

# Index

Page numbers for BTWs are in **bold**, problems in *italics*

- active and passive transformations, 368–372
- adjoint
  - formal definition, 344
  - of a product, 342, 365
- affine transformation, 325
- Airy disk, 579
- Airy functions, 599, 644, 655–657, 661, 755
- aliasing, 609
- Ampère’s law, 171, 219, 227
- analytic, 242
  - at infinity, 242
  - Cauchy–Riemann equations, 240
  - continuation, 248
  - functions, 239–243
- angular momentum, 4, 37, 38
  - commutation relation, 398
- ansatz, 438
- Archimedes’ principle, 470
- Arg, 262
  - see also* logarithm, principal branch
- argument principle, 313
- asymptotic expansions, 95
- BAC-CAB identity, 138
- Baker–Campbell–Hausdorff formula, 72
- basis, 330–335
  - complete orthonormal, 359
  - completeness, 356–362
  - continuous, 587
  - curvilinear coordinates, 729
  - cylindrical, 7
  - of Pauli matrices, 352
- sheared, 473
- spherical, 8
- standard, 331
- beats, 451
- Bernoulli number, 58
- Bessel functions
  - cylindrical  $J$ , 552–555, 653, 746–750
    - as normal modes of hanging rope, 667
    - asymptotic form, 314, 553, 746, 747
    - completeness, 750
    - generating function, 749
    - identities, 748
    - integral representations, 252, 553
    - linear independence, 636
    - orthogonality, 654, 685, 750
    - recursion relations, 749, 750
    - series representation, 634, 747
    - Wronskian, 748
    - zeros, 654, 747
  - Hankel, *see* Hankel functions
  - modified, *see* modified Bessel functions
  - Neumann, *see* Neumann functions
  - spherical  $j$ , 565, 752–754
    - asymptotic form, 755
    - closed form, 752
    - completeness relation, 754
    - from cylindrical, 752
    - orthogonality, 754
    - recursion relations, 753
    - zeros, 754
- Bessel’s equation, 634, 699, 725, 746
  - from Laplacian in cylindrical coordinates, 672
  - from Laplacian in spherical coordinates, 674
  - modified, 751
  - modified spherical, 754
  - spherical, 752
- Bessel’s inequality, 544
- beta function, 101, 312
- binomial
  - coefficient, 42
  - distribution, 43
  - large  $n$  limit, 50
  - expansion, 63
  - theorem, 42
- block diagonal, 435
- Born series, 722
- Bose–Einstein condensation, 125
- boson gas, ideal, 122
- boundary conditions
  - Cauchy, 676
  - Dirichlet, 676
  - homogeneous, 649, 676
  - inhomogeneous, 681, 710
  - Neumann, 676
  - periodic, 649
  - Robin, 676
  - separated, 649
  - Sturm–Liouville, 647
- bra, *see* bra-ket notation
- bra-ket notation, 343–345
  - bra vector, 343
  - ket vector, 336
  - zero vector, 337
- Bragg scattering, 579

