## Table of Contents

Preface to the Second Edition .......................... xv
Preface to the First Edition ......................... xvii

### Part I Judgments and Rules

1 Abstract Syntax .................................. 3
   1.1 Abstract Syntax Trees ..................... 3
   1.2 Abstract Binding Trees ................... 6
   1.3 Notes ..................................... 10

2 Inductive Definitions .............................. 12
   2.1 Judgments .................................. 12
   2.2 Inference Rules ............................ 12
   2.3 Derivations ................................ 14
   2.4 Rule Induction ............................. 15
   2.5 Iterated and Simultaneous Inductive Definitions .... 17
   2.6 Defining Functions by Rules ............... 18
   2.7 Notes ..................................... 19

3 Hypothetical and General Judgments ............... 21
   3.1 Hypothetical Judgments .................... 21
   3.2 Hypothetical Inductive Definitions ....... 24
   3.3 General Judgments .......................... 26
   3.4 Generic Inductive Definitions .......... 27
   3.5 Notes ..................................... 28

### Part II Statics and Dynamics

4 Statics ........................................... 33
   4.1 Syntax ..................................... 33
   4.2 Type System ................................ 34
   4.3 Structural Properties ..................... 35
   4.4 Notes ..................................... 37
### Contents

5 Dynamics 39
- 5.1 Transition Systems 39
- 5.2 Structural Dynamics 40
- 5.3 Contextual Dynamics 42
- 5.4 Equational Dynamics 44
- 5.5 Notes 46

6 Type Safety 48
- 6.1 Preservation 48
- 6.2 Progress 49
- 6.3 Run-Time Errors 50
- 6.4 Notes 52

7 Evaluation Dynamics 53
- 7.1 Evaluation Dynamics 53
- 7.2 Relating Structural and Evaluation Dynamics 54
- 7.3 Type Safety, Revisited 55
- 7.4 Cost Dynamics 56
- 7.5 Notes 57

**Part III Total Functions**

8 Function Definitions and Values 61
- 8.1 First-Order Functions 61
- 8.2 Higher-Order Functions 62
- 8.3 Evaluation Dynamics and Definitional Equality 65
- 8.4 Dynamic Scope 66
- 8.5 Notes 67

9 System $T$ of Higher-Order Recursion 69
- 9.1 Statics 69
- 9.2 Dynamics 70
- 9.3 Definability 71
- 9.4 Undefinability 73
- 9.5 Notes 75

**Part IV Finite Data Types**

10 Product Types 79
- 10.1 Nullary and Binary Products 79
- 10.2 Finite Products 81
- 10.3 Primitive Mutual Recursion 82
- 10.4 Notes 83
11 Sum Types
  11.1 Nullary and Binary Sums 85
  11.2 Finite Sums 86
  11.3 Applications of Sum Types 88
  11.4 Notes 91

Part V Types and Propositions

12 Constructive Logic 95
  12.1 Constructive Semantics 95
  12.2 Constructive Logic 96
  12.3 Proof Dynamics 100
  12.4 Propositions as Types 101
  12.5 Notes 101

13 Classical Logic 104
  13.1 Classical Logic 105
  13.2 Deriving Elimination Forms 109
  13.3 Proof Dynamics 110
  13.4 Law of the Excluded Middle 111
  13.5 The Double-Negation Translation 113
  13.6 Notes 114

Part VI Infinite Data Types

14 Generic Programming 119
  14.1 Introduction 119
  14.2 Polynomial Type Operators 119
  14.3 Positive Type Operators 122
  14.4 Notes 123

15 Inductive and Coinductive Types 125
  15.1 Motivating Examples 125
  15.2 Statics 128
  15.3 Dynamics 130
  15.4 Solving Type Equations 131
  15.5 Notes 132

Part VII Variable Types

16 System F of Polymorphic Types 137
  16.1 Polymorphic Abstraction 137
  16.2 Polymorphic Definability 140
  16.3 Parametricity Overview 142
  16.4 Notes 144
17 Abstract Types 146
  17.1 Existential Types 146
  17.2 Data Abstraction 149
  17.3 Definability of Existential Types 150
  17.4 Representation Independence 151
  17.5 Notes 153

18 Higher Kinds 154
  18.1 Constructors and Kinds 155
  18.2 Constructor Equality 156
  18.3 Expressions and Types 157
  18.4 Notes 158

Part VIII Partiality and Recursive Types

19 System PCF of Recursive Functions 161
  19.1 Statics 162
  19.2 Dynamics 163
  19.3 Definability 165
  19.4 Finite and Infinite Data Structures 167
  19.5 Totality and Partiality 167
  19.6 Notes 169

20 System FPC of Recursive Types 171
  20.1 Solving Type Equations 171
  20.2 Inductive and Coinductive Types 172
  20.3 Self-Reference 174
  20.4 The Origin of State 176
  20.5 Notes 177

Part IX Dynamic Types

21 The Untyped $\lambda$-Calculus 181
  21.1 The $\lambda$-Calculus 181
  21.2 Definability 182
  21.3 Scott’s Theorem 184
  21.4 Untyped Means Uni-Typed 186
  21.5 Notes 187

