Numerical Linear Algebra

This self-contained introduction to Numerical Linear Algebra provides a comprehensive, yet concise, overview of the subject. It includes standard material such as direct methods for solving linear systems and least-squares problems, error, stability and conditioning, basic iterative methods and the calculation of eigenvalues. Later chapters cover more advanced material, such as Krylov subspace methods, multigrid methods, domain decomposition methods, multipole expansions, hierarchical matrices and compressed sensing.

The book provides rigorous mathematical proofs throughout, and gives algorithms in general-purpose language-independent form. Requiring only a solid knowledge in linear algebra and basic analysis, this book will be useful for applied mathematicians, engineers, computer scientists and all those interested in efficiently solving linear problems.

HOLGER WENDLAND holds the Chair of Applied and Numerical Analysis at the University of Bayreuth. He works in the area of Numerical Analysis and is the author of two other books, *Scattered Data Approximation* (Cambridge, 2005) and *Numerische Mathematik* (Springer 2004, with Robert Schaback).

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Numerical Linear Algebra

An Introduction

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Preface

Numerical Linear Algebra (NLA) is a subarea of Applied Mathematics. It is mainly concerned with the development, implementation and analysis of numerical algorithms for solving linear problems. In general, such linear problems arise when discretising a continuous problem by restricting it to a finite-dimensional subspace of the original solution space. Hence, the development and analysis of numerical algorithms is almost always problem-dependent. The more is known about the underlying problem, the better a suitable algorithm can be developed.

Nonetheless, many of the so-derived methods are more general in the sense that they can be applied to larger classes of problems than initially intended. One of the challenges in Mathematics is deciding how to describe the necessary assumptions, under which a certain method works, in the most general way. In the context of NLA, this means finding for each method the most general description of matrices to which the method can be applied. It also means extracting the most general methods from the vast number of available algorithms. Particularly for users with new problems this is crucial, as it allows them to apply and test well-established algorithms first, before starting to develop new methods or to extend existing ones.

In this book, I have attempted to use this *matrix-driven* approach rather than the *problem-driven* one. Naturally, the selection of the material is biased by my own point of view. Also, a book on NLA without any examples would be rather dire, so there are typical examples and applications included to illustrate the methods, but I have tried to restrict myself to simple examples, which do not require much previous knowledge on specific problems and discretisation techniques.

During the past years, I have given courses on Numerical Linear Algebra at advanced BSc and early MSc level at the University of Sussex (UK), the University of Oxford (UK) and the University of Bayreuth (Germany). I have also given courses on Numerical Analysis which covered parts of the NLA material in Oxford, Göttingen (Germany) and Bayreuth.

This book on Numerical Linear Algebra is based on these courses and the material of these courses. It covers the standard material, as well as more recent and more specific techniques, which are usually not found in standard textbooks on NLA. Examples include the multigrid method, the domain decomposition method, multipole expansions, hierarchical matrices and applications to compressed or compressive sensing. The material on each of these topics fills entire books so that I can obviously present only a selection. However, this selection should allow the readers to grasp the underlying ideas of each topic and enable them to understand current research in these areas.

Each chapter of this book contains a small number of theoretical exercises. However, to really understand NLA one has to implement the algorithms by oneself and test them on some of the matrices from the examples. Hence, the most important exercises intrinsic to each chapter are to implement and test the proposed algorithms.

All algorithms are stated in a clean pseudo-code; no programming language is preferred. This, I hope, allows readers to use the programming language of their choice and hence yields the greatest flexibility.

Finally, NLA is obviously closely related to Linear Algebra. However, this is not a book on Linear Algebra, and I expect readers to have a solid knowledge of Linear Algebra. Though I will review some of the material, particularly to introduce the notation, terms like linear space, linear mapping, determinant etc. should be well-known.