

## Modern Condensed Matter Physics

Modern Condensed Matter Physics brings together the most important advances in the field from recent decades. It provides instructors teaching graduate-level condensed matter courses with a comprehensive and in-depth textbook that will prepare graduate students for research or further study alongside reading more advanced and specialized books and research literature in the field.

This textbook covers the basics of crystalline solids as well as analogous optical lattices and photonic crystals, while discussing cutting-edge topics such as disordered systems, mesoscopic systems, many-body systems, quantum magnetism, Bose–Einstein condensates, quantum entanglement, and superconducting quantum bits.

Students are provided with the appropriate mathematical background to understand the topological concepts that have been permeating the field, together with numerous physical examples ranging from the fractional quantum Hall effect to topological insulators, the toric code, and Majorana fermions. Exercises, commentary boxes, and appendices afford guidance and feedback for beginners and experts alike

**Steven M. Girvin** received his BS in 1971 from Bates College and his PhD in 1977 from Princeton University. He joined the Yale faculty in 2001, where he is Eugene Higgins Professor of Physics and Professor of Applied Physics. From 2007 to 2017 he served as Deputy Provost for Research. His research interests focus on theoretical condensed matter physics, quantum optics, and quantum computation; he is co-developer of the circuit QED paradigm for quantum computation.

Honors: Fellow of the American Physical Society, the American Association for the Advancement of Science, and the American Academy of Arts and Sciences; Foreign Member of the Royal Swedish Academy of Sciences, Member of the US National Academy of Sciences; Oliver E. Buckley Prize of the American Physical Society (2007); Honorary doctorate, Chalmers University of Technology (2017); Conde Award for Teaching Excellence (2003).

**Kun Yang** received his BS in 1989 from Fudan University and his PhD in 1994 from Indiana University. In 1999 he joined the faculty of Florida State University, where he is now McKenzie Professor of Physics. His research focuses on many-particle physics in condensed matter and trapped-cold-atom systems.

Honors: Fellow of the American Physical Society and the American Association for the Advancement of Science; Alfred Sloan Research Fellowship (1999); Outstanding Young Researcher Award, Overseas Chinese Physics Association (2003).



# Modern Condensed Matter Physics

STEVEN M. GIRVIN

Yale University, Connecticut

KUN YANG

Florida State University







Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05-06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment, a department of the University of Cambridge.

We share the University's mission to contribute to society through the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9781107137394

DOI: 10.1017/9781316480649

© Steven M. Girvin and Kun Yang 2019

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press & Assessment.

First published 2019 (version 2, August 2023)

A catalogue record for this publication is available from the British Library

Library of Congress Cataloging-in-Publication data

Names: Girvin, Steven M., author. | Yang, Kun, 1967- author.

Title: Modern condensed matter physics / Steven M. Girvin (Yale University,

Connecticut), Kun Yang (Florida State University).

Description: Cambridge ; New York, NY : Cambridge University Press, [2019]

Identifiers: LCCN 2018027181 | ISBN 9781107137394

Subjects: LCSH: Condensed matter. | Electronic structure. | Atomic structure.

Classification: LCC QC173.454 .G57 2019 | DDC 530.4/1-dc23

LC record available at https://lccn.loc.gov/2018027181

ISBN 978-1-107-13739-4 Hardback

Additional resources for this publication at www.cambridge.org/Girvin&Yang

Cambridge University Press & Assessment has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.



#### Brief contents

	Preface Acknowledgments	<i>page</i> xvii xix
1	Overview of Condensed Matter Physics	1
2	Spatial Structure	9
3	Lattices and Symmetries	20
4	Neutron Scattering	44
5	<b>Dynamics of Lattice Vibrations</b>	64
6	Quantum Theory of Harmonic Crystals	78
7	<b>Electronic Structure of Crystals</b>	98
8	Semiclassical Transport Theory	164
9	Semiconductors	198
10	Non-local Transport in Mesoscopic Systems	222
11	Anderson Localization	252
12	Integer Quantum Hall Effect	301
13	Topology and Berry Phase	331
14	<b>Topological Insulators and Semimetals</b>	362
15	Interacting Electrons	376
16	Fractional Quantum Hall Effect	430



vi	BRI	EF CONTENTS	
17	Ma <sub>2</sub>	gnetism	480
18	Bos	e–Einstein Condensation and Superfluidity	531
19	Sup	erconductivity: Basic Phenomena and Phenomenological Theories	549
20	) Mic	roscopic Theory of Superconductivity	592
Appen	dix A.	Linear-Response Theory	632
Appen	dix B.	The Poisson Summation Formula	640
Appen	dix C.	Tunneling and Scanning Tunneling Microscopy	642
Appen	dix D.	Brief Primer on Topology	647
Appen	dix E.	Scattering Matrices, Unitarity, and Reciprocity	657
Appen	dix F.	<b>Quantum Entanglement in Condensed Matter Physics</b>	659
Appen	dix G.	Linear Response and Noise in Electrical Circuits	665
Appen	dix H.	Functional Differentiation	673
Appen	dix I.	Low-Energy Effective Hamiltonians	675
Appen	dix J.	Introduction to Second Quantization	680
	Refe Inde:	rences	685 692



#### Contents

	Preface Acknowledgments	page xvii xix
1	Overview of Condensed Matter Physics	1
1.1	Definition of Condensed Matter and Goals of Condensed Matter Physics	1
1.2	Classification (or Phases) of Condensed Matter Systems	3
	1.2.1 Atomic Spatial Structures	4
	1.2.2 Electronic Structures or Properties	4
	1.2.3 Symmetries	5
	1.2.4 Beyond Symmetries	6
1.3	Theoretical Descriptions of Condensed Matter Phases	6
1.4	Experimental Probes of Condensed Matter Systems	8
2	Spatial Structure	9
2.1	Probing the Structure	9
2.2	Semiclassical Theory of X-Ray Scattering	10
	<b>(2)</b>	
2.3	Quantum Theory of Electron–Photon Interaction and X-Ray Scattering	13
2.4	X-Ray Scattering from a Condensed Matter System	15
2.5	Relationship of $S(\vec{q})$ and Spatial Correlations	16
2.6	Liquid State versus Crystal State	17
3	Lattices and Symmetries	20
3.1	The Crystal as a Broken-Symmetry State	20
3.2	Bravais Lattices and Lattices with Bases	24
	3.2.1 Bravais Lattices	24
	3.2.2 Lattices with Bases	26
2.2	