

Contents

<i>Preface to the Second Edition</i>	<i>page xv</i>
<i>Preface to the First Edition</i>	xvii
1 Events and Probability	1
1.1 Application: Verifying Polynomial Identities	1
1.2 Axioms of Probability	3
1.3 Application: Verifying Matrix Multiplication	8
1.4 Application: Naïve Bayesian Classifier	12
1.5 Application: A Randomized Min-Cut Algorithm	15
1.6 Exercises	17
2 Discrete Random Variables and Expectation	23
2.1 Random Variables and Expectation	23
2.1.1 Linearity of Expectations	25
2.1.2 Jensen's Inequality	26
2.2 The Bernoulli and Binomial Random Variables	27
2.3 Conditional Expectation	29
2.4 The Geometric Distribution	33
2.4.1 Example: Coupon Collector's Problem	35
2.5 Application: The Expected Run-Time of Quicksort	37
2.6 Exercises	40
3 Moments and Deviations	47
3.1 Markov's Inequality	47
3.2 Variance and Moments of a Random Variable	48
3.2.1 Example: Variance of a Binomial Random Variable	51

CONTENTS

3.3	Chebyshev's Inequality	51
3.3.1	Example: Coupon Collector's Problem	53
3.4	Median and Mean	55
3.5	Application: A Randomized Algorithm for Computing the Median	57
3.5.1	The Algorithm	58
3.5.2	Analysis of the Algorithm	59
3.6	Exercises	62
4	Chernoff and Hoeffding Bounds	66
4.1	Moment Generating Functions	66
4.2	Deriving and Applying Chernoff Bounds	68
4.2.1	Chernoff Bounds for the Sum of Poisson Trials	68
4.2.2	Example: Coin Flips	72
4.2.3	Application: Estimating a Parameter	72
4.3	Better Bounds for Some Special Cases	73
4.4	Application: Set Balancing	76
4.5	The Hoeffding Bound	77
4.6*	Application: Packet Routing in Sparse Networks	79
4.6.1	Permutation Routing on the Hypercube	80
4.6.2	Permutation Routing on the Butterfly	85
4.7	Exercises	90
5	Balls, Bins, and Random Graphs	97
5.1	Example: The Birthday Paradox	97
5.2	Balls into Bins	99
5.2.1	The Balls-and-Bins Model	99
5.2.2	Application: Bucket Sort	101
5.3	The Poisson Distribution	101
5.3.1	Limit of the Binomial Distribution	105
5.4	The Poisson Approximation	107
5.4.1*	Example: Coupon Collector's Problem, Revisited	111
5.5	Application: Hashing	113
5.5.1	Chain Hashing	113
5.5.2	Hashing: Bit Strings	114
5.5.3	Bloom Filters	116
5.5.4	Breaking Symmetry	118
5.6	Random Graphs	119
5.6.1	Random Graph Models	119
5.6.2	Application: Hamiltonian Cycles in Random Graphs	121
5.7	Exercises	127
5.8	An Exploratory Assignment	133
6	The Probabilistic Method	135
6.1	The Basic Counting Argument	135

CONTENTS

6.2	The Expectation Argument	137
6.2.1	Application: Finding a Large Cut	138
6.2.2	Application: Maximum Satisfiability	139
6.3	Derandomization Using Conditional Expectations	140
6.4	Sample and Modify	142
6.4.1	Application: Independent Sets	142
6.4.2	Application: Graphs with Large Girth	143
6.5	The Second Moment Method	143
6.5.1	Application: Threshold Behavior in Random Graphs	144
6.6	The Conditional Expectation Inequality	145
6.7	The Lovász Local Lemma	147
6.7.1	Application: Edge-Disjoint Paths	150
6.7.2	Application: Satisfiability	151
6.8*	Explicit Constructions Using the Local Lemma	152
6.8.1	Application: A Satisfiability Algorithm	152
6.9	Lovász Local Lemma: The General Case	155
6.10*	The Algorithmic Lovász Local Lemma	158
6.11	Exercises	162
7	Markov Chains and Random Walks	168
7.1	Markov Chains: Definitions and Representations	168
7.1.1	Application: A Randomized Algorithm for 2-Satisfiability	171
7.1.2	Application: A Randomized Algorithm for 3-Satisfiability	174
7.2	Classification of States	178
7.2.1	Example: The Gambler's Ruin	181
7.3	Stationary Distributions	182
7.3.1	Example: A Simple Queue	188
7.4	Random Walks on Undirected Graphs	189
7.4.1	Application: An s - t Connectivity Algorithm	192
7.5	Parrondo's Paradox	193
7.6	Exercises	198
8	Continuous Distributions and the Poisson Process	205
8.1	Continuous Random Variables	205
8.1.1	Probability Distributions in \mathbb{R}	205
8.1.2	Joint Distributions and Conditional Probability	208
8.2	The Uniform Distribution	210
8.2.1	Additional Properties of the Uniform Distribution	211
8.3	The Exponential Distribution	213
8.3.1	Additional Properties of the Exponential Distribution	214
8.3.2*	Example: Balls and Bins with Feedback	216
8.4	The Poisson Process	218
8.4.1	Interarrival Distribution	221

