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C. T. C. Wall is Emeritus Professor in the Division of Pure Mathematics at the University of Liverpool. During his career he has held positions at Oxford and Cambridge and been invited as a major speaker to numerous conferences in Europe, the USA, and South America. He was elected Fellow of the Royal Society in 1969.

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