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Figures are in **bold**. Individual singularity or bifurcation types are listed under ‘singularity type’ or ‘bifurcation type’ respectively

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