

Index

- $\langle \cdot, \cdot \rangle$, 18
- $\langle \cdot, \cdot \rangle_2$, 10
- π_n , 196
- A*-conjugate, 190
- ACA, 280, 297
- adaptive cross approximation, 280, 297
- additive Schwarz method, 310
- adic
 - B*-, 30
- adjoint, 320
- admissible function, 288
- AINV, 352
- algebraic multigrid, 251, 258
- algorithm, 12
 - Strassen, 15
 - thresholding, 390
- angle, 326
- approximation
 - best, 23
- approximation error, 97
- approximation property, 253
- Arnoldi method, 212, 213
- associated preconditioner, 335
- B*-adic, 30
- back substitution, 60
- backward Gauss–Seidel method, 126
- backward-stable, 54, 61, 72, 77
- Banach’s fixed point theorem, 102
- band-matrix, 11
- base, 30
- basis
 - orthonormal, 19
- basis function, 5, 261
- basis pursuit, 372
 - robust, 376
- Bauer–Fike theorem, 139
- best approximation, 23
- BiCG, 230
- BiCG method, 234
 - stabilised, 242
- BiCGSTAB, 242
- biconjugate gradient method, 230
- bidiagonal, 172
- binary tree, 270
- block cluster tree, 289
- block matrix, 283
- block preconditioner, 357
- bottom-up, 271
- bounding box, 266
- Bregman distance, 386
- Bregman iteration, 386
- cancellation, 48
- Cauchy–Schwarz inequality, 18, 33
- cell, 264
- CGNR method, 203
- CGS method, 239
- chain, 131
- characteristic polynomial, 132
- Chebyshev polynomial, 199
- child, 270
- Cholesky factorisation, 76
 - incomplete, 346
- cluster tree, 286
- coarse-grid correction operator, 250
- coherence, 381
- compactly supported radial basis function, 7
- companion matrix, 208
- compatible, 37
- compressible, 375
- compressive sampling matching pursuit, 390
- computational cost, 13

404

condition number, 39, 46
 absolute, 47
 relative, 47
 reduced, 202
 conjugate directions, 190
 conjugate gradient method, 192
 conjugate gradient squared method, 239
 consistent, 102
 consistently ordered, 116, 122
 contraction mapping, 102
 CoSaMP, 390
 cost
 computational, 13
 cubic, 15
 linear, 15
 quadratic, 15
 Courant–Fischer theorem, 136
 Cramér’s inequality, 280
 cross validation
 generalised, 98
 data error, 98
 decomposition
 stable, 312
 deflation, 171
 degenerate kernel, 261
 δ_{ij} , 11
 dense, 11
 depth, 270
 diagonal matrix, 11
 diagonalisable, 20
 diagonally dominant, 67, 133
 weakly, 108
 dimension reduction, 370
 direct method, 4
 discrepancy principle
 Morozov’s, 98
 double precision, 30
 edge, 270
 eigenvalue, 4, 132
 dominant, 141
 generalised, 21
 eigenvector, 4, 132
 frequency, 245
 generalised, 21
 elementary lower triangular matrix, 62
 elementary permutation matrix, 74
 energy norm, 194
 equality
 parallelogram, 29
 equiangular, 383
 equilibration, 331

Index

error
 approximation, 97
 cancellation, 48
 data, 98
 input, 32
 overflow, 31
 relative, 36
 Euclidean norm, 17, 33
 evaluation point, 263
 expansion
 far-field, 263, 275
 multipole, 266
 unipole, 264
 exponent, 30
 extension matrix, 316
 factorisation
 LU-, 66
 Cholesky, 76
 far-field expansion, 263, 275
 fast Gauss transform, 279
 field of values, 208
 fill distance, 266
 fill in, 114
 fill level, 343
 fixed point, 101
 fixed point theorem, 102
 floating point arithmetic, 30
 floating point number, 30
 floating point number system, 30
 floating point operations, 13
 flops, 13
 FOM, 259
 forward substitution, 61
 Frobenius norm, 46, 295
 full matrix, 6, 11
 full orthogonal method, 259
 full weighting, 247
 function
 admissible, 288
 basis, 5, 261
 generating, 279
 gradient, 48
 Hermite, 279
 Lagrange, 6
 Lebesgue, 305
 positive definite, 7
 radial, 7
 Galerkin approximation, 9
 Gauss–Seidel
 relaxation, 120
 Gauss–Seidel method, 110, 126

