowledge of 3GPP-LTE is not required, but familiarity with the LTE standard will help the reader in understanding the applications of the results presented in the book. An outline of the different releases in the LTE standard and the features relevant to HetNets in each one is provided in Appendix B.

Acknowledgments

It is a remarkable testament to modern communications that four authors on three continents were able to collaborate so amiably and productively on this book, having met collectively but once, and we are grateful to Professors Jeffrey G. Andrews, François Baccelli, and Gustavo de Veciana of the University of Texas at Austin, and the Simons Foundation for making that singular occasion possible, at the Simons Conference on Networks and Stochastic Geometry in Austin in May 2015 that they organized.

We are grateful to Naoto Miyoshi for his careful reading of the manuscript and his precious remarks, in particular in Chapter 8. We are also grateful to Julie Lancashire and Heather Brolly, our editor and content manager, respectively, at Cambridge University Press for their patience as deadlines for the finished manuscript came and went as the authors juggled their work and teaching schedules with writing.

B. B. would like to express his gratitude to François Baccelli for introducing him to the art and science of modeling of communication networks, as well as to the Inria–ENS team where, since 1999, B. B. had the pleasure and honor of collaborating with François Baccelli on the foundations of stochastic geometric modeling of wireless networks. Without these early, visionary scientific choices and the leadership of François Baccelli, as well as the working environment provided by Inria and ENS, the author's more recent contribution to this book would not have been possible. The author is also grateful to Mohamed Kadhem Karray for a long, close collaboration and very many stimulating discussions. Finally, but equally importantly, B. B. would like to thank his lovely family – Mira, Klara, Karol, and Antoni – for their understanding and support while he worked on his share of the manuscript.

M. H. thanks Béla Bollobás for sparking his interest in random geometric graphs and stochastic geometry in the early 2000s, when the application of stochastic geometry to wireless networks modeling and analysis was still in its infancy. Since then, the annual number of papers published on the topic has grown by almost 30 dB! He is grateful to his PhD students, postdoc, and other collaborators; most of the credit for the results and insights obtained are due to their tireless efforts and stimulating discussions. And he would like to express his deep gratitude to his family – Roxana and little Kevin – for their love, support, and understanding that this book project would take time away from them.

P. K. thanks his PhD supervisor, Peter G. Taylor, at the University of Melbourne, who set him on the path of using stochastic geometry models, which ultimately led to doing interesting projects with B. B. in the Inria–ENS team, then headed by François Baccelli.

This journey continued by working at the Weierstrass Institute in a team headed by Wolfgang König and focused on applying stochastic methods to wireless networks. P. K. is grateful to the aforementioned researchers and research institutes. None of this would have started without his family who long ago inspired in him interests in mathematics, science, and engineering.

S. M. would like to express his gratitude once more to Professor Jeffrey G. Andrews of the University of Texas at Austin, for inviting him to speak at the 2015 Simons Conference, an occasion on which he met for the first time not only two of his co-authors, but also nearly all the top researchers in stochastic geometry, most of whom he knew until then only from their publications. S. M. also thanks the management at DOCOMO Innovations, Inc., for their support, and the corporate general counsel, Jeremy Tucker, Esq., for negotiating the book contract with Cambridge University Press. Last but certainly not least, he would like to thank his family – Chandreyee, Rik, Inika, and the four-legged tail-wagging distraction Johnny – for their patience while he worked on his share of the manuscript.

Notations

$\sim$	distributed as
${}_2F_1(a,b;c;z)$	Gaussian hypergeometric function
α	slope of path loss model (path loss exponent)
δ	$= 2/\alpha$
$\mathcal{B}_{[0,1]}$	the Borel σ-algebra on $[0,1]$
τ	SINR threshold
$\Gamma(z)$	gamma function
$\gamma(z,a)$	lower incomplete gamma function
Φ, Ψ	PPP of base station locations
$\tilde{\Phi}, \tilde{\Psi}$	PPP of received powers from the base stations of the PPP Φ or Ψ
x	scalar
$\boldsymbol{x}$	vector
x	a point of a point process
$\mathbf{A}$	matrix
X	random variable
$f_X(x)$	probability density function of X evaluated at x
$\boldsymbol{X}$	random vector
$f_{\boldsymbol{X}}(\boldsymbol{x})$	joint probability density function of $\boldsymbol{X}$ evaluated at $\boldsymbol{x}$
$\Lambda(\cdot)$	intensity measure
I	interference power
K	intercept of path loss model
$\ell(\cdot)$	path loss function
L_{x}	propagation loss between the user at the origin and the base station x
$\mathcal{L}_X(s)$	Laplace transform of X evaluated at s: $\mathbb{E}[\exp(-sX)]$
$\mathcal{N}$	the space of all sequences
N_φ	counting measure of φ
ν	Lebesgue measure
φ	a point pattern
B	set or event
$\mathbf{1}_B(\cdot)$	indicator function of the set B
$\mathbb{E}[X]$	expectation of the random variable X
$\mathbb{P}(B)$	probability of the event B, also equal to $\mathbb{E}[\mathbf{1}_B]$
$\mathbb{R}$	the set of real numbers, also written $(-\infty,\infty)$
$\mathbb{R}_+$	the set of nonnegative reals, also written $[0,\infty)$
$\mathbb{R}_{++}$	the set of positive reals, also written $(0,\infty)$

Acronyms and Abbreviations

3GPP	Third Generation Partnership Project
ASAPPP	approximate SIR analysis based on the Poisson point process
AWGN	additive white Gaussian noise
BLER	block error rate
CapEx	capital expenses
CCDF	complementary cumulative distribution function
CDF	cumulative distribution function
CoMP	coordinated multipoint
EFIR	expected fading-to-interference ratio
GPP	Ginibre point process
HetNet	heterogeneous cellular network
IA	interference alignment
ICIC	intercell interference coordination
ICSC	interference cancellation and signal combination
IIC	independent interference cancellation
iid	independent and identically distributed
ISR	interference-to-signal ratio
LTE	long-term evolution
LTE-A	long-term evolution-advanced
MIMO	multiple-input multiple-output
MISR	mean interference-to-signal ratio
MRC	maximal ratio combining
pgfl	probability generating functional
PPP	Poisson point process
RDP	relative distance process
REB	range expansion bias
SC	signal combination
SER	symbol error rate
SIC	successive interference cancelation
SINR	signal-to-interference-plus-noise ratio
SIR	signal-to-interference ratio
SISO	single-input single-output
SNR	signal-to-noise ratio
STINR	signal-to-total-interference-plus-noise ratio
STIR	signal-to-total-interference ratio
UMTS	universal mobile telecommunications system
ZFBF	zero-forcing beamforming