

ANALYTICAL GROUNDWATER MECHANICS

Groundwater mechanics is the study of fluid flow in porous media. Focusing on applications and case studies, this book explains the basic principles of groundwater flow using mathematical expressions to describe a wide range of different aquifer configurations. Emphasis is placed throughout on the importance of developing simplified models that can be solved analytically to provide insight into complex groundwater flow scenarios and to allow better interpretation of the full numerical solution.

Focusing first on identifying the important features of a problem, the book explains how to translate practical questions into mathematical form and discusses the interpretation of the results. Illustrated with numerous real-world examples and graphical results, this is an ideal textbook for advanced undergraduate and graduate courses in earth science, geological engineering, and environmental engineering, as well as a useful reference for researchers and professionals.

OTTO D. L. STRACK is Professor of Civil Engineering and Geomechanics at the University of Minnesota. With almost 50 years' experience, he is an expert in groundwater mechanics with a particular interest in regional modeling of groundwater flow and transport, and he is the original developer of the analytic element method. He has received the Lifetime Achievement Award granted by the Minnesota Groundwater Association and is a correspondent of the Royal Dutch Academy of Sciences. He has published widely in the field, including the successful textbook *Groundwater Mechanics* (1989).

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OTTO D. L. STRACK

University of Minnesota



CAMBRIDGE
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University Printing House, Cambridge CB2 8BS, United Kingdom
One Liberty Plaza, 20th Floor, New York, NY 10006, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
4843/24, 2nd Floor, Ansari Road, Daryaganj, Delhi – 110002, India
79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9781107148833

DOI: 10.1017/9781316563144

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First published 2017

Printed in the United Kingdom by Clays, St Ives plc

A catalogue record for this publication is available from the British Library.

Library of Congress Cataloging-in-Publication Data

Names: Strack, Otto D. L., 1943– author.

Title: Analytical groundwater mechanics / Otto D.L. Strack.

Description: New York, NY : Cambridge University Press, 2017. | Includes bibliographical references and index.

Identifiers: LCCN 2017032438 | ISBN 9781107148833 (Hardback : alk. paper)

Subjects: LCSH: Groundwater flow—Mathematical models.

Classification: LCC GB1197.7 .S765 2017 | DDC 551.49—dc23

LC record available at <https://lccn.loc.gov/2017032438>

ISBN 978-1-107-14883-3 Hardback

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Preface

The subject matter covered in this text is the mathematical description of fluid flow through porous media. Some parts of this book have been taken from Strack [1989], but much of the material is newly written. The two primary objectives are as follows. The first is to instruct the reader in approximating groundwater flow problems in such a manner that they can be solved analytically. The analytic solution will help us to gain insight, prior to constructing a complete solution to the problem using some numerical method if a more elaborate model is required.

The second objective is to explain how to simplify a practical problem so that it is analytically tractable. It requires considerable skill and understanding to approximate an actual flow problem by a simpler one that can be solved analytically, yet provide insight into the essence of the original problem. Even if the problem cannot be handled adequately by simple means, and recourse to a numerical solution is necessary, the determination and interpretation of relatively crude approximate solutions often provides crucial insight. The understanding thus gained can be used with advantage in selecting and setting up a numerical model that may ultimately be used to solve the problem. Modern computational environments exist that are suitable for implementing analytical solutions with relative ease, and are capable of displaying the results in a variety of manners, often in graphical form. The availability of such environments greatly enhances the use of analytic solutions as compared with in the past.

In view of the primary two objectives, emphasis is placed on a detailed coverage of methods for solving a variety of problems, rather than on providing a catalogue of existing solutions. Application of complex variable methods greatly simplifies the method of solution of many groundwater flow problems. Although complex variable methods carry with them a certain level of intimidation for many, once understood, complex variables make major simplification possible, as compared with real variables. We introduce in this text complex variables using Wirtinger calculus (Wirtinger [1927]), which extends the use of complex variables to general

two-dimensional problems. The implementation of complex variables in the majority of modern computational engines makes their use for obtaining analytic solutions attractive, especially in view of the primary objectives of this text.

The subject matter is organized in ten chapters. Each chapter covers one main topic. Chapter 1 is concerned with the basic concepts and the derivation of the governing equations. Chapter 2 deals with steady flow in a single aquifer. Steady shallow interface flow is the topic of Chapter 3, and two-dimensional flow in the vertical plane is covered in Chapter 4. Steady flow in leaky aquifers is covered in Chapter 5 and three-dimensional flow in Chapter 6. Transient flow is discussed in Chapter 7. A substantial part of the text, Chapter 8, is devoted to the use of complex variables for solving a variety of practical problems. It will become apparent to the reader that using complex variables instead of real ones leads to significant simplification, visible not only in the mathematical treatment, but also in the implementation in simple computer codes. An additional advantage of the use of complex variables is that for many of the problems flow nets can be easily produced, giving a graphical illustration of the flow patterns. The reader who completed the exercises will have produced a simple but flexible computer model capable of modeling features such as rivers, wells, lakes of various shapes, areas of recharge, and inhomogeneities in the hydraulic conductivity. Fluid particle paths and contaminant transport are covered in Chapter 9. An overview of numerical methods for solving groundwater flow problems is presented in Chapter 10; the reader is introduced to the basics of both finite difference and finite element methods. Applications as well as exercises are included in Chapter 2 through 10. There are two appendixes.

Many groundwater flow problems, in particular problems of regional aquifer flow, are not solved exactly in this text; approximate governing equations are derived for a variety of aquifer configurations. The mathematical formulations for these configurations are unified by the use of discharge potentials, expressed differently for each type of flow that corresponds to a particular aquifer configuration. The mathematical formulations thus become similar, and solutions and techniques are often transferable from one type of flow to another. An additional, but related, simplification of major practical interest is the application of the concept of vertically integrated flow, which results in accurate predictions of flow rates, while approximating the hydraulic heads.

Acknowledgments

The author is grateful to Dr. Erik Anderson, Dr. Henk M. Haitjema, and Dr. Kees A. Maas for their careful reading of portions of the text. They identified ambiguity in explanations, noted missing examples and inconsistencies, and suggested additions and deletions. The author is indebted to all three experts in the field for their contributions to this work.

The text could not have been created without the help of the author's wife, Andrine D. Strack, who unwaveringly lent her support to the author in his efforts. She created figures and identified, through multiple readings of the text, many places where the explanations lacked in clarity.

Work on this text proceeded over the past 10 years. The text was used in the classroom as it grew to completion. Many of the students attending the classes identified errors and inconsistencies, and the author is grateful for their comments.