- 
- branch cuts, 256, 301–303
  - branch points, 255
    - algebraic, 264
    - at infinity, 257, 267
    - logarithmic, 264
    - order of, 259
  - Bromwich integral, *see* Laplace transform
  - calculus of residues, 287
    - branch cuts, 301
    - integrating along real axis, 295
    - integrating around a circle, 293
    - poles on the contour, 303
    - residue at infinity, 291
    - residue theorem, 288
  - calculus of variations, 169, 194, 493
  - catenary, 70
  - Cauchy
    - integral formula, 244
    - integral theorem, 240
    - principal value, 84
    - sequence, 56
  - Cauchy–Riemann equations, 240
  - causal structure, 318
  - Chebyshev polynomials, 520
    - generating function, 516
    - recursion relations, 519
    - Rodrigues formula, 516
  - Chebyshev’s equation, 658
  - chemical potential, 120
  - Christoffel symbol, 489–493
    - in terms of derivatives of the metric, 498
  - circulation, 170–181
  - closure relation, *see* completeness relation
  - commutator, 72, 396
    - and simultaneous diagonalization, 425
    - and the uncertainty principle, 428
  - Jacobi identity, 403
    - relation to cross product, 398
  - complete orthonormal basis, 359
  - completeness relation, 357, 366, 546
  - complex algebra
    - as rotations in the plane, 19
    - de Moivre’s theorem, 20
    - Euler’s identity, 18
    - polar representation, 18
    - pure phase, 18
    - real and imaginary parts, 19
    - roots of unity, 21
  - complex potential, 279
  - conformal mapping
    - and Laplace’s equation, 270, 280
    - and the Jacobian, 270
    - critical points, 270
  - inversion, 272
  - linear, 270
  - list, 271
  - Möbius transformation, 272
  - stereographic projection, 272, 277
  - conservative fields, 177
    - see also* path independence
  - continuity
    - and conservation, 219
  - continuity equation
    - differential form, 223
    - integral form, 183, 219
  - contour integral, 239
  - convergence
    - absolute, 57
    - alternating series, 57
    - comparison test, 54
    - conditional, 57
    - integral test, 55
    - mean-square ( $\mathbb{L}^2$ ), 543–546
    - orthogonal functions
      - Dirichlet conditions, 540
    - pointwise, 537
    - radius of, 58, 59, 249
    - ratio test, 53
      - new and improved, 67
    - uniform, 538
      - Weierstrass  $M$  test, 539
  - convolution, 589
    - convolution theorem, 590
    - discrete, 69
  - coordinates
    - curvilinear, 10, 71, 141, 729–731
    - elliptical, 16
    - hyperspherical, 14, 17
    - integration measures, 36
    - oblate spheroidal, 16
  - cosmic microwave background (CMB), 562
  - covariant and invariant, 458
  - covariant derivative, 490
  - curl, 137
    - and Dirac delta, 208
    - as circulation density, 205
    - in curvilinear coordinates, 142
    - in cylindrical coordinates, 143
    - in spherical coordinates, 143
    - swirl, 149–151
  - curvilinear coordinates, *see* coordinates
  - cyclic permutations, 32
  - cycloid, 164
  - de Rham cohomology, 234
  - degeneracy, *see* eigenvalues
  - del, *see* gradient
  - dense space, 573
  - determinant, 35
  - $e^{\text{Tr}}$ , 68, 433
    - as triple scalar product, 35
    - product of eigenvalues, 421
  - diagonalization, 394, 419
    - and matrix powers, 421
    - simultaneous, 425
    - unitary, 421
  - differential equations, 615
    - linear, 615
    - ordinary, *see* ordinary differential equations
    - partial, *see* partial differential equations
  - diffraction, 576, 591
    - grating, 602
  - diffusion equation, *see* heat equation
  - Dirac comb, 581, 592, 606, 646
  - Dirac delta function, 105–113
    - and curl, 208
    - and divergence, 202
    - as limit of sequence of functions, 106, 107, 115
    - integral representation, 106, 115
    - relation to step function, 110
  - Dirac notation, *see* bra-ket notation
  - directional derivative, 133
  - Dirichlet conditions
    - and differential equations, *see* boundary conditions
    - and orthogonal functions, *see* convergence
  - dispersion relations, 309
  - divergence, 136
    - and Dirac delta, 202
    - as flux density, 199
    - in curvilinear coordinates, 142
    - in cylindrical coordinates, 143
    - in spherical coordinates, 143
    - surge and spread, 146–149
  - divergence theorem, 197–203, 222
    - alternate forms, 199
  - dog bone contour, 302
  - drumhead normal modes
    - circular, 684
    - elliptical, 691
    - hearing the shape of a drum, 687
    - rectangular, 683
    - spherical, 686
    - timpani, 691
    - whispering mode, 686
  - duality of electromagnetism, 369, 400
  - Duhamel’s principle, 696, 710
  - eigenvalue problem, 405–412
    - generalized, 429
    - secular determinant, 406
    - spectral decomposition, 423

