

Index of notations

Set theory

$\emptyset$ is the empty set.

$x \in X$ denotes that x is an element of the set X .

$X \subseteq Y$ means that X is a subset of Y .

$\bar{X}$ denotes the closure of X (in Y).

$\{x \mid A(x)\}$ is the set of objects x satisfying a condition $A(x)$.

$X \times Y$ is the set of pairs $\{(x, y) \mid x \in X, y \in Y\}$.

$\Delta(X)$ is the diagonal subset $\Delta(X) := \{(x, y) \in X \times X \mid x = y\}$ of $X \times X$.

$f : X \rightarrow Z$ means that f is a map from X to Z .

$\text{Im } f$ denotes the image of f : $\{f(x) \mid x \in X\}$.

$|$ denotes restriction, for example, for $Y \subseteq X \xrightarrow{f} Z$, $f|Y$ is the restriction of f to Y .

$\circ$ denotes composite: the composite $g \circ f$ is given by $g \circ f(x) := g(f(x))$.

$\mathbb{R}$ is the set of real numbers,

$\mathbb{R}^n$ is the vector space of n -tuples $x = (x_1, \dots, x_n)$ with each $x_k \in \mathbb{R}$,

$\|x\| := \sqrt{(x_1^2 + \dots + x_n^2)}$.

$\mathbb{R}_+^n$ is the subset with $x_1 \geq 0$, $\mathbb{R}_{++}^n$ the subset with $x_1 \geq 0, x_2 \geq 0$.

$[a, b]$ is the closed interval $\{x \in \mathbb{R} \mid a \leq x \leq b\}$;

$[a, b)$ is the half-open interval $a \leq x < b$ (allowing $b = \infty$); similarly $(a, b]$.

$D_x^n(r)$ is the closed disc $\{y \in \mathbb{R}^n \mid \|x - y\| \leq r\}$,

$S_x^{n-1}(r)$ the sphere $\{y \in \mathbb{R}^n \mid \|x - y\| = r\}$,

$\dot{D}_x^n(r)$ the open disc $\{y \in \mathbb{R}^n \mid \|x - y\| < r\}$,

$D_+^n(r) := D_x^n(r) \cap \mathbb{R}_+^n$ is the closed half-disc

$\dot{D}_+^n(r) := \dot{D}_x^n(r) \cap \mathbb{R}_+^n$ the open half-disc.

If x is omitted, the centre is the origin; if r is omitted, the radius is $r = 1$.

$D^k(a, b) := \{x \in \mathbb{R}^k \mid a \leq \|x\| \leq b\}$.

Thus $I := [0, 1] = D_+^1$ and $\mathbb{R}^+ := \mathbb{R}_+^1 = [0, \infty)$.
 $u : \mathbb{R}^n \setminus \{0\} \rightarrow S^{n-1}$ is defined by $u(x) := x/\|x\|$. (§4.2).

Groups, fields, etc.

$\mathbb{Z}$ is the ring of integers.

$\mathbb{Z}_n$ is the additive group of integers modulo the natural number n .

$\mathbb{R}$ is the field of real numbers.

$\mathbb{Q}$ is the field of rational numbers.

$\mathbb{C}$ is the field of complex numbers.

$\sigma(q)$ is the signature of a quadratic form q defined over $\mathbb{R}$,

$\text{Arf}(\mu)$ is the Arf invariant of a quadratic form μ defined over $\mathbb{Z}_2$.

$\mathfrak{S}(\mu)$ is the Gauss sum of the quadratic form μ on a finite group.

$P(A_*; F)(t) := \sum_0^\infty \dim_F(A_n)t^n$ is the Poincaré series of A_* .

$|G|$ is the order of the finite group G .

$\text{Tors}(A)$ is the torsion subgroup of the abelian group A .

$A \oplus B$ is the direct sum of A and B ;

$A \otimes B$ is the tensor product of A and B .

$G^\vee$ is the dual group to G .

$\text{Ker}(\phi)$ is the kernel of the group homomorphism $\phi : A \rightarrow B$;

$\text{Coker}(\phi)$ is the cokernel of $\phi : A \rightarrow B$.

G/H is the quotient (space) of (right cosets) of a group G by a subgroup H . If

H is a normal subgroup of G , this is the quotient group.

$\mathbf{GL}_m(K)$ is the group of nonsingular $(m \times m)$ matrices over the field K .

$\mathbf{SL}_m(K)$ is the subgroup of matrices of determinant 1.

