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A. P. S. Selvadurai , A. P. Suvorov
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Thermo-Poroelasticity and Geomechanics

Investigations of multiphysical processes in geomaterials have gained increasing attention due to the ongoing interest in solving complex geoenvironmental problems. This book provides a comprehensive exposition of the classical theory of thermo-poroelasticity, complemented by complete analytical solutions to problems in thermo-poromechanics that are used to validate computational results from multiphysics codes used in practice. The methodologies offer an insight into real-life problems related to modern environmental geosciences, including nuclear waste management, geologic sequestration of greenhouse gases to mitigate climate change, and the impact of energy resources recovery on groundwater resources. A strong focus is placed on analytical approaches to benchmark the accuracy of the computational approaches that are ultimately used in real-life problems. The extensive coverage of both theory and applications in thermo-poroelasticity and geomechanics provides a unified presentation of the topics, making this an accessible and invaluable resource for researchers, students or practitioners in the field.

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Thermo-Poroelasticity and Geomechanics

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This book is dedicated to the memory of our parents
K.S. Selvadurai and W.M.A. Selvadurai
and
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Preface

Investigations of multiphysical processes in geomaterials have come to the forefront due to the continuing interests in the application of such theories for solving complex geoenvironmental problems. The topic of geologic disposal of heat-emitting nuclear fuel requires the consideration of the simultaneous influences of mechanical deformations, fluid flow, heat transfer and chemical alterations. Similar processes are encountered in the geologic sequestration of greenhouse gases in fluidized forms, the extraction of geothermal energy from both shallow and deep sources, the fluid pressure-induced influences of faults during slip at a fault or rock fracture, the mechanics of frozen soils, and geomechanical influences of glacial advances. The complete consideration of all possible multiphysical interactions in geomaterials is a daunting task and clearly an unrealistic expectation. For example, the study of thermo-hydro-mechanical effects in geologic media has to consider the mechanics of typical porous skeletons due to elastic, plastic and creep behavior, the fluid transport characteristics of the intact skeleton, and how these processes in turn can be influenced by the mechanical alterations of the porous skeleton and the heat transfer processes. In addition, chemical actions can alter the mechanical, fluid flow and thermal properties of the geological medium. The coupled multiphysical influences can be made more complicated by introducing considerations such as anisotropy, heterogeneity of the porous fabric and non-saturation of the porous medium. Clearly, the consideration of all these processes is formidable when the constitutive functionals and parameters have to be determined experimentally to make the results meaningful for geoscientific applications. The situation becomes even more complex because the interpretation of material data gathered for geoscientific applications can vary from the laboratory miniature specimen scale through the bench scale to the field scale. The prudent approach is to keep the multiphysical interactions to a bare minimum so that the processes that are critical to geoscientific applications can be emphasized. Such an approach requires a great deal of experience and patience. Although this volume largely focuses on geomaterial behavior, the range of applications of the theories of thermo-poroelastic behavior extends beyond the geosciences and can provide useful approaches to modeling problems in diverse areas, from biomechanics, notably the mechanics of bone materials and soft tissues, to industrial processes, involving drying of porous materials.

Once a multiphysical model of a geomaterial is selected, the application of the theory to any practical problem requires the use of computational approaches to represent the physical geoenvironmental situation in a realistic form. The area of computational multiphysics modeling of thermo-hydro-mechanical processes in geomaterials is a major area of research in the geosciences, and several computational codes have been developed by commercial organizations, university researchers, industrial organizations and research institutions. A common theme in most of these developments is the ubiquitous caveat that the developers do not accept responsibility for the accuracy of the computational treatments for the problems posed by the user. It is therefore the responsibility of the user to engage in validation exercises that can build confidence in the computational modeling capabilities of a particular code. This problem is sometimes addressed through inter-code calibrations, where computational schemes developed by various organizations are used to examine the same benchmark problem and to provide informative evaluations of the computational capabilities and accuracy. An alternative approach is to compare the computational results with known solutions to both linear and nonlinear multiphysics problems. Such attempts are rare in situations where thermo-hydro-mechanical processes in their entirety are taken into consideration. The main objective of this volume is to present a set of analytical solutions that can be used as elementary representations of practical situations involving thermo-hydro-mechanical problems that can also serve as benchmarks for the calibration of computational schemes. The analytical treatments presented are restricted to linear initial boundary value problems in thermo-poroelasticity, the rationale being that the computational developments for examining fully non-linear problems in thermo-hydro-mechanics should, whenever possible, be first calibrated against exact analytical solutions for linear problems.