## INDEX

761

- eigenvalues, 405  
 and determinant, 421  
 and trace, 421  
 degeneracy, 409  
     lifting, 426  
     of hermitian operator, 413  
     of similar matrices, 421  
     of unitary operators, 417
- eigenvectors, 405  
     and diagonalization, 420  
     and normal matrices, 421  
     and normal modes, 439  
     common, 411  
     normalized, 408  
     of Hermitian operator, 413  
     of unitary operators, 417  
     *see also* Sturm–Liouville
- Einstein solid, 119
- Einstein summation convention, 478
- electromagnetic field tensor, 483  
     and invariants, 496
- entire function, 241
- entropy, 181
- equipotential, 134, 155, 279, 701
- equivalence principle, 494
- error function, 91
- Euclidean space, 141, 160–162, 506  
     as tangent plane, 493  
     rotations, 374, 376
- Euler’s rotation theorem, 412, 733
- Euler–Mascheroni constant, 56
- exponential integral function, 648
- extended complex plane, *see*  
     Riemann sphere
- faithful representation, 337, 375, 544,  
     739
- Faraday’s law, 196, 217, 221, 223,  
     230
- Fibonacci sequence, 433
- flux, 182–193  
     net, 183
- four-vector, 329, 388
- Fourier series, 335, 532  
     “ringing”, 530  
     *see also* Gibbs phenomenon  
     complex basis, 527  
     continuum limit, *see* Fourier  
         transform  
     cosine series, 526, 532–534  
     generalized, 652  
     in space and time, 666  
     musical instruments, 528  
     of band-limited spectrum, 605  
     of crystal electron distribution, 579  
     orthogonality, 523  
     over a disk, 552  
     over a rectangle, 550
- sine series, 532–534, 555, 649,  
     670, 678  
     solution to differential equation,  
     646, 649
- Fourier transform, 316, 570  
     and Green’s function, 712–716  
     as continuum limit of Fourier  
         series, 569
- Bragg scattering, 579
- complementary space, 580
- conventions, 573
- diffraction, 576
- Dirac comb, 581
- eigensystem, 601
- in  $n$  dimensions, 573
- of delta function, 571
- of Gaussian, 581
- of wave packet, 574
- pairs, 571
- Plancherel’s theorem, 572
- reciprocal spectral widths, 575,  
     581
- of trumpet blast, 576
- uncertainty principle, 576
- symmetries, 581
- unitarity, 572
- Fourier–Bessel series, 654
- Fraunhofer diffraction, *see* diffraction
- Fredholm alternative, 431
- frequency modulation, 553
- fundamental theorem  
     of algebra, 98, 408  
     of arithmetic, 52  
     of calculus, 175, 197, 210–212  
         boundary of the boundary, 211
- gamma function, 94, 95, 249  
     reflection formula, 312
- gauge  
     equivalence of fields, 209  
     invariance, 225  
     transformation, 225
- Gauss’ law, 130, 184, 202, 222  
     for gravity and magnetism, 202  
     in different dimensions, 185
- Gauss’ theorem, *see* divergence  
     theorem
- Gaussian, 47, 50
- general covariance, 484
- generalized Fourier series, *see*  
     Sturm–Liouville
- generating functions, 515–519
- orthogonal polynomials, 516
- generator, 403  
     as vector space, 398
- Hermitian, 397
- of rotation, 396
- of rotations, 733
- angular momentum, 398  
     commutation relation, 398
- of time evolution, 418
- of translation, 62
- geodesics, 492  
     and extremal path, 492
- Gibbs phenomenon, 530, 540, 541
- gimbal lock, 737
- gradient  
     in curvilinear coordinates, 142  
     in cylindrical coordinates, 143  
     in spherical coordinates, 143  
     interpretation, 133  
     operator, 136
- Gram–Schmidt orthogonalization,  
     353
- Green’s function, 662, 694  
     advanced, 318, 319  
     and Fourier transforms, 712–716  
     and method of images, 701  
     and the variation of constants, 697  
     as projection operator, 704  
     eigenfunction expansion, 703  
         complete orthogonal basis, 708  
         driven string, 705  
     for Poisson equation, 698  
     for Schrödinger equation, 320  
     fundamental solution, 701, 702  
     heat equation, 709, 713  
     inhomogeneous boundary  
         conditions, 710  
     modified, 705–708  
     of the Laplacian, 695  
     Poisson equation, 695  
     retarded, 318, 319, 715  
     Schrödinger equation, 717  
     symmetry property, 700  
     translation invariance, 700  
     wave equation, 699, 714
- Green’s identities, 215
- Green’s theorem in the plane, 204,  
     215  
     *see also* Stokes’ theorem
- groups  
      $O(n)$  and  $SO(n)$ , 374  
      $U(1)$  and  $SO(2)$ , 377  
      $U(n)$  and  $SU(n)$ , 377  
     continuous, connected, compact,  
         397  
     formal definition, 381
- Hankel functions, 700  
     cylindrical  $H$ , 750  
         asymptotic form, 750  
         recursion relations, 750
- spherical  $h$ , 726  
     from cylindrical, 753  
     recursion relations, 753