22 Dynamic Typing 189
  22.1 Dynamically Typed PCF 189
  22.2 Variations and Extensions 192
  22.3 Critique of Dynamic Typing 194
  22.4 Notes 195
# Table of Contents

23 Hybrid Typing 198
   23.1 A Hybrid Language 198
   23.2 Dynamic as Static Typing 200
   23.3 Optimization of Dynamic Typing 201
   23.4 Static versus Dynamic Typing 203
   23.5 Notes 204

Part X Subtyping

24 Structural Subtyping 207
   24.1 Subsumption 207
   24.2 Varieties of Subtyping 208
   24.3 Variance 211
   24.4 Dynamics and Safety 215
   24.5 Notes 216

25 Behavioral Typing 219
   25.1 Statics 220
   25.2 Boolean Blindness 226
   25.3 Refinement Safety 228
   25.4 Notes 229

Part XI Dynamic Dispatch

26 Classes and Methods 235
   26.1 The Dispatch Matrix 235
   26.2 Class-Based Organization 238
   26.3 Method-Based Organization 239
   26.4 Self-Reference 240
   26.5 Notes 242

27 Inheritance 245
   27.1 Class and Method Extension 245
   27.2 Class-Based Inheritance 246
   27.3 Method-Based Inheritance 248
   27.4 Notes 249

Part XII Control Flow

28 Control Stacks 253
   28.1 Machine Definition 253
   28.2 Safety 255
   28.3 Correctness of the K Machine 256
   28.4 Notes 259
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>Exceptions</td>
<td>260</td>
</tr>
<tr>
<td>29.1</td>
<td>Failures</td>
<td>260</td>
</tr>
<tr>
<td>29.2</td>
<td>Exceptions</td>
<td>262</td>
</tr>
<tr>
<td>29.3</td>
<td>Exception Values</td>
<td>263</td>
</tr>
<tr>
<td>29.4</td>
<td>Notes</td>
<td>264</td>
</tr>
<tr>
<td>30</td>
<td>Continuations</td>
<td>266</td>
</tr>
<tr>
<td>30.1</td>
<td>Overview</td>
<td>266</td>
</tr>
<tr>
<td>30.2</td>
<td>Continuation Dynamics</td>
<td>268</td>
</tr>
<tr>
<td>30.3</td>
<td>Coroutines from Continuations</td>
<td>269</td>
</tr>
<tr>
<td>30.4</td>
<td>Notes</td>
<td>272</td>
</tr>
</tbody>
</table>

## Part XIII Symbolic Data

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>Symbols</td>
<td>277</td>
</tr>
<tr>
<td>31.1</td>
<td>Symbol Declaration</td>
<td>277</td>
</tr>
<tr>
<td>31.2</td>
<td>Symbol References</td>
<td>280</td>
</tr>
<tr>
<td>31.3</td>
<td>Notes</td>
<td>282</td>
</tr>
<tr>
<td>32</td>
<td>Fluid Binding</td>
<td>284</td>
</tr>
<tr>
<td>32.1</td>
<td>Statics</td>
<td>284</td>
</tr>
<tr>
<td>32.2</td>
<td>Dynamics</td>
<td>285</td>
</tr>
<tr>
<td>32.3</td>
<td>Type Safety</td>
<td>286</td>
</tr>
<tr>
<td>32.4</td>
<td>Some Subtleties</td>
<td>287</td>
</tr>
<tr>
<td>32.5</td>
<td>Fluid References</td>
<td>288</td>
</tr>
<tr>
<td>32.6</td>
<td>Notes</td>
<td>289</td>
</tr>
<tr>
<td>33</td>
<td>Dynamic Classification</td>
<td>291</td>
</tr>
<tr>
<td>33.1</td>
<td>Dynamic Classes</td>
<td>291</td>
</tr>
<tr>
<td>33.2</td>
<td>Class References</td>
<td>293</td>
</tr>
<tr>
<td>33.3</td>
<td>Definability of Dynamic Classes</td>
<td>294</td>
</tr>
<tr>
<td>33.4</td>
<td>Applications of Dynamic Classification</td>
<td>295</td>
</tr>
<tr>
<td>33.5</td>
<td>Notes</td>
<td>296</td>
</tr>
</tbody>
</table>

## Part XIV Mutable State

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>34</td>
<td>Modernized Algol</td>
<td>301</td>
</tr>
<tr>
<td>34.1</td>
<td>Basic Commands</td>
<td>301</td>
</tr>
<tr>
<td>34.2</td>
<td>Some Programming Idioms</td>
<td>306</td>
</tr>
<tr>
<td>34.3</td>
<td>Typed Commands and Typed Assignables</td>
<td>307</td>
</tr>
<tr>
<td>34.4</td>
<td>Notes</td>
<td>310</td>
</tr>
<tr>
<td>35</td>
<td>Assignable References</td>
<td>313</td>
</tr>
<tr>
<td>35.1</td>
<td>Capabilities</td>
<td>313</td>
</tr>
<tr>
<td>35.2</td>
<td>Scoped Assignables</td>
<td>314</td>
</tr>
</tbody>
</table>
Table of Contents