Chapter 1 presents an introduction to the range of applications of thermo-poroelasticity. The topic of deep geologic disposal of heat-emitting nuclear waste represents one of the well-developed areas of applications of thermo-hydro-mechanics. These developments have had a powerful influence on other areas of application, including geologic sequestration of greenhouse gases and geothermal energy extraction. Other topics, such as enhanced oil recovery, frictional heating during slip and rupture along geological faults, and discontinuities and cold regions geomechanics, invariably require the consideration of thermo-hydro-mechanical effects. The authors have attempted to provide references to a reasonably comprehensive set of articles, but this list is not meant to be exhaustive. These topics all contain an extensive body of literature of varying rigor, quality and utility; the interested reader is well advised to consult the commonly accessible information retrieval systems to supplement the references cited here. The main objective of Chapter 1 is to illustrate that the mathematical developments in the areas indicated have common threads and that full advantage can be taken of the complementary aspects when organizing future research directions.

Chapter 2 presents the constitutive equations and field equations governing non-isothermal quasi-static thermo-poroelastic processes. The restriction to quasi-static processes is governed primarily by the range of potential applications indicated in Chapter 1. Issues related to the influence of earthquake loads and other dynamic phenomena are also important to all the problem areas discussed in Chapter 1, but these would require a separate volume to treat them adequately. The constitutive relationships outlined in this book consider only Hookean elastic materials and porous media for which Darcy's law provides an adequate description for fluid flow through the accessible saturated pore space. In the case of the heat transfer process, attention is restricted to heat conduction phenomena, and other modes of heat transfer are omitted. The justification for excluding convective heat transfer processes is largely dictated by the areas of application of the developments, notably to low-porosity intact geological materials where the process of heat transfer by convection of the saturating pore fluid is less important. The developments outlined can also be extended to examine sparsely fractured geological media but not to situations where the geological medium is extensively fractured. In highly fractured geomaterials, convective heat transfer will be important, but whether such geological locations provide the best setting for deep disposal of heat-emitting radioactive waste or geologic sequestration of fluidized greenhouse gases is debatable. The chapter culminates in the generalized presentation of the initial boundary value problem governing the thermo-poroelasticity problem.

The presentation of any viable set of benchmark problems must necessarily include the one-dimensional problem. A comprehensive study of the one-dimensional initial boundary value problem in thermo-poroelasticity is given in Chapter 3. The initial conditions and boundary conditions are varied to generate solutions to problems that can constitute experimental situations either for determining the material parameters or for the calibration of experiments. This chapter also presents, separately, thermoelastic, hydro-mechanical and thermo-hydro-mechanical problems, and includes the influence of both the fluid-phase and solid-phase compressibilities in the mathematical developments. Both numerical results and results of computational estimates are presented for various initial boundary value problems with reduced couplings. The results presented in this chapter are also instructive for educational purposes, where the treatments of the various forms of couplings are self-contained.

In Chapter 4 we examine the thermo-poroelastic problem of a fluid-filled rigid cavity that is contained between semi-infinite one-dimensional poroelastic regions. Since the cavity material is rigid, the expansion is due only to the physical process of thermal expansion of the cavity region. Useful approximations are invoked to develop analytical results for the pressure within the cavity due to either pressure or thermal pulses applied to the cavity region. The similarities in the hydraulic pulse tests are demonstrated using the analytical results and compared with computational results obtained from a finite element modeling.

Chapter 5 considers the classical one-dimensional axisymmetric analysis of the thermo-poroelasticity problem where a cylinder is subjected to a non-zero temperature change over the lateral cylindrical surface; this surface is maintained free of radial effective stress and the pore pressures are maintained at a zero value. The entire cylindrical element is also subjected to an axial strain during the application of the temperature changes. Complete mathematical solutions are developed for various combinations of thermal and mechanical loadings, and the mathematical results are used to validate computational models of the different problems. The development of Mandel–Cryer type responses in the pore pressure field within the cylinder is used as a guide to establish the acceptability of the results. Chapter 6 likewise considers the exterior plane strain problem related to a fluid-filled cylindrical cavity in a poroelastic medium of infinite extent. When modeling problems related to fluid regions, the porosity of the cavity region is set to unity and the elastic stiffness and thermal expansion properties of the cavity region are assigned small values. The applicability of the physical situation to transient pressure pulse tests that can be conducted along a section of a borehole is also discussed. The results of the analytical solutions are used to validate a computational procedure. The solutions are also used to demonstrate the distinctions that can arise between coupled and uncoupled approaches; the distinct solutions can influence the experimental estimation of hydro-thermo-mechanical parameters of fluid-saturated geologic media subjected to heating.