- harmonic conjugates, 279
- harmonic function, 158, 279
- heat equation, 226, 663, 669
  - Green's function, 709, 713
  - inhomogeneous boundary conditions, 681, 711
  - source term, 710
- helix, 163
- Helmholtz equation, 322, 670, 721
  - general solutions, 676
- Helmholtz theorem, 212
- Hermite polynomials, 513
  - generating function, 516, 517
  - recursion relations, 519
  - Rodrigues formula, 516
  - series solution, 638
- Hermite's equation, 519
- Hermitian operator
  - formal definition, 344, 645
  - matrix, 342
  - orthogonal basis of eigenfunctions, 413, **708**
  - real eigenvalues, 413
  - vs. self-adjoint, **651**
- Hessian, 423
- Hilbert space, 506, **545**
  - rigged, 588
- Hilbert transform, 310
- holomorphic, *see* analytic functions
- homotopy, *see* winding number
- Huygens' principle, 565, 576
- hydrogen atom, 659
- hyperbolic functions, 22
- hyperboloid, 166, 167
- idempotent, 326
- incompressible field, 130, 149, 155–158
- index algebra
  - collapsing a sum, 31
  - contracted index, 29
  - cross product, 33
  - determinant, 35
  - dot product, 33
  - dummy index, 29
  - orthonormality, 31
  - trace, 35
  - upstairs/downstairs, *see* tensors
- infinities, countable and uncountable, **506**
- inner product, 340
  - $2 \times 2$  matrices, 352
  - and projection, 350
  - as a projection, 345, 347, 348
  - of functions, 505
- inner product space
  - formal definition, 345
- Schwarz inequality, 346
- triangle inequality, 346
- integrals
  - circulation, 170
  - flux, 182, 183
  - line, 159
  - surface, 164
- integration techniques
  - axial and spherical symmetry, 88
  - by parts, 80, 210, 215
  - change of variables, 81
  - completing the square, 92
  - differentiation with respect to a parameter, 89
  - expanding the integrand, 94
  - Gaussian, 90, 300
  - parity (even/odd), 83
  - partial fractions, 96
  - powers of sin and cos, 86
- interpolation formula, 608
- inverse functions in the complex plane
  - $\cos^{-1}$ , 267
  - $\sin^{-1}$ , 267
  - $\tan^{-1}$ , 262
- inversion, *see* parity
- inverted pendulum, 642
- irrotational field, 149, 155–158
- Jacobian, 9–12
  - and conformal mapping, 270
- Jordan's lemma, 297
- Joukowski airfoil, **283**
- Kepler's equation, 77, **103**, **115**, 554
- Kepler's laws, 73
- ket, *see* bra-ket notation
- keyhole contour, 301, 302, **312**, **313**
- Klein–Gordon equation, 320
- Kronecker delta, 30, 31
- LACMA, 339
- Lagrange multiplier, 121, 135, **153**
- Laguerre polynomials, 511
  - associated, 521, 522
  - generating function, 516
  - orthogonality, 648
  - recursion relations, 519
  - Rodrigues formula, 516
  - series solution, 644
- Laguerre's equation, 519
- Laplace transform, 594–598
  - Bromwich integral, 595
  - driven oscillator, 597
  - from Fourier, 602
  - properties, 596
- Laplace's equation, 158
  - and conformal mapping, 270
- general solutions, 676
- Laplacian, 136
  - eigenfunctions, **674**
  - in curvilinear coordinates, 142
  - in cylindrical coordinates, 143
  - in spherical coordinates, 143
  - interpretation, 145
  - of a vector, 144
- lattice
  - Bravais, 551
  - direct, 551
  - reciprocal, 552, 582, 592
- Laurent series, 244, 250–254
- Legendre functions
  - associated, 521, 556, 648
  - of the second kind, 627, 647
- Legendre polynomials, 509
  - and electric potential, 515
  - from Gram–Schmidt
  - orthogonalization, 508
  - generating function, 516
  - orthogonality, 647
  - recursion relations, 519
  - Rodrigues formula, 516
  - series solution, 636
- Legendre's equation, 519
- Leibniz rule, 48
- Levi-Civita  $\epsilon$ , 31
  - and the cross product, 33
  - and the determinant, 35
  - beyond three dimensions, 34
  - identities, 32, 39
  - tensor density, 497
- Liénard–Wiechert potential, 716
- Lie algebra, 398
- line integrals, 159–164, 170
- linear independence, 330
- linear operator, 325
- linear translation-invariant (LTI), 593
- Liouville's theorem, 245
- Lippmann–Schwinger equation, 724
- Ln, *see* logarithm, principal branch
- logarithm
  - In vs. Ln, 262
  - in the complex plane, 261–264
  - principal argument, 262
  - principal branch, 262
  - series expansion, 65
- logarithmic derivative, 120
- Lorentz
  - boosts, 388
  - four-vector, 329, 388
  - group, 381, 389, 398
  - invariant, 114
  - matrix, 379–381, 388
  - scalar, 482
  - tensors, 482
  - transformations, 4, 329, 373, 379

