

Index

References to definitions have page numbers in roman font, whereas others (such as to theorems, examples, exercises) are in *slant* font. Of course, some results appear on the same page as the definition.

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