Connections in Discrete Mathematics

Discrete mathematics has been rising in prominence in the past fifty years, both as a tool with practical applications and as a source of new and interesting mathematics. The topics in discrete mathematics have become so well developed that it is easy to forget that common threads connect the different areas, and it is through discovering and using these connections that progress is often made.

For more than fifty years, Ron Graham has been able to illuminate some of these connections and has helped bring the field of discrete mathematics to where it is today. To celebrate his contribution, this volume brings together many of the best researchers working in discrete mathematics, including Fan Chung, Erik Demaine, Persi Diaconis, Peter Frankl, Al Hales, Jeffrey Lagarias, Allen Knutson, Janos Pach, Carl Pomerance, Neil Sloane, and of course Ron Graham himself.

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Connections in Discrete Mathematics

A Celebration of the Work of Ron Graham

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Preface

The last fifty years have seen rapid growth in the rise of discrete mathematics, in areas ranging from the classics of number theory and geometry to the modern tools in computation and algorithms, with hundreds of topics in between. Part of this growth is driven by the increasing availability and importance of computational power, and part is due to the guiding influence and leadership of mathematicians in this field who have helped to encourage generations of mathematicians to pursue research in this area.

Among these mathematicians who have played a leadership role, Ron Graham stands out for his contributions to theory, his visibility to the larger community, his role in mentoring many young mathematicians, and for his longevity.

In 1962, Ron Graham finished his dissertation in combinatorial number theory under the leadership of Derrick Lehmer. He soon found himself at Bell Labs, where he would spend the next thirty-seven years, including as director of the Mathematical Sciences Research Center and as Chief Scientist of AT&T Labs – Research, before his (first) retirement in 1999. During this time he helped bring together many of the best and brightest young minds in discrete mathematics to work in the Labs and would guide their research and prepare them for their later careers. After Bell Labs, Ron went to University of California, San Diego to become the Irwin and Joan Jacobs Chair in Information and Computer Science in the Department of Computer Science and Engineering until his (second) retirement in 2016. He is now emeritus professor at University of California, San Diego and remains mathematically active.

Over the past 50 years, he has had a constant flow of new ideas, with more than 300 papers, a half dozen books, and hundreds of editorial assignments. He has also traveled the world giving talks on mathematics (sometimes demonstrating some of his acrobatic skills such as one-armed handstands and juggling, showing students that mathematicians aren't the stereotypical introverts). Thanks to his friendship with Paul Erdős (30 joint papers!) and his frequent

travel, Ron played an important role as a bridge to mathematics in Hungary and several other countries at a time when communication and collaboration were limited. As president of the American Mathematical Society and later the Mathematical Association of America, Ron led the largest mathematical societies in the world. In addition to all of this, Ron Graham has been recognized with numerous awards, accolades, and honorary degrees. It is hard to believe that one individual was able to accomplish so much, but as Ron likes to point out "there are twenty-four hours in the day, and if that's not enough, there are also the nights."

In 2015, Ron Graham turned eighty, and to help mark this occasion a special conference was organized, *Connections in Discrete Mathematics*. This was a chance to bring together many of his friends and colleagues, the best and brightest in discrete mathematics, to celebrate Ron, and also to celebrate discrete mathematics. A major theme of the conference was connections, both the personal connections (as Ron had with so many speakers and participants) as well as the connections between mathematical topics. Both types of connections are what lead to advances in mathematics and open up new ideas for exploration.

This book came out of the conference, with many of the authors having been featured speakers. The chapters here are across the spectrum of discrete mathematics, with topics in number theory, probability, graph theory, Ramsey theory, discrete geometry, algebraic combinatorics, and, of course, juggling. A beautiful mix of topics and also of writing styles, this book has something for everyone.

We thank the many authors for their excellent contributions, including Joe Buhler, Fan Chung, Erik Demaine, Persi Diaconis, Peter Frankl, Al Hales, Jeffrey Lagarias, Allen Knutson, Jarik Nešetřil, Janos Pach, Carl Pomerance, Vojtěch Rödl, Neil Sloane, Tom Trotter, Catherine Yan, and of course Ron Graham. We were happy to see the wealth of ideas that were in these chapters and hope that readers will find something that inspires them.

All three of the editors have been heavily influenced by Ron Graham and his friendship, and like many people in our field, we would not be where we are today without him. Thank you, Ron.

Fig. 0.1. Ron Graham demonstrating a large Rubik's cube to C. K. Cheng, Kevin Milans, and Joel Spencer at the *Connections in Discrete Mathematics* conference in June 2015. Image courtesy of IRMACS. Used with permission.