$\mathbf{GL}_m^+(\mathbb{R}) \subset \mathbf{GL}_m(\mathbb{R})$ is the subgroup of matrices with positive determinant.

$\mathbf{O}_m \subset \mathbf{GL}_m(\mathbb{R})$ is the orthogonal group, $\{A \in \mathbf{GL}_m(\mathbb{R}) \mid AA^t = I\}$.

$\mathbf{U}_m \subset \mathbf{GL}_m(\mathbb{C})$ is the unitary group, $\{A \in \mathbf{GL}_m(\mathbb{C}) \mid AA^{\overline{t}} = I\}$.

$\mathbf{SO}_m := \mathbf{O}_m \cap \mathbf{SL}_m(\mathbb{R})$.

$\mathbf{SU}_m := \mathbf{U}_m \cap \mathbf{SL}_m(\mathbb{C})$.

G_n is the monoid of maps of S^{n-1} to itself of degree ± 1 .

$F_n \subset G_{n+1}$ is the set of base-point preserving maps $S^n \rightarrow S^n$.

Top_n is defined in §8.9.

$SG_n, SF_n, \text{STop}_n$ are the corresponding subsets of orientation-preserving maps.

For each of the above groups and monoids C_n ,

$B(C_n)$ is the classifying space of C_n ;

$B(G_n)$ is the classifying space for spherical fibrations with fibre S^{n-1} ;

C is the union of the C_n ; and

$B(C)$ is the inductive limit of the sequence $B(C_n)$.

Manifolds, etc.

$Bp(x)$ is the bump function (§1.1).

$T_P M$ is the tangent space at $P \in M$ to the smooth manifold M ;

$T_P^\vee M$ is the dual vector space.

$\mathbb{T}(M)$ is the tangent vector bundle of M ; the dual is $\mathbb{T}^\vee(M)$.

$\mathbb{T}^0(M)$ is the zero cross-section.

$\mathbb{N}(M/V)$ is the normal bundle of the smooth submanifold $V \subset M$.

∂M is the boundary of M : for example, $\partial D_x^n(r) = S_x^{n-1}(r)$.

$\angle M$ is the corner of M .

$\overset{\circ}{M} := M \setminus \partial M$ is the interior of M .

$\partial_- W$, $\partial_+ W$ and $\partial_c W$ are the lower, upper, and middle parts of the boundary ∂W of a cobordism W .

$D(M)$ is the double of M .

$M_1 \# M_2$ is the connected sum of manifolds M_1 and M_2 .

$M_1 + M_2$ is the boundary sum of M_1 and M_2 (§2.7).

$P^m(\mathbb{R}) = P(\mathbb{R}^{m+1})$ is the set of lines through the origin in $\mathbb{R}^{m+1}$;

$P^m(\mathbb{C}) = P(\mathbb{C}^{m+1})$ the set of lines in $\mathbb{C}^{m+1}$.

$P^\infty(\mathbb{R}) := \bigcup_{n \in \mathbb{N}} P^n(\mathbb{R})$; $P^\infty(\mathbb{C}) := \bigcup_{n \in \mathbb{N}} P^n(\mathbb{C})$.

$Gr_{m,k}$ is the Grassmann manifold of k -dimensional subspaces of $\mathbb{R}^m$.

$V_{m,k} \cong O_m/O_k$ is the Stiefel manifold of isometric embeddings $\mathbb{R}^k \rightarrow \mathbb{R}^m$.

$V'_{m,k} \cong GL_m(\mathbb{R})/GL_k(\mathbb{R})$ is the set of linear embeddings $\mathbb{R}^k \rightarrow \mathbb{R}^m$.

$J^k(V, M)$ is the space of k -jets of maps $V \rightarrow M$;

$j^k f : V \rightarrow J^k(V, M)$ is the k -jet of the map $f : V \rightarrow M$;

$V^{(r)}$ is the subset of V^r consisting of r -tuples of *distinct* points of V .

${}_r J^k(V, M)$ is the subset of $(J^k(V, M))^r$ lying over $V^{(r)}$.

${}_r j^k f : V^{(r)} \rightarrow {}_r J^k(V, M)$ is the multijet of $f : V \rightarrow M$. §4.4.

$C^r(V, M)$ is the set of C^r maps $V \rightarrow M$ ($0 \leq r \leq \infty$);

$C_{pr}^r(V, M)$ is the set of proper C^r maps.

