Convergence of One-Parameter Operator Semigroups

This book presents a detailed and contemporary account of the classical theory of convergence of semigroups and its more recent developments treating the case where the limit semigroup, in contrast to the approximating semigroups, acts merely on a subspace of the original Banach space (this is the case, for example, with singular perturbations). The author demonstrates the far-reaching applications of this theory using real examples from various branches of pure and applied mathematics, with a particular emphasis on mathematical biology. These examples also serve as short, nontechnical introductions to biological concepts involved, allowing readers to develop intuitions underlying mathematical results.

The book may serve as a useful reference, containing a significant number of new results ranging from the analysis of fish populations to signaling pathways in living cells. It comprises many short chapters, which allows readers to pick and choose those topics most relevant to them, and it contains 160 end-of-chapter exercises so that readers can test their understanding of the material as they go along.

ADAM BOBROWSKI is a professor and Chairman of the Department of Mathematics at Lublin University of Technology, Poland. He has authored over 50 scientific papers and two books *Functional Analysis for Probability and Stochastic Processes* and *An Operator Semigroup in Mathematical Genetics*.

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Convergence of One-Parameter Operator Semigroups

In Models of Mathematical Biology and Elsewhere

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To the most enthusiastic drummer, boxer, and eater ever – my son Marek

Contents

Preface

Now lift up your eyes, and look from the place where you are, northward and southward and eastward and westward; for all the land that you see I will give to you and to your seed forever.

(Gen. 13:14b–15. Holy Bible. Recovery Version. © 2003 Living Stream Ministry, Anaheim, California.)

The question of convergence and the related Trotter–Kato–Neveu–Sova–Kurtz Theorem lie in the center of the theory of one-parameter semigroups of operators. In fact, many key results, including the fundamental Hille–Yosida Theorem, are proved by approximating a semigroup involved by a family of semigroups that are easier to handle. On the other hand, the most elegant proof of the Trotter–Kato–Neveu–Sova–Kurtz Theorem is obtained by applying the Hille–Yosida Theorem to a certain, coordinate-wise acting operator in the space of convergent sequences [163, 211, 229]. Hence, all of the basic books devoted to the theory, except, of course, of the classics [115] and [180] published before the Trotter–Kato–Neveu–Sova–Kurtz Theorem was known, treat the subject at least roughly (see, e.g., [9, 101, 128, 163, 284, 334, 353]). Other books, like [132], take advantage of the theory and show its beautiful and far-reaching applications.

This book aims to present the classic theory of convergence of semigroups and also to discuss its applicability to models of mathematical biology and other branches of mathematics. At the same time, it is an attempt at expressing my long fascination with phenomena accompanying such convergence, and my continuous amazement at the variety of examples, especially of the stochastic type, found in the literature.

The main difficulty in applying the theory to the models mentioned here is that the Trotter–Kato–Neveu–Sova–Kurtz-type theorems characterize convergence that is almost uniform in time $t \in [0, \infty)$, whereas in the models one

often encounters a situation in which convergence is not uniform in the neighborhood of $t = 0$. Therefore, this book splits naturally into two parts. In the first part, I discuss the models in which the classical theory applies. In the second part, I present methods developed for treating convergence that is not uniform near $t = 0$, such as singular perturbations. Therefore, in particular, the book is an outgrowth of Tosio Kato's project [199] to incorporate singular perturbations in the theory of convergence of semigroups of operators.

As already stated, this book is equally devoted to the general theory and to examples, which are selected from various branches of applied and pure mathematics with an emphasis on models of mathematical biology. These examples serve to exemplify applications of the main theorems, but at the same time are meant to be nontechnical, short introductions to related biological models. Deprived of these examples, the book loses more than half of its worth (if it possesses any). However, for readers who want to know the structure of the book, the main theoretical chapters in Part I are:

- Chapter 2, where the basic convergence theorem is presented
- Chapter 7, where the relations between the limit pseudoresolvent and the extended limit are presented
- Chapter 8, devoted to the characterization of the regularity subspace where the semigroups converge in a regular way
- Chapter 14, in which the question of convergence outside of the regularity space is discussed
- Chapter 15, in which the Hasegawa condition is introduced
- Chapter 18, which is devoted to discrete-time approximations
- Chapter 24, which is devoted to the case in which the approximating semigroups act in different spaces than the limit semigroup
- Chapter 25, in which the stability condition is discussed

In Part II, the most important theoretical chapters include:

- Chapters 26, 28, and 29, in which basic examples and fundamental properties of irregular convergence are presented
- Chapter 31, providing tools for dealing with parabolic problems
- Chapter 42, presenting the important Kurtz's Theorem, which allows dealing with variety of convergence questions, including problems of hyperbolic type
- Chapters 54 and 55, giving some insight into alternative approaches

In Part III, devoted to convergence of cosine families in addition to basic theory (which is in many aspects parallel to the theory of convergence of semigroups), we reveal the stunning result that there is no singular perturbation theory for cosine families.