## INDEX

763

- magnetic monopoles, 227, 235  
 malice aforethought, 378  
 Mathieu functions, 691  
 Mathieu's equation, 643, 660  
 matrices  
     anti-Hermitian, 416  
     antisymmetric, 30  
     defective, 411  
     determinant, 35  
     function of, 61, 433  
     Hermitian, 342, 413  
     normal, 412, 419  
     orthogonal, 374  
         improper, 381  
         proper, 374  
     positive, 424  
     similar, 392  
     symmetric, 30  
     trace, 35  
     unitary, 376, 416  
 Maxwell speed distribution, 118  
 Maxwell stress tensor, 470  
 Maxwell's equations, 218, 225  
     and charge conservation, 218, 223  
     and electromagnetic waves, 221, 223  
 mean value theorem, 98  
 meromorphic, 241  
 method of images, 701  
 metric  
     cartesian  $\mathbb{R}^3$ , 476  
     inverse  $g^{ab}$ , 479  
     Minkowski spacetime  $\eta_{\mu\nu}$ , 476  
     non-Euclidean, 487, 488  
     Schwarzschild, 497  
     shear  $\mathbb{R}^2$ , 477  
     spherical  $\mathbb{R}^3$ , 476  
     tensor  $g_{ab}$ , 475  
         tensor transformation, 486  
 metric space, 481  
 minimal surface, 169  
 Minkowski space, *see* relativity  
 modified Bessel functions  
     cylindrical, 261, 719, 751  
         generating function, 755  
     spherical, 754  
 moment of inertia, 37, 38  
     principal axes, 405, 414  
     tensor, 462  
 multipole expansion, 511  
  