Chapters 7 and 8 are devoted to the study of spherically symmetric thermo-poroelasticity problems dealing with the solid sphere and the spherical cavity, respectively. Here, again, analytical results are developed for situations where either the boundary of the solid sphere is subjected to mechanical and/or thermal loads, or the internal cavity is subjected to heating. Despite the spatially one-dimensional nature, the sphere and spherical cavity problems are three-dimensional problems that allow validation of computational approaches, which can then be used to examine real-life problems. An extensive range of analytical solutions are presented, and these are again used to test the accuracy of computational schemes.

Finally, Chapter 9 examines the thermo-hydro-mechanical processes that manifest in a sparsely fractured rock mass during the advance of a glacier. The problem has no convenient analytical result that can be used to generate even approximate results. Hence a real-life example similar to the one posed in this chapter can be solved only by using a computational approach. In view of the reasonable correlation that exists between the analytical and computational results for benchmark problems presented in the previous chapters, there is justification for examining the problem related to the glacial advance using a robust computational approach. The results of the computational modeling provide information that can be used to further refine the computational approach at locations critically important to a safety assessment scenario. The sparsely fractured rock mass contains sessile fractures, and, in the case of the computational modeling, it has the added advantage that nonlinear thermo-hydro-mechanical effects can be included in the extension of the work.

In this volume, we have attempted to present a comprehensive study of a set of benchmark problems that can be used to validate the thermo-poromechanical aspects of coupled processes. This volume follows the highly successful Cambridge University Press volumes *Elasticity and Geomechanics* (1996) by R.O. Davis and A.P.S. Selvadurai and *Plasticity and Geomechanics* (2002) by R.O. Davis and A.P.S. Selvadurai, where the emphasis was on the introduction of the subject matter through a comprehensive exposition of the relevant material complemented by a set of exercises in each chapter. This volume follows the same basic philosophy, except that the examples are presented within the chapters. In some instances, the examples provided constitute original research on the subject. An extensive set of references is presented in each chapter to enable the researcher, student or practitioner to obtain an appreciation of the wider applicability of the results. The mathematical results presented in the volume have been verified to the best abilities of the authors, who would appreciate readers drawing our attention to any errors or omissions.

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The work presented in this volume grew out of the longstanding association of the first author with problems related to environmental geomechanics and, in particular, the topic of deep geologic disposal of heat-emitting nuclear fuel wastes. The research activities in this area, and allied topics, involved theoretical developments, computational modeling and experimental research, and were supported by Atomic Energy of Canada Limited, the Canadian Nuclear Safety Commission, the Geological Survey of Canada, the National Energy Board, the Nuclear Waste Management Organization of Ontario, and the Natural Sciences and Engineering Research Council of Canada. This research support was instrumental in maintaining a viable research program that enabled the developments presented in the volume. The Fellowship support provided by the Killam Program of the Canada Council for the Arts, the Humboldt Senior Scientist and the Max Planck Awards of the Humboldt Foundation is also gratefully acknowledged. A number of past students and researchers have contributed significantly to the research activities involving thermo-porophysics and thermo-poromechanics of geomaterials, including Professor Z.Q. Yue, Dr. M.C. Au, Dr. J. Hu, Dr. T.S. Nguyen, Dr. K. Sepehr, Dr. A.T. Mahyari, Dr. G. Armand, Dr. Q. Lan, Dr. A. Shirazi, Dr. Q. Yu, Dr. W. Dong, Dr. H. Ghiabi, Dr. M. Najari, Dr. P.A. Selvadurai, Dr. J. Kim, Mr. P. Carnaffan, Mr. A. Letendre, Mr. L. Jenner, Mr. B. Hekimi, Mr. A. Głowacki, Mr. C. Couture, Mr. K.C. McMartin and Mr. N. Vannelli. It is a pleasure to acknowledge their contributions to the research activities in environmental geomechanics both at Carleton University and at McGill University. McGill's support in terms of the James McGill Professorship is also gratefully acknowledged. The enthusiastic support for the volume provided by Professor A.H.-D. Cheng, University of Mississippi, Professor Y. Abousleiman, University of Oklahoma, and Emeritus Professor R.O. Davis, University of Canterbury, is particularly appreciated. The authors are grateful to Dr. Susan Francis and the staff at Cambridge

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