- backward, 126
- Gauss–Seidel relaxation, 120, 318
- Gaussian, 7, 279
- generalised cross validation, 98
- generalised eigenvalue, 21
- generalised eigenvector, 21
- generating function, 279
- Gershgorin circle theorem, 133
- Givens rotation, 100, 155, 222
- GMRES method, 203
 - restarted, 220
- gradient, 48
- Gram–Schmidt, 19, 212
- grid
 - coarse, 246
- H_n , 279
- \mathcal{H} -matrix, 291
- Hölder inequality, 33
- hard-thresholding operator, 387
- Hermite function, 279
- Hermite polynomial, 279
- Hessenberg form, 160
- Hessenberg matrix, 214
- hierarchical partitioning, 285
- hierarchical bisectional partitioning, 274
- hierarchical matrix, 291
- hierarchical partitioning, 264
- hierarchical subdivision, 264
- Hilbert space, 325
- Horner scheme, 351
- Householder, 174
- Householder matrix, 79
- I , 11
- identity matrix, 11
- IEEE 754, 30
- ill-conditioned, 47
- ILU(p), 344
- in situ*, 64
- inequality
 - Cauchy–Schwarz, 18, 33
 - Hölder, 33
 - Kantorovich, 186
- inertia, 137
- Inf, 31
- inner product, 18
- input error, 32
- inverse
 - pseudo-, 27
- inverse iteration, 144
- inverse multiquadric, 7
- invertible, 12
- irreducible, 109, 131, 176
- iteration
 - inverse, 144
 - Richardson, 131, 318
 - von Wielandt, 144
 - iteration matrix, 102, 103
 - iterative method, 4
- Jacobi, 316
- Jacobi preconditioning, 331
- Jacobi method, 106, 244
 - block relaxation, 316
 - for computing eigenvalues, 156
 - cyclic, 158
 - relaxation, 117
- Jacobi relaxation, 117
- kd -tree, 275
- Kantorovich inequality, 186
- kernel, 7, 9
 - degenerate, 261
 - positive definite, 7
- Kronecker symbol, 11
- Krylov space, 194
- LU factorisation, 66
 - incomplete, 339
- ℓ_p -norm, 33
- L-curve principle, 99
- Lagrange function, 6
- Lanczos method, 222
 - biorthogonalisation, 227
 - regular termination, 229
 - two-sided, 227
 - look-ahead, 230
 - serious breakdown, 229
- Lanczos orthogonalisation method, 222
- Landau symbols, 14
- leaf, 270
- least-squares problem, 87
 - penalised, 95
- Lebesgue function, 305
- level, 264
- lower triangular matrix, 11
 - normalised, 63, 66
- M -matrix, 131
- machine number, 30
- machine precision, 31
- mantissa, 30
- mapping
 - contraction, 102
- matrix
 - M -, 131

- elementary lower triangular, 62
- normalised lower triangular, 66
- permutation, 73
- positive definite, 19
- positive semi-definite, 19
- band-, 11
- bidiagonal, 172
- block, 283
- companion, 208
- dense, 11
- diagonal, 11
- elementary permutation, 74
- full, 6, 11
- Hessenberg, 160, 214
- hierarchical, 291
- Householder, 79
- identity, 11
- irreducible, 176
- iteration, 102, 103
- lower triangular, 11
- normal, 45, 135, 139
- normalised lower triangular, 63
- null space, 19
- pattern, 339
- range, 19
- rank, 19
- reducible, 109
- sparse, 8, 11
- square, 11
- square root, 20, 312
- trace, 46
- tridiagonal, 11, 15
- unitary, 84
- upper triangular, 11
- Vandermonde, 6
- matrix norm, 37
 - compatible, 37
 - induced, 37
 - multiplicative, 38
 - subordinate, 37
- method
 - Arnoldi, 212, 213
 - BiCG, 230, 234
 - BiCGSTAB, 242
 - block Jacobi relaxation, 316
 - CGNR, 203
 - CGS, 239
 - conjugate gradient, 192
 - Gauss–Seidel, 110
 - GMRES, 203
 - Gram–Schmidt, 212
 - Jacobi, 106, 244
 - Lanczos, 222
 - Lanczos biorthogonalisation, 227
 - look-ahead Lanczos, 230
 - MINRES, 203, 222, 225
 - multiplicative Schwarz, 318
 - power, 124, 142
 - QMR, 235
 - von Mises, 142
- MINRES method, 203, 222, 225
- Morozov’s discrepancy principle, 98
- multigrid, 244
 - algebraic, 251
 - approximation property, 253
 - smoothing property, 253
- multiplicative matrix norm, 38
- multipole expansion, 266
- multiquadric, 7, 261
- NaN, 31
- near neighbours, 272
- node, 270
 - depth, 270
- non-zero pattern, 338
- norm, 17
 - energy, 194
 - Euclidean, 17, 33
 - Frobenius, 46, 295
 - matrix, 37
 - outer, 154
- normal, 45, 135, 139
- normal equations, 87
- normalised, 63
- normalised lower triangular matrix, 63, 66
- null space, 19
 - null space property, 373
 - robust, 377
 - stable, 375
- number
 - machine, 30
- numerical radius, 209
- O , 14
- oct-tree, 270
- operator
 - coarse-grid correction, 250
 - hard-thresholding, 387
 - prolongation, 246
 - restriction, 246
 - Schwarz, 309
 - Schwarz additive, 309
- ordered
 - consistently, 116