 Neumann conditions, *see* boundary conditions  
 Neumann functions  
     cylindrical  $N$ , 653, 748  
         recursion relations, 750  
     definition, 748  
     spherical  $n$ , 752  
  
     asymptotic form, 755  
     closed form, 752  
     from cylindrical, 752  
     recursion relations, 753  
 Newton's law of cooling, 621  
 normal modes, 439–451  
     LC circuit, 452  
      $N$ -mass chain, 448  
     bracelet, 446  
     continuum limit, 503  
     double pendulum, 442  
     three-mass system, 443  
     two-mass system, 437  
     zero modes, 444  
     *see also* drumhead normal modes  
 Nyquist  
     frequency, 605  
     rate, 605  
  
 observables, 415  
     complete set of commuting (CSCO), 426  
 operator spectrum, 406  
 orbital anomalies (angles), 76  
 ordinary differential equations (ODE)  
     first order, 616–621  
         isobaric, 622  
     second order, 623  
         complementary solution, 628  
         constant coefficients, 623  
         particular solution, 628  
         series solutions, 630  
         Sturm–Liouville form, 647  
         variation of constants, 628  
         Wronskian, 625  
 orthogonal polynomials  
     generating function, 516  
     recursion relations, 518, 519  
     Rodrigues formula, 516  
 orthogonality, 348  
     continuous basis, 587  
 orthonormal basis, 348  
 oscillations, 3, 61  
     coupled, 437–451  
     damped, 23  
         impulse response, 316  
         interpretation of poles, 317  
 driven, 24, 315, 585, 597, 629  
 equivalent representations, 27  
 resonance, 24  
 transverse, 441  
  
 parametric equations, 162  
 parity  
     and Fourier series, 523  
     and reflections in  $\mathbb{R}^3$ , 384  
 even and odd, 83  
 in Fourier sin and cos series, 531  
  
 of differential operators, 633  
 of spherical harmonics, 557  
 operator, 384  
 pseudotensors, 465  
 violation in weak nuclear force, 385  
 Parseval's theorem, 543  
 partial differential equations (PDE)  
     elliptic, hyperbolic, parabolic, 664  
     separation of variables, 665  
         cartesian, 671  
         cylindrical, 672  
         space and time, 665  
         spherical, 673  
 partial wave expansion, 565, 725  
 partition function, 117  
 Pascal's triangle, 42, 44  
 path independence, 173  
     equivalent statements, 176  
     in the complex plane, 240  
     of thermodynamic state variables, 179  
     *see also* Stokes' theorem  
 Pauli matrices, 326, 334, 352  
     algebra, 366  
     and  $\mathbb{R}^3$  rotations, 377  
 perturbation theory, 722  
 Picard's theorem, 254  
 Plancherel's theorem, *see* Fourier transform  
 Poisson equation, 158, 585  
     Green's function  
         in one dimensions, 695  
         in three dimensions, 695, 698  
 Poisson summation formula, 599  
 pole, 253  
     on the contour, 303  
     simple, 253  
     *see also* singularities  
 polylogarithm, 100, 123, 125  
 potential, 133  
     and conservation of energy, 181  
     complex, 279  
     electromagnetic, 224  
     scalar, 175  
     *see also* path independence, Stokes' theorem  
     vector, 188  
     *see also* surface independence, divergence theorem  
 power spectrum, 545  
     angular, 562  
 Poynting's theorem, 227  
 principal axes, *see* moment of inertia  
 principal branch, 256  
 principal value, 318  
     Ln, 262  
      $\sin^{-1}, \cos^{-1}$ , 267

