HADRONS AT FINITE TEMPERATURE

High energy laboratories are performing experiments in heavy ion collisions to explore the structure of matter at high temperature and density. This elementary book explains the basic ideas involved in the theoretical analysis of these experimental data. It first develops two topics needed for this purpose, namely hadron interactions and thermal field theory. Chiral perturbation theory is developed to describe hadron interactions and thermal field theory is formulated in the real time method. In particular, spectral form of thermal propagators is derived for fields of arbitrary spin and used to calculate loop integrals. These developments are then applied to find quark condensate and hadron parameters in medium, including dilepton production. Finally, the non-equilibrium method of statistical field theory to calculate transport coefficients is reviewed. With technical details explained in the text and appendices, this book should be accessible to researchers as well as graduate students interested in thermal field theory.

Samirnath Mallik is Emeritus Scientist at Saha Institute of Nuclear Physics. He worked at the University of Berne, the University of Karlsruhe and Tata Institute of Fundamental Research. His research interests are thermal field theory and heavy ion collisions.

Sourav Sarkar is Scientist at Variable Energy Cyclotron Centre, Kolkata and Associate Professor, Homi Bhabha National Institute. His research interests concern applications of thermal field theory, mainly to study transport phenomena and hadronic properties in hot and dense medium.

CAMBRIDGE MONOGRAPHS ON MATHEMATICAL PHYSICS

General Editors: P. V. Landshoff, D. R. Nelson, S. Weinberg

S. J. Aarseth *Gravitational N-Body Simulations: Tools and Algorithms*†
J. Ambjørn, B. Durhuus and T. Jonsson *Quantum Geometry: A Statistical Field Theory Approach*†
A. M. Anile *Relativistic Fluids and Magneto-fluids: With Applications in Astrophysics and Plasma Physics*
J. A. de Azcárraga and J. M. Izquierdo *Lie Groups, Lie Algebras, Cohomology and Some Applications in Physics*†
O. Babelon, D. Bernard and M. Talon *Introduction to Classical Integrable Systems*†
F. Bastianelli and P. van Nieuwenhuizen *Path Integrals and Anomalies in Curved Space*†
D. Baumann and L. McAllister *Inflation and String Theory*
V. Belinski and E. Verdaguer *Gravitational Solitons*†
J. Bernstein *Kinetic Theory in the Expanding Universe*†
G. F. Bertsch and R. A. Broglia *Oscillations in Finite Quantum Systems*†
N. D. Birrell and P. C. W. Davies *Quantum Fields in Curved Space*†
K. Bolejko, A. Krasiński, C. Hellaby and M-N. Célérier *Structures in the Universe by Exact Methods: Formation, Evolution, Interactions*
D. M. Brink *Semi-Classical Methods for Nucleus-Nucleus Scattering*†
M. Burgess *Classical Covariant Fields*†
E. A. Calzetta and B.-L. B. Hu *Nonequilibrium Quantum Field Theory*
S. Carlip *Quantum Gravity in 2+1 Dimensions*†
P. Cartier and C. DeWitt-Morette *Functional Integration: Action and Symmetries*†
J. C. Collins *Renormalization: An Introduction to Renormalization, the Renormalization Group and the Operator-Product Expansion*†
P. D. B. Collins *An Introduction to Regge Theory and High Energy Physics*†
M. Creutz *Quarks, Gluons and Lattices*†
P. D. D'Eath *Supersymmetric Quantum Cosmology*†
J. Dereziński and C. Gérard *Mathematics of Quantization and Quantum Fields*
F. de Felice and D. Bini *Classical Measurements in Curved Space-Times*
F. de Felice and C. J. S Clarke *Relativity on Curved Manifolds*†
B. DeWitt *Supermanifolds, 2^{nd} edition*†
P. G. O. Freund *Introduction to Supersymmetry*†
F. G. Friedlander *The Wave Equation on a Curved Space-Time*†
J. L. Friedman and N. Stergioulas *Rotating Relativistic Stars*
Y. Frishman and J. Sonnenschein *Non-Perturbative Field Theory: From Two Dimensional Conformal Field Theory to QCD in Four Dimensions*
J. A. Fuchs *Affine Lie Algebras and Quantum Groups: An Introduction, with Applications in Conformal Field Theory*†
J. Fuchs and C. Schweigert *Symmetries, Lie Algebras and Representations: A Graduate Course for Physicists*†
Y. Fujii and K. Maeda *The Scalar-Tensor Theory of Gravitation*†
J. A. H. Futterman, F. A. Handler and R. A. Matzner *Scattering from Black Holes*†
A. S. Galperin, E. A. Ivanov, V. I. Ogievetsky and E. S. Sokatchev *Harmonic Superspace*†
R. Gambini and J. Pullin *Loops, Knots, Gauge Theories and Quantum Gravity*†
T. Gannon *Moonshine beyond the Monster: The Bridge Connecting Algebra, Modular Forms and Physics*†
M. Göckeler and T. Schücker *Differential Geometry, Gauge Theories, and Gravity*†
C. Gómez, M. Ruiz-Altaba and G. Sierra *Quantum Groups in Two-Dimensional Physics*†
M. B. Green, J. H. Schwarz and E. Witten *Superstring Theory Volume 1: Introduction*
M. B. Green, J. H. Schwarz and E. Witten *Superstring Theory Volume 2: Loop Amplitudes, Anomalies and Phenomenology*
V. N. Gribov *The Theory of Complex Angular Momenta: Gribov Lectures on Theoretical Physics*†
J. B. Griffiths and J. Podolský *Exact Space-Times in Einstein's General Relativity*†
S. W. Hawking and G. F. R. Ellis *The Large Scale Structure of Space-Time*†
F. Iachello and A. Arima *The Interacting Boson Model*†
F. Iachello and P. van Isacker *The Interacting Boson-Fermion Model*†
C. Itzykson and J. M. Drouffe *Statistical Field Theory Volume 1: From Brownian Motion to Renormalization and Lattice Gauge Theory*†
C. Itzykson and J. M. Drouffe *Statistical Field Theory Volume 2: Strong Coupling, Monte Carlo Methods, Conformal Field Theory and Random Systems*†
G. Jaroszkiewicz *Principles of Discrete Time Mechanics*
C. V. Johnson *D-Branes*†
P. S. Joshi *Gravitational Collapse and Spacetime Singularities*†