CONTENTS

8.4.2	Combining and Splitting Poisson Processes	222
8.4.3	Conditional Arrival Time Distribution	224
8.5	Continuous Time Markov Processes	226
8.6	Example: Markovian Queues	229
8.6.1	$M/M/1$ Queue in Equilibrium	230
8.6.2	$M/M/1/K$ Queue in Equilibrium	233
8.6.3	The Number of Customers in an $M/M/\infty$ Queue	233
8.7	Exercises	236
9	The Normal Distribution	242
9.1	The Normal Distribution	242
9.1.1	The Standard Normal Distribution	242
9.1.2	The General Univariate Normal Distribution	243
9.1.3	The Moment Generating Function	246
9.2*	Limit of the Binomial Distribution	247
9.3	The Central Limit Theorem	249
9.4*	Multivariate Normal Distributions	252
9.4.1	Properties of the Multivariate Normal Distribution	255
9.5	Application: Generating Normally Distributed Random Values	256
9.6	Maximum Likelihood Point Estimates	258
9.7	Application: EM Algorithm For a Mixture of Gaussians	261
9.8	Exercises	265
10	Entropy, Randomness, and Information	269
10.1	The Entropy Function	269
10.2	Entropy and Binomial Coefficients	272
10.3	Entropy: A Measure of Randomness	274
10.4	Compression	278
10.5*	Coding: Shannon's Theorem	281
10.6	Exercises	290
11	The Monte Carlo Method	297
11.1	The Monte Carlo Method	297
11.2	Application: The DNF Counting Problem	300
11.2.1	The Naïve Approach	300
11.2.2	A Fully Polynomial Randomized Scheme for DNF Counting	302
11.3	From Approximate Sampling to Approximate Counting	304
11.4	The Markov Chain Monte Carlo Method	308
11.4.1	The Metropolis Algorithm	310
11.5	Exercises	312
11.6	An Exploratory Assignment on Minimum Spanning Trees	315

CONTENTS

12	Coupling of Markov Chains	317
12.1	Variation Distance and Mixing Time	317
12.2	Coupling	320
12.2.1	Example: Shuffling Cards	321
12.2.2	Example: Random Walks on the Hypercube	322
12.2.3	Example: Independent Sets of Fixed Size	323
12.3	Application: Variation Distance Is Nonincreasing	324
12.4	Geometric Convergence	327
12.5	Application: Approximately Sampling Proper Colorings	328
12.6	Path Coupling	332
12.7	Exercises	336
13	Martingales	341
13.1	Martingales	341
13.2	Stopping Times	343
13.2.1	Example: A Ballot Theorem	345
13.3	Wald's Equation	346
13.4	Tail Inequalities for Martingales	349
13.5	Applications of the Azuma–Hoeffding Inequality	351
13.5.1	General Formalization	351
13.5.2	Application: Pattern Matching	353
13.5.3	Application: Balls and Bins	354
13.5.4	Application: Chromatic Number	355
13.6	Exercises	355
14	Sample Complexity, VC Dimension, and Rademacher Complexity	361
14.1	The Learning Setting	362
14.2	VC Dimension	363
14.2.1	Additional Examples of VC Dimension	365
14.2.2	Growth Function	366
14.2.3	VC dimension component bounds	368
14.2.4	ϵ -nets and ϵ -samples	369
14.3	The ϵ -net Theorem	370
14.4	Application: PAC Learning	374
14.5	The ϵ -sample Theorem	377
14.5.1	Application: Agnostic Learning	379
14.5.2	Application: Data Mining	380
14.6	Rademacher Complexity	382
14.6.1	Rademacher Complexity and Sample Error	385

CONTENTS

14.6.2	Estimating the Rademacher Complexity	387
14.6.3	Application: Agnostic Learning of a Binary Classification	388
14.7	Exercises	389
15	Pairwise Independence and Universal Hash Functions	392
15.1	Pairwise Independence	392
15.1.1	Example: A Construction of Pairwise Independent Bits	393
15.1.2	Application: Derandomizing an Algorithm for Large Cuts	394
15.1.3	Example: Constructing Pairwise Independent Values Modulo a Prime	395
15.2	Chebyshev's Inequality for Pairwise Independent Variables	396
15.2.1	Application: Sampling Using Fewer Random Bits	397
15.3	Universal Families of Hash Functions	399
15.3.1	Example: A 2-Universal Family of Hash Functions	401
15.3.2	Example: A Strongly 2-Universal Family of Hash Functions	402
15.3.3	Application: Perfect Hashing	404
15.4	Application: Finding Heavy Hitters in Data Streams	407
15.5	Exercises	411
16	Power Laws and Related Distributions	415
16.1	Power Law Distributions: Basic Definitions and Properties	416
16.2	Power Laws in Language	418
16.2.1	Zipf's Law and Other Examples	418
16.2.2	Languages via Optimization	419
16.2.3	Monkeys Typing Randomly	419
16.3	Preferential Attachment	420
16.3.1	A Formal Version	422
16.4	Using the Power Law in Algorithm Analysis	425
16.5	Other Related Distributions	427
16.5.1	Lognormal Distributions	427
16.5.2	Power Law with Exponential Cutoff	428
16.6	Exercises	429
17	Balanced Allocations and Cuckoo Hashing	433
17.1	The Power of Two Choices	433
17.1.1	The Upper Bound	433
17.2	Two Choices: The Lower Bound	438
17.3	Applications of the Power of Two Choices	441
17.3.1	Hashing	441
17.3.2	Dynamic Resource Allocation	442
17.4	Cuckoo Hashing	442
17.5	Extending Cuckoo Hashing	452
17.5.1	Cuckoo Hashing with Deletions	452

CONTENTS

17.5.2 Handling Failures	453
17.5.3 More Choices and Bigger Bins	454
17.6 Exercises	456
<i>Further Reading</i>	463
<i>Index</i>	464

Note: Asterisks indicate advanced material for this chapter.