

STRUCTURED DEPENDENCE BETWEEN STOCHASTIC PROCESSES

The relatively young theory of structured dependence between stochastic processes has many real-life applications in areas including finance, insurance, seismology, neuroscience, and genetics. With this monograph, the first to be devoted to the modeling of the structured dependence between random processes, the authors not only meet the demand for a solid theoretical account but also develop a stochastic processes counterpart of the classical copula theory that exists for finite-dimensional random variables.

Presenting both the technical aspects and the applications of the theory, this is a valuable reference for researchers and practitioners in the field, as well as for graduate students in pure and applied mathematics programs. Numerous theoretical examples are included, alongside examples of both current and potential applications, aimed at helping those who need to model the structured dependence between dynamic random phenomena.

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