- principal value (cont.)
  - $\tan^{-1}$ , 263
  - $z^\alpha$ , 263
  - and residues, 303
  - $\text{Arg}$ , 262
  - Cauchy, 84, 295, 303–307
- projection
  - along vs. onto, 473
  - and Gram–Schmidt
    - orthogonalization, 353
  - and inner product, 345, 347
  - and matrix elements, 362
  - Green’s function, 704
  - in quantum mechanics, 361
  - Mercator, 277
  - of vector components, 349–351
  - on Pauli basis, 352
  - onto a continuous basis, 570, 587
  - operator, 366, 547
  - spectral decomposition, 423
  - stereographic, 242, 247, 272
- projection operator, 326
- propagator, *see also* Green’s functions
  - heat equation
    - and thermodynamic arrow of time, 717
  - Klein–Gordon, 321
  - Schrödinger, 320
- quantum measurement, 415
- quantum scattering
  - Born approximation, 721
  - cross section, 721
  - hard sphere, 728
  - scattering amplitude, 721
- quarkonium, 655
- quaternions, 326, 378, **742**
  - see also* rotations
- raindrop, 617, 619, 620
- Rayleigh quotient, 40
- reciprocal space, 552
- reflections, 381–384
- relativity, *see also* Lorentz, 64, 219
  - and hyperbolic geometry, 25
  - four-velocity, 482, 483
  - general covariance, 484
  - Minkowski space, 5, 379, 388, 476, 482, 495
  - principle of, 457
  - proper time, 482, 483, 497
  - rapidity, 379
  - spacetime, 5, 162, 329, 379, 388, 482
  - Thomas precession, 389
- representation, 337
  - faithful, 337, 375, 544, 739
- irreducible, 435
  - matrix, 362
- residue theorem, 288
- residues, 244
  - see also* calculus of residues at infinity, 291
  - formulas, 289
  - general formula, 287
  - of a quotient, 289
  - of an essential singularity, 289
  - partial fractions, 292
  - simple pole, 288
  - Sommerfeld–Watson transformation, 308
- Riemann sphere, 242
- Riemann surface, **264**
- rotations, 9, 368
  - $2\pi$  vs.  $4\pi$  rotation, 740
  - angle-axis, 733
  - Euler angles, 735–738
  - generators, 395–399
  - improper, 381
  - in  $\mathbb{R}^n$ , 386
  - in complex Euclidean space, 376
  - in real Euclidean space, 374
  - in spacetime, 379
    - boosts, 388
    - rapidity, 379
  - in the complex plane, 19
  - in three space, 385–388
    - interpretation of eigensystem, 417
  - non-commutativity, 336, 387
  - orthogonal vs. unitary representations, 742
  - proper, 374
  - quaternions, 739–743
  - Rodrigues formula, 734
  - spin, nutation, precession, 738
  - Runge–Lenz vector, 73
  - sampling theorem, 605
  - scale factors, 9–12
  - Schrödinger equation, 320
    - and square integrability, 660
    - equivalent integral representation, 722
    - free particle, 649
    - Green’s function, 320, 717
    - harmonic oscillator, 638
    - hydrogen atom, 659
    - particle in a box, 533
    - probability current, 226
    - quantum pendulum, 659
    - quarkonium, 655
    - radial equation, 725
    - retarded propagator, 320
    - scattering, 721
  - time independent, 418, 653, 688
  - Schwarz inequality, 346
    - see also* inner product space
  - Schwarzschild, *see* metric
  - secular determinant, *see* eigenvalue problem
  - self-adjoint, *see* Hermitian
  - separation of variables, 665
  - series
    - geometric, 51
    - harmonic, 54, **55**
    - power, 58
    - Taylor, 60
  - sgn function, 100
  - Shah function, *see* Dirac comb
  - shape of space, **740**
  - shear, **151**, 205
  - similarity transformations, 391–395
  - simply connected region, 228–235
  - sinc function, 575
  - single valued, 232, 279
  - singularities, 252–254
    - branch point, 255
    - isolated, 252
      - essential, 253
      - pole, 253
        - removable, 253
      - regular and irregular, 633
    - skew coordinates, 13
    - solid angle, 11
    - Sommerfeld–Watson transformation, 308
    - spacetime, *see* relativity
    - spectral decomposition, *see* eigenvalue problem
    - spectral theorem, **651**, 658
    - sphere, *n*-dimensional, **13**, 102
    - spherical harmonics, 557
      - in Poisson Green’s function, 698, 701
      - addition theorem, 561
      - as orbitals, 559
      - Jeans rule, 566
      - ladder operators, 568
      - normal modes on a sphere, 558
      - properties, 560
    - spin, 361, 367, 399, 740
    - square integrable functions, 505, 507, **573**, 660
    - standard basis, 331
    - stationary phase approximation, 314, 747
    - steepest descent, method of, 103
    - step function, 110, 114
      - integral representation, 312
    - stereographic projection, 242, 247, 272, **277**
    - Stirling approximation, 50, 103