$\text{Imm}(V, M)$ is the set of (smooth) immersions $V \rightarrow M$;

$\text{Emb}(V, M)$ is the set of (smooth) embeddings $V \rightarrow M$;

$\text{Diff}(M)$ is the set of diffeomorphisms of M .

Σ^i , $\Sigma^i(V, M) \subset J^1(V, M)$, $\Sigma^i f$ are Thom-Boardman sets: see §4.5.

$I(\phi)$ is the number of double points of an immersion $\phi : V^k \rightarrow M^{2k}$.

Cobordism theory

For ξ an orthogonal bundle (or spherical fibration), we write

A_ξ for the associated disc bundle,

S_ξ for the sphere bundle,

$T(\xi) = A_\xi/S_\xi$ for the Thom space,

B_ξ for the base.

For ξ the universal bundle over $B(C_n)$, these become $A(C_n)$, $S(C_n)$, $T(C_n)$.

$\Omega_m(X, v)$ is the set of normal cobordism classes of maps of degree 1 to X .

$P_m := \Omega_m(D^m, \varepsilon)$.

$\text{Kerv}(\phi, v, T)$ is the Kervaire invariant of a normal cobordism class.

L_m is $\mathbb{Z}$, 0, $\mathbb{Z}_2$ or 0 according as $m \equiv 0, 1, 2$ or $3 \pmod{4}$.

Ω_m^G is the cobordism group of m -manifolds with (weak) G -structure.

Ω_m^{fr} is the framed cobordism group.

Θ_m^k is the group of homotopy spheres $\Sigma^m \subset S^{m+k}$,

$F\Theta_m^k$ is the group of framed homotopy spheres $\Sigma^m \subset S^{m+k}$,

Σ_m^k is the group of embeddings $S^m \subset S^{m+k}$.

B_k is the k th Bernoulli number.

Homology theory

$H_r(X, Y; A)$ is the r th homology group of (X, Y) with coefficients in A .

If A is omitted, it is taken as $\mathbb{Z}$.

$\tilde{H}^k(X, Y)$ is the reduced cohomology group.

$K_k(M) := \text{Ker}(\phi_* : H_k(M) \rightarrow H_k(X))$ for $\phi : M \rightarrow X$ a normal map.

$[M]$ is the fundamental homology class of the manifold M .

$\beta_p : H^k(X; \mathbb{Z}_p) \rightarrow H^{k+1}(X; \mathbb{Z}_p)$ is the Bockstein homomorphism.

$\mathcal{S}_p$ is the mod p Steenrod algebra,

χ its canonical anti-automorphism,

$\overline{\mathcal{S}}_p := \mathcal{S}_p / \langle \beta_p \rangle$.

$K(\pi, n)$ is the Eilenberg–MacLane space.

$J_k : \pi_k(SO) \rightarrow \pi_k^S$ is the stable J homomorphism.

$w_k(\xi), v_k(\xi) \in H^k(X; \mathbb{Z}_2)$ are the Stiefel–Whitney, Wu classes of a bundle ξ .

If $\xi \oplus \eta$ is trivial, $\overline{w}_k(\xi) = w_k(\eta)$.

$c_k \in H^{2k}(X; \mathbb{Z})$ is the k th Chern class,

$p_{4k} \in H^{4k}(X; \mathbb{Z})$ are the Pontrjagin classes.

Homotopy theory

$*$ is the base point.

X^+ is the disjoint union of X and $*$.

$X \wedge Y$ is the smash product of X and Y .

$X * Y$ is the joint of X and Y .

- $[X : Y]$ is the set of (based) homotopy classes of maps $X \rightarrow Y$.
- $\pi_r(X, Y)$ is the r th homotopy group of (X, Y) .
- $K^{(n)}$ is the n -skeleton of K .
- $X^{(k)}$ is a $(k - 1)$ -connected cover of X .
- $SX := S^1 \wedge X$ is the suspension of X .
- ΩX is the loop space of X .
- $\{X : Y\} = \lim_{n \rightarrow \infty} [S^n X : S^n Y]$.
- $\pi_r^S(X) := \{S^r : X\}$.
- $\mathbb{S}$ is the sphere spectrum.
- $\mathbb{K}(A, k)$ is the Eilenberg–MacLane spectrum.
- $\mathbb{TG}$ is the classifying spectrum of the stable group G (in the sense of §8.2).
- $\mathbb{BU}$ and $\mathbb{BO}$ are the Bott spectra, with connective versions $\mathbb{BU}\langle k \rangle$ and $\mathbb{BO}\langle k \rangle$.

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