J. I. Kapusta and C. Gale *Finite-Temperature Field Theory: Principles and Applications, 2^{nd} edition*†
V. E. Korepin, N. M. Bogoliubov and A. G. Izergin *Quantum Inverse Scattering Method and Correlation Functions*†
M. Le Bellac *Thermal Field Theory*†
Y. Makeenko *Methods of Contemporary Gauge Theory*†
N. Manton and P. Sutcliffe *Topological Solitons*†
N. H. March *Liquid Metals: Concepts and Theory*†
I. Montvay and G. Münster *Quantum Fields on a Lattice*†
L. O'Raifeartaigh *Group Structure of Gauge Theories*†
T. Ortín *Gravity and Strings, 2^{nd} edition*
A. M. Ozorio de Almeida *Hamiltonian Systems: Chaos and Quantization*†
L. Parker and D. Toms *Quantum Field Theory in Curved Spacetime: Quantized Fields and Gravity*
R. Penrose and W. Rindler *Spinors and Space-Time Volume 1: Two-Spinor Calculus and Relativistic Fields*†
R. Penrose and W. Rindler *Spinors and Space-Time Volume 2: Spinor and Twistor Methods in Space-Time Geometry*†
S. Pokorski *Gauge Field Theories, 2^{nd} edition*†
J. Polchinski *String Theory Volume 1: An Introduction to the Bosonic String*†
J. Polchinski *String Theory Volume 2: Superstring Theory and Beyond*†
J. C. Polkinghorne *Models of High Energy Processes*†
V. N. Popov *Functional Integrals and Collective Excitations*†
L. V. Prokhorov and S. V. Shabanov *Hamiltonian Mechanics of Gauge Systems*
A. Recknagel and V. Schiomerus *Boundary Conformal Field Theory and the Worldsheet Approach to D-Branes*
R. J. Rivers *Path Integral Methods in Quantum Field Theory*†
R. G. Roberts *The Structure of the Proton: Deep Inelastic Scattering*†
C. Rovelli *Quantum Gravity*†
W. C. Saslaw *Gravitational Physics of Stellar and Galactic Systems*†
R. N. Sen *Causality, Measurement Theory and the Differentiable Structure of Space-Time*
M. Shifman and A. Yung *Supersymmetric Solitons*
H. Stephani, D. Kramer, M. MacCallum, C. Hoenselaers and E. Herlt *Exact Solutions of Einstein's Field Equations, 2^{nd} edition*†
J. Stewart *Advanced General Relativity*†
J. C. Taylor *Gauge Theories of Weak Interactions*†
T. Thiemann *Modern Canonical Quantum General Relativity*†
D. J. Toms *The Schwinger Action Principle and Effective Action*†
A. Vilenkin and E. P. S. Shellard *Cosmic Strings and Other Topological Defects*†
R. S. Ward and R. O. Wells, Jr *Twistor Geometry and Field Theory*†
E. J. Weinberg *Classical Solutions in Quantum Field Theory: Solitons and Instantons in High Energy Physics*
J. R. Wilson and G. J. Mathews *Relativistic Numerical Hydrodynamics*†
T. Ortín *Gravity and Strings*
D. Baumann and L. McAllister *Inflation and String Theory*
S. Raychaudhuri and K. Sridhar *Particle Physics of Brane Worlds and Extra Dimensions*
J. Kroon *Conformal Methods in General Relativity*