## INDEX

765

- Stokes' theorem, 190, 203–209, 223
  - alternate forms, 205
- strain tensor, 460
- stream function, 155, 279–285
- streamline, 190
  - vector field representation, 129, 131
- stress tensor, 458
- Sturm–Liouville, 645
  - and the Schrödinger equation, 653
  - eigenvalue equation, 647
  - general properties, 652
  - Wronskian, 641
- superposition, 322, 325
  - and linear differential equations, 666
  - and quantum mechanics, 415, 565
  - and vector space, 328
  - of electric fields, 130
  - of gravitational fields, 178
  - of normal modes, 443
  - of plane waves, 106
- surface independence, 188–193
  - equivalent statements, 192
  - see also* divergence theorem
- surface integrals, 164–168
- tangent vectors, **168**
- Taylor expansion, 60
  - in more than one dimension, 63
- tensors
  - covariant derivative, 490
  - densities, 497
  - derivatives, 488
  - duals, 467
  - dyad, 461
  - general transformation, 463
  - integration, 487
  - invariant, 466
  - irreducible, **468**
  - Lorentz, 482
  - metric, 475
- pseudotensors, 465
- tensor product, 461
- upstairs/downstairs indices, 477
- thermodynamics, 179–181
- top hat function, 578, 591, 594, 607
- trace, 35
  - $\ln \det$ , 68, 433
  - as inner product, 352
  - sum of eigenvalues, 421
- triangle inequality, *see* inner product space
- Trotter formula, 72
- uncertainty principle, **428**, 576
- unitary operator
  - and stationary states, 418
  - basis of eigenvectors, 417
  - eigenvalues, 416
- vector, *see also* basis
  - adjoint, 340
  - and arrows, **336**
  - as equivalence class of rotation, 460
  - as scalar function, 334
  - contravariant and covariant, 481
  - fields, 129
    - as gradient of a scalar, 133
    - circulation, 170
    - flux, 182
    - incompressible, 130, 149, 155–158
    - irrotational, 149, 155–158
    - Laplacian, 144
    - path independent, 176
    - surface independent, 192
    - tangent to streamline, 130, 131
      - see also* gradient, divergence, curl
  - identities
    - derivative, 141
  - in  $\mathbb{R}^2$  as a complex number, 239
- index algebra, 28
- dot and cross products, 33
- label vs. index, 31
- inner product
  - continuous, 505
  - discrete, 340
- norm, 341
- position-dependent basis, 6
- projection, 345–348
- vector space
  - $\mathbb{L}^2$ , 505
  - basic description, 328–329
  - complete, *see* Hilbert space
  - dual spaces, 340
  - formal definition, **338**
  - infinite dimensional, 504
  - linearity, 325
  - of functions
    - polynomials, 334
    - sine and cosine, 335
- wave equation, 221, 318, 663
  - d'Alembert solution, 689
  - for electric and magnetic fields, 221
  - Green's function, 699, 714
- wave packet, *see* Fourier transform
- waveforms, 21
- Weierstrass *M* test, *see* convergence
- Wigner *D* matrix, 435
- winding number, **231**, 232, *313*
- Wronskian, 626
  - of Bessel *J*, 748
- Young's modulus, 459
- Yukawa potential, 216, 724
- zero modes, 444
- zeros of a function, 252
- zeta function, 52, 94, 95, 249
  - analytic continuation, 249