- orthogonal, 19
- orthogonal matching pursuit, 386
- orthogonal projection, 22, 251, 326
- orthonormal basis, 19
- Ostrowski, 138
- outer norm, 154
- outer product, 79
- over-determined, 4, 87
- overflow, 31
- panel, 263, 264
- parallelogram equality, 29
- parameter
 - regularisation, 95
 - relaxation, 117
- partitioning, 283
 - hierarchical, 264, 285
 - hierarchical bisectional, 274
 - tensor product, 283
 - uniform hierarchical box, 267
- penalised least-squares problem, 95
- penalty term, 95
- Penrose conditions, 27
- permutation, 73
- permutation matrix, 73
 - elementary, 74
- pivoting, 72
 - partial, 73
 - total, 73
- point
 - evaluation, 263
 - source, 263
- polynomial
 - characteristic, 132
 - Chebyshev, 199
 - Hermite, 279
- polynomials, 196
- positive definite, 19
- positive definite function, 7
- positive semi-definite, 19
- post-order, 270
- power method, 124, 142
- pre-Hilbert space, 18
- pre-order, 270
- preconditioner
 - associated, 335
 - block, 357
 - incomplete LU , 339
 - polynomial, 346, 348
 - sparse approximate, 352
- preconditioning
 - Jacobi, 331
- problem
 - least-squares, 87
- projection, 22, 251
 - orthogonal, 22, 251
- prolongation, 246
- pseudo-inverse, 27
- QR -method with shifts, 170
- QMR method, 235
 - transpose-free, 243
- QMRCGSTAB, 243
- quad-tree, 270
- quasi-minimal residual method, 235
- quasi-uniform, 267
- radial, 7
- radius
 - numerical, 209
 - separation, 266
 - spectral, 103, 244
- range, 19
- rank, 19
- rank deficiency, 94
- Rayleigh, 135
- Rayleigh quotient, 42, 145
- reduced singular value decomposition, 26, 172
- reduced vector, 308
- reducible, 109
- regular termination of Lanczos method, 229
- regularisation
 - Tikhonov, 95
- relative error, 36
- relaxation, 116
 - block Jacobi, 316
 - Gauss–Seidel, 120, 318
 - Jacobi method, 117
 - symmetric Gauss–Seidel, 127
- relaxation parameter, 117
- residual, 116
- residual correction, 131
- restarted GMRES, 220
- restricted isometry constant, 378
- restricted isometry property, 378
- restriction, 246
- Richardson iteration, 131, 318
- RIP, 378
- robust basis pursuit, 376
- robust null space property, 377
- root of a tree, 270
- s -sparse, 371
- Sassenfeld criterion, 111
- Schwarz method

408

additive, 310
 multiplicative, 318
 Schwarz operator, 309
 additive, 309
 self-adjoint, 251
 separation radius, 266
 serious breakdown of Lanczos method, 229
 signal, 370
 singular value decomposition, 25, 172
 reduced, 26, 172
 singular values, 26
 slack variable, 384
 smoothing property, 253
 SOR, 120
 source panel, 263
 source point, 263
 space
 Hilbert, 325
 Krylov, 194
 pre-Hilbert, 18
 SPAI, 352
 sparse, 8, 11
 s -, 371
 sparse solution, 371
 spectral radius, 103, 244
 splitting
 symmetric, 126
 splitting dimension, 274
 splitting preconditioner, 335
 splitting value, 274
 square matrix, 11
 SSOR, 127
 stabilised BiCG method, 242
 stable, 312
 backward, 54, 61, 72, 77
 steepest descent, 185
 stopping criterion, 107
 Strassen algorithm, 15
 strictly row diagonally dominant, 67, 108
 sub-diagonal, 15
 sub-tree, 270
 subdivision
 hierarchical, 264
 substitution
 back, 60
 forward, 61
 super-diagonal, 15
 support, 371
 SVD, 25, 172
 reduced, 26, 172
 Sylvester's theorem of inertia, 137

Index

symmetric, 12
 symmetric Gauss–Seidel relaxation, 127
 symmetric order, 270
 symmetric splitting, 126
 SYMMLQ, 225
 T_n , 199
 tensor product partitioning, 283
 TFQMR, 243
 theorem
 Bauer–Fike, 139
 Courant–Fischer, 136
 quotient, 140
 Sylvester's, 137
 thresholding algorithm, 390
 Tikhonov regularisation, 95
 top-down, 271
 trace, 46
 transpose-free QMR, 243
 tree, 270
 balanced, 270
 block cluster, 289
 child, 270
 cluster, 286
 depth, 270
 post-order, 270
 pre-order, 270
 symmetric order, 270
 triangular matrix
 elementary lower, 62
 tridiagonal, 11
 tridiagonal matrix, 15
 under-determined, 4
 uniform hierarchical box partitioning, 267
 unipole expansion, 264
 unisolvent, 306
 unit ball, 33
 unit vector, 33
 unitary, 84
 upper triangular matrix, 11
 V-cycle, 255, 257
 Vandermonde matrix, 6
 vector iteration, 142
 von Mises method, 142
 von Wielandt iteration, 144
 W-cycle, 257
 weakly diagonally dominant, 108
 well-conditioned, 47, 49
 well-separated, 263, 264, 272, 290
 Welsh bound, 382