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KEY: Numbers in **bold** indicate where terms are defined

- ∂ -pseudomanifold, *see* pseudomanifold, ∂ -pseudomanifold
- ∂ -stratified pseudomanifold, *see* stratified pseudomanifold, ∂ -stratified pseudomanifold
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