† Available in paperback

Hadrons at Finite Temperature

SAMIRNATH MALLIK
Saha Institute of Nuclear Physics

SOURAV SARKAR
Variable Energy Cyclotron Center, Kolkata

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781107145313

First published 2016

Printed in the United Kingdom by Clays, St Ives plc

A catalogue record for this publication is available from the British Library

Library of Congress Cataloging-in-Publication Data
Names: Mallik, Samirnath, author. | Sarkar, S. (Sourav), author.
Title: Hadrons at finite temperature / Samirnath Mallik (Saha Institute of Nuclear Physics), Sourav Sarkar (Variable Energy Cyclotron Center, Kolkata).
Other titles: Cambridge monographs on mathematical physics.
Description: Cambridge, United Kingdom ; New York, NY : Cambridge University Press, 2016. |
Series: Cambridge monographs on mathematical physics |
Includes bibliographical references and index.
Identifiers: LCCN 2016032396 | ISBN 9781107145313 (hardback) |
ISBN 1107145317 (hardback)
Subjects: LCSH: Hadrons. | Thermodynamics. | Field theory (Physics) | Broken symmetry (Physics) | Heavy ion collisions.
Classification: LCC QC793.5.H32 M35 2016 | DDC 539.7/216–dc23
LC record available at https://lccn.loc.gov/2016032396

ISBN 978-1-107-14531-3 Hardback

To the memory of my parents —S.M.

Contents

Cambridge University Press
978-1-107-14531-3 — Hadrons at Finite Temperature
Samirnath Mallik , Sourav Sarkar
Frontmatter
More Information

Cambridge University Press
978-1-107-14531-3 — Hadrons at Finite Temperature
Samirnath Mallik , Sourav Sarkar
Frontmatter
More Information

Cambridge University Press
978-1-107-14531-3 — Hadrons at Finite Temperature
Samirnath Mallik , Sourav Sarkar
Frontmatter
More Information

Preface

This book is an elementary introduction to hadronic properties at finite temperature (and chemical potential). Its contents may be divided into three parts.

The first part, comprising the first three chapters, develops the (vacuum) theory of hadronic interactions at low energy. As propagators play a major role in applications, we review in Chapter 1 in detail the vacuum propagators for fields of different spins. This provides the background in which to discuss later (in Chapter 4) the form of thermal propagators. The next two chapters are devoted to strong interaction dynamics. Chapter 2 describes the phenomenon of spontaneous symmetry breaking leading to Goldstone bosons. It prepares the ground for chiral perturbation theory in Chapter 3. We review this theory in some detail, including Goldstone as well as (heavy) non-Goldstone fields interacting with the Goldstone fields. Chapters 2 and 3 constitute an elementary yet general introduction to the effective theory of strong interactions at low energy.

The second part consists of the next two chapters and presents the equilibrium thermal field theory. Broadly speaking, there are two formulations of this theory, concerning so-called real and imaginary time. Though the imaginary time formulation is used more often than that of real time, we choose the latter in this book and try to show some of its advantages. In Chapter 4 we derive in elementary ways the thermal matrix propagators for fields of low spin and go on to obtain the spectral representation of complete propagators for fields of arbitrary spin. In Chapter 5 the thermal perturbation theory is developed in relation to the matrix structure of the propagators.

The third part consists of the last four chapters. Here we apply the methods developed to study different thermal one- and two-point functions in the hadronic phase using chiral perturbation theory. In Chapter 6 we obtain the temperature dependence of a number of hadronic parameters to one loop. Some two-loop results are presented in Chapter 7. In Chapter 8 we derive the dilepton cross-section measured in heavy ion collisions. Finally, Chapter 9 describes the (non-equilibrium) transport phenomenon, whose discussion can be reduced to fluctuations in thermal equilibrium. Clearly the range of applications of thermal field theory in particle physics is much wider. Many such applications are covered in books (mentioned in Chapter 4), though mostly in imaginary time formulation.

The first two parts are independent of each other, but it is the thread of spectral representations which binds all the three parts together. In the first part it appears in the rigorous proof of the Goldstone theorem. In the second part the thermal propagators for general fields are derived as spectral representations, which are then applied to problems in the third part. Our treatment of general fields demonstrates the belief that spin is not an essential complication in the thermal context, once its structure is understood in vacuum.