Perhaps it should be added here that in Part II, Chapters 33 through 40 (with the exception of Chapter 37) form an essay on singular limits involving fast diffusion, including the treatment of the homogenization theorem of Conway, Hoff, and Smoller discussed in Chapter 38.

This book suffers from the omission of important theorems and examples, most of which I am simply ignorant of. Of those I am aware of, I should mention the beautiful phenomenon ascribed to Bafico and Baldi [13]. The reason for not including it here is that it appears to be related to convergence in weaker topology, and I determined to restrict myself to strong topology of Banach spaces. Other examples include the Janas–Berezin approximation method [33, 189], state-space collapse in the theory of queues (see [295] and the references given there), models of adaptive dynamics [79, 113, 285] (which are in most interesting cases nonlinear), diffusion processes on an open book [152], and definitely many, many more. See, for example, the recent book by J. Banasiak and M. Lachowicz [25] written in the spirit of asymptotic analysis, and the paper by S. N. Evans and R. B. Sowers [133] – a rich source of examples of stochastic nature (I thank T. G. Kurtz and S. N. Evans for this reference). Fortunately, there are other, perhaps more apt accounts of the geography of the vast land of singular-perturbation theory, including [205, 263, 280, 311, 316, 333], to mention just a few that I have on my bookshelf. (On April 30, 2015, MathSciNet returned 2,404 matches to the "singular perturbation" query.) The other side of this coin is that the lion's share of the examples presented here have not been available in book form as of this writing.

I have benefited from delivering a series of lectures at the Institute of Mathematics of the Polish Academy of Sciences and teaching two courses at Maria Curie-Skłodowska University (UMCS) in Lublin, Poland, all based on the material in this book. Judging by the feedback that I have received, audiences have benefited, too, but you can never be sure of things like that. I would like to thank both institutions for allowing me to teach these courses, despite the danger of having a new generation of students interested in the theory of semigroups of operators. Financially, beginning in 2011 and until 2014, I was supported by the Polish Government, grant number 6081/B/H03/2011/40.

Moreover, I would like to express my thanks to Professor Jan Kisyński, my teacher and my friend, who introduced me to the world of convergence of semigroups and for years has been an invaluable "point of reference," and to my students, particularly to Radek Bogucki, without whom this book would have been completely different, and Adam Gregosiewicz (a victim of my course at UMCS), who solved many exercises – his solution manual will be available on the web in due time, "but you can never be sure of things like that." Special thanks are also due to Jacek Banasiak for stimulating discussions, hints for

solving problems, and references leading to new and interesting ones. Moreover, I would like to acknowledge friendly support I have received from A. Bátkai, K. Bogdan, D. Mugnolo, W. Chojnacki, J. Goldstein, M. Kimmel, T. Komorowski, M. Lachowicz, H. Leszczyński, T. Lipniacki, A. Marciniak-Czochra, R. Rudnicki, Y. Tomilov, and D. Wrzosek, who in various ways contributed to my understanding of the subjects covered in the book. They have done it in the unfeigned hope that I will be able to comprehend some complex issues in mathematics and biology; if their hopes are shattered by the book, they should not be held responsible for all or any misrepresentations, as they were acting in good faith.

Last but not least, thanks are due to my family: I would like to thank my wife Beata for her constant encouragement and help, without which it would be very difficult to devote so much time to writing this book. Special thanks go also to my sons, Radek and Marek, for supplementing this monograph with unique pictures.[1,2]

[1] Approximately 10 years ago, after I had completed writing my first book [49], I heartily wanted to thank Roger Astley from Cambridge University Press for his help and encouragement. However, he did not allow me, by saying something to the effect that Cambridge University Press does not approve of such acknowledgments. By looking at other books recently, to my astonishment I discovered that this was a flat lie. Hence, I am thanking him now in a footnote, which is printed in footnote-size, lest he should notice.

[2] Hey, I can also thank my friends Chyl and Jarecki for, hm, being my friends.