35.3 Free Assignables 316
35.4 Safety 318
35.5 Benign Effects 320
35.6 Notes 321

36 Lazy Evaluation 323
36.1 PCF By-Need 323
36.2 Safety of PCF By-Need 326
36.3 FPC By-Need 328
36.4 Suspension Types 329
36.5 Notes 331

Part XV Parallelism

37 Nested Parallelism 335
37.1 Binary Fork-Join 335
37.2 Cost Dynamics 338
37.3 Multiple Fork-Join 341
37.4 Bounded Implementations 342
37.5 Scheduling 346
37.6 Notes 348

38 Futures and Speculations 350
38.1 Futures 350
38.2 Speculations 351
38.3 Parallel Dynamics 352
38.4 Pipelining with Futures 354
38.5 Notes 356

Part XVI Concurrency and Distribution

39 Process Calculus 359
39.1 Actions and Events 359
39.2 Interaction 361
39.3 Replication 363
39.4 Allocating Channels 364
39.5 Communication 366
39.6 Channel Passing 369
39.7 Universality 371
39.8 Notes 372

40 Concurrent Algol 375
40.1 Concurrent Algol 375
40.2 Broadcast Communication 378
40.3 Selective Communication 380
40.4 Free Assignables as Processes 382
40.5 Notes 383

41 Distributed Algol 385
41.1 Statics 385
41.2 Dynamics 388
41.3 Safety 390
41.4 Notes 391

Part XVII  Modularity

42 Modularity and Linking 395
42.1 Simple Units and Linking 395
42.2 Initialization and Effects 396
42.3 Notes 398

43 Singleton Kinds and Subkinding 399
43.1 Overview 399
43.2 Singletons 400
43.3 Dependent Kinds 402
43.4 Higher Singletons 405
43.5 Notes 407

44 Type Abstractions and Type Classes 409
44.1 Type Abstraction 410
44.2 Type Classes 412
44.3 A Module Language 414
44.4 First- and Second-Class 418
44.5 Notes 419

45 Hierarchy and Parameterization 422
45.1 Hierarchy 422
45.2 Abstraction 425
45.3 Hierarchy and Abstraction 427
45.4 Applicative Functors 429
45.5 Notes 431

Part XVIII  Equational Reasoning

46 Equality for System $T$ 435
46.1 Observational Equivalence 435
46.2 Logical Equivalence 439
46.3 Logical and Observational Equivalence Coincide 440
46.4 Some Laws of Equality 443
46.5 Notes 444
## Contents

<table>
<thead>
<tr>
<th>Part</th>
<th>Chapter</th>
<th>Title</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>47</td>
<td>Equality for System <strong>PCF</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>47.1</td>
<td>Observational Equivalence</td>
<td>445</td>
</tr>
<tr>
<td></td>
<td>47.2</td>
<td>Logical Equivalence</td>
<td>445</td>
</tr>
<tr>
<td></td>
<td>47.3</td>
<td>Logical and Observational Equivalence Coincide</td>
<td>446</td>
</tr>
<tr>
<td></td>
<td>47.4</td>
<td>Compactness</td>
<td>449</td>
</tr>
<tr>
<td></td>
<td>47.5</td>
<td>Lazy Natural Numbers</td>
<td>452</td>
</tr>
<tr>
<td></td>
<td>47.6</td>
<td>Notes</td>
<td>453</td>
</tr>
<tr>
<td>48</td>
<td>Parametricity</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>48.1</td>
<td>Overview</td>
<td>454</td>
</tr>
<tr>
<td></td>
<td>48.2</td>
<td>Observational Equivalence</td>
<td>455</td>
</tr>
<tr>
<td></td>
<td>48.3</td>
<td>Logical Equivalence</td>
<td>456</td>
</tr>
<tr>
<td></td>
<td>48.4</td>
<td>Parametricity Properties</td>
<td>461</td>
</tr>
<tr>
<td></td>
<td>48.5</td>
<td>Representation Independence, Revisited</td>
<td>464</td>
</tr>
<tr>
<td></td>
<td>48.6</td>
<td>Notes</td>
<td>465</td>
</tr>
<tr>
<td>49</td>
<td>Process Equivalence</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>49.1</td>
<td>Process Calculus</td>
<td>467</td>
</tr>
<tr>
<td></td>
<td>49.2</td>
<td>Strong Equivalence</td>
<td>467</td>
</tr>
<tr>
<td></td>
<td>49.3</td>
<td>Weak Equivalence</td>
<td>469</td>
</tr>
<tr>
<td></td>
<td>49.4</td>
<td>Notes</td>
<td>472</td>
</tr>
<tr>
<td></td>
<td>49.5</td>
<td>Notes</td>
<td>473</td>
</tr>
</tbody>
</table>

### Part XIX Appendices

- A Background on Finite Sets | 477
- Bibliography | 479
- Index | 487