At the end of each chapter there are in general one or two problems, which are fully worked out. These are of different varieties, like important side results or pieces of calculation needed in the same or later chapters, which, if included in text, could affect the continuity of the presentation. The appendices attempt to derive all the results, mostly elementary, which are needed or mentioned in the text. Numerical evaluations are not reported, except on two occasions; where necessary, they are referred to in the literature.

Though a monograph, the book should be accessible to all graduate students interested in relativistic thermal field theory. The only prerequisite is acquaintance with the conventional perturbative framework using the canonical method of field theory. All other technical details are developed in the text and appendices.

Acknowledgements by S.M.

This book owes a great deal to Professor Heinrich Leutwyler. Through countless discussions with him, I could straighten out my understanding and hence the presentation of this book in many places. I take this opportunity to express my deep gratitude to him. Of course, we authors alone are responsible for any errors that may still remain. It is a pleasure for me to thank Professor Leutwyler's wife, Ursula, for her kind and warm hospitality during my stays with them. At this point I thank Professor A. Ereditato (Albert Einstein Centre for Fundamental Physics) for supporting my visits to the University of Bern. I also thank Professor H. Bebie, Professor J. Gasser and Professor P. Minkowski for their encouragement to complete the book.

I have lectured at several institutions on different parts of the book. For these invitations I thank Professor B. Ananthanarayan (Indian Institute of Science, Bangalore), Professor A. Harindranath (Saha Institute of Nuclear Physics, Kolkata), Professor H. Mishra (Physical Research Laboratory, Ahmedabad) and Professor H.S. Sharatchandra (Institute of Mathematical Sciences, Chennai). For help and suggestions with LaTex I thank Professor P.B. Pal (Saha Institute of Nuclear Physics). I also thank Professor M.K. Sanyal and Professor B.K. Chakrabarti (Saha Institute of Nuclear Physics) for extending academic facilities of the Institute during the period of writing the book. Finally I thank my daughter, Joyita and son-in-law, Saket for help with the computer.

This book was written under a project granted by the Department of Science and Technology, Government of India, during the period from May 2011 to April 2013. I am glad to have corresponded with Mr S.S. Kohli (Scientist F) during this period and thank him for his patience in awaiting the book's publication.

Notation

Greek indices run over t, x, y, z or 0, 1, 2, 3. The indices α, β, γ may also run over the generators of a symmetry group. Latin indices are used to run over components of other quantities. Repeated indices are summed over, unless stated otherwise.

The Lorentz metric is diagonal, $g_{\mu\nu} = g^{\mu\nu} = diag(1, -1, -1, -1)$. The d'Alembertian is $\Box = g^{\mu\nu}\partial^2/\partial x^\mu x^\nu = \partial_t^2 - \nabla^2$, where ∇^2 is the Laplacian $\partial^2/\partial x^i \partial x^i$.

Spatial coordinates and momentum three-vectors are denoted by boldface small letters. Isospin three-vectors are denoted by an arrow over the letters.

The complex conjugate of any quantity A is denoted A^*. The transpose and hermitian adjoint of a matrix or vector R is denoted R^T and $R^\dagger = R^{*T}$ respectively. The hermitian adjoint of an operator $\mathcal{O}$ is denoted $\mathcal{O}^\dagger$. A bar on a Dirac spinor ψ is defined by $\overline{\psi} = \psi^\dagger \gamma_0$. The conventions for Dirac gamma matrices are spelt out in Section 1.3.

Vacuum propagators are denoted by Δ_F for bosonic fields and by S_F for fermionic fields with necessary indices.

Thermal matrices are written in boldface capital letters. In particular, thermal propagators are denoted by $\boldsymbol{D}$ and $\boldsymbol{S}$ for bosonic and fermionic ones respectively, again with necessary indices. Thus, for bosonic propagators we make a notational distinction between the vacuum (Δ_F) and thermal (D) ones, which we do not maintain for fermionic propagators. When the matrices are diagonalised, the diagonal elements are written with a bar on the symbols, such as $\overline{D}$, $\overline{S}$ etc.

The mass of the pion is denoted by M to lowest order in chiral pertubation theory. Masses of other particles will be denoted by m with a subscript for the particle.

Following convention, we use natural units, $\hbar = c = 1$. So the fine structure constant is $\alpha = e^2/(4\pi) = 1/137$.

The equations in the text are numbered as usual, prefixed with chapter and section numbers. The equations in Problems are prefixed with P and chapter number. The appendices are denoted by A, B etc. and their equations are prefixed with the corresponding letter.