QUINE, NEW FOUNDATIONS, AND THE PHILOSOPHY OF SET THEORY

W. V. Quine’s set theory, New Foundations, has often been treated as an anomaly in the history and philosophy of set theory. In this book, Sean Morris shows that it is, in fact, well motivated, emerging in a natural way from the early development of set theory. Morris introduces and explores the notion of set theory as explication: the view that there is no single correct axiomatization of set theory but rather various axiomatizations that all serve to explicate the notion of set and are judged largely according to pragmatic criteria. Morris also brings out the important interplay between New Foundations, Quine’s philosophy of set theory, and his philosophy more generally. We see that Quine’s early technical work in logic foreshadows his later famed naturalism, with his philosophy of set theory playing a crucial role in his primary philosophical project of clarifying our conceptual scheme and, specifically, its logical and mathematical components.

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This book is dedicated to Bill Hart and to Haewon and John.
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Preface

This book emerged out of my doctoral dissertation at the University of Illinois at Chicago, written some years ago under the direction of W. D. Hart. After completing the dissertation, I moved on to researching topics of a more general nature in Quine’s philosophy, though always with an eye toward how his early work in logic and the foundations of mathematics shaped his general approach to philosophy. From time to time, I continued to present some of my views on Quine’s philosophy of set theory and its contrasts with much contemporary philosophy of set theory. I remained surprised by – in discussions of the philosophy of set theory – how set theory was just about always and without question identified with Zermelo-Fraenkel set theory and its variants and the associated iterative conception of set. From this, it seemed to me that there might still be something to contribute to contemporary discussions of the philosophy of set theory by focusing on Quine’s alternative approach – an approach that ignores the iterative conception, at least as a metaphysical view about sets, and that is a good deal more experimental and exploratory in nature, seeing set theory as a still largely unsettled area of the mathematical sciences.

In assembling the following list of acknowledgments, I could not help being reminded of the words of a recent Nobel laureate:

Some are mathematicians
Some are carpenters’ wives …
We always did feel the same
We just saw it from a different point of view.¹

The support and inspiration I drew throughout the writing of this book, teachers, friends, and colleagues supported and inspired me in ways that were as varied as they were indispensable. The paths that many of these

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