Introduction to Magnetohydrodynamics

Magnetohydrodynamics (MHD) plays a crucial role in astrophysics, planetary magnetism, engineering and controlled nuclear fusion. This comprehensive textbook emphasises physical ideas, rather than mathematical detail, making it accessible to a broad audience. Starting from elementary chapters on fluid mechanics and electromagnetism, it takes the reader all the way through to the latest ideas in more advanced topics, including planetary dynamos, stellar magnetism, fusion plasmas and engineering applications.

With the new edition, readers will benefit from additional material on MHD instabilities, planetary dynamos and applications in astrophysics, as well as a whole new chapter on fusion plasma MHD. The development of the material from first principles and its pedagogical style makes this an ideal companion for both undergraduate students and postgraduate students in physics, applied mathematics and engineering. Elementary knowledge of vector calculus is the only prerequisite.

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Second Edition

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Dedicated to the memory of Henri

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Preface to the Second Edition

Some 15 years have passed since the first edition of this book was published, and it seems natural to revisit the subject after so long a break, reacquainting oneself with an old friend, so to speak.

If an excuse were required to revisit MHD after such a prolonged absence, then the recent advances in geophysical and astrophysical applications provide ample motivation. Astrophysical MHD, for example, has made great progress, partially as a result of the extraordinary observational data gathered from spacecraft-based instruments. On the other hand the relentless rise in computing power has, for the first time, made it possible to compute certain (but certainly not all) aspects of planetary dynamos, heralding a new wave of dynamo theories. As a result, the geophysical and astrophysical applications of MHD are now more thought-provoking and inviting than ever before.

So how should one update a book in the light of these developments? Clearly there is a need to place more emphasis on the geophysical and astrophysical applications in a second edition, which in any event provides the perfect excuse for offering a more balanced presentation of MHD. So Chapter 14, on planetary dynamos, and Chapter 15, on astrophysical applications, are largely new. Another omission in the first edition was an absence of fusion plasma MHD, and it is hoped that this has been remedied by the addition of Chapter 16. Between them, Chapters 14 through 16 provide an introduction to many of the applications of MHD in physics, and the author thanks Felix Parra Diaz and Gordon Ogilvie for providing helpful comments on draft versions of Chapters 15 and 16. Perhaps the final major addition is an extended treatment of turbulence (in Chapter 8) and MHD turbulence (in Chapter 9), which reflect recent progress in theories of MHD turbulence. Despite this shift in emphasis, the engineering applications, which were a particular feature of the first edition, have been largely retained as they are sadly underrepresented elsewhere in textbooks on MHD.

Despite these changes, the ambition of the text remains largely the same: to provide a self-contained introduction to MHD for graduate and advanced undergraduate students, with background material on electromagnetism and fluid mechanics developed from first principles, and with the fundamental theory illustrated through a broad range of applications.

Preface to the First Edition

Prefaces are rarely inspiring and, one suspects, seldom read. They generally consist of a dry, factual account of the content of the book, its intended readership, and the names of those who assisted in its preparation. There are, of course, exceptions, of which Den Hartog's preface to a text on mechanics is amongst the wittiest. Musing whimsically on the futility of prefaces in general, and on the inevitable demise of those who, like Heaviside, use them to settle old scores, Den Hartog's preface contains barely a single relevant fact. Only in the final paragraph does he touch on more conventional matters with the observation that he has 'placed no deliberate errors in the book, but he has lived long enough to be quite familiar with his own imperfections'.

We, for our part, shall stay with a more conventional format. This work is more of a text than a monograph. Part I (the larger part of the book) is intended to serve as an introductory text for (advanced) undergraduates and post-graduate students in physics, applied mathematics and engineering. Part II, on the other hand, is more of a research monograph and we hope that it will serve as a useful reference for professional researchers in industry and academia. We have at all times attempted to use the appropriate level of mathematics required to expose the underlying phenomena. Too much mathematics can, in our opinion, obscure the interesting physics and needlessly frighten the student. Conversely, a studious avoidance of mathematics inevitably limits the degree to which the phenomena can be adequately explained.

It is our observation that physics graduates are often well versed in the use of Maxwell's equations, but have only a passing acquaintance with fluid mechanics. Engineering graduates often have the opposite background. Consequently, we have decided to develop, more or less from first principles, those aspects of electromagnetism and fluid mechanics which are most relevant to our subject, and which are often treated inadequately in elementary courses.

The material in the text is heavily weighted towards incompressible flows and to engineering (as distinct from astrophysical) applications. There are two reasons for this. The first is that there already exist several excellent texts on astrophysical, geophysical and plasma MHD, whereas texts oriented towards engineering applications are somewhat thinner on the ground. Second, in recent years we have witnessed a rapid growth in the application of MHD to metallurgical processes. This has spurred a great deal of fruitful research, much of which has yet to find its way into textbooks or monographs. It seems timely to summarise elements of this research. We have not tried to be exhaustive in our coverage of the metallurgical MHD, but we hope to have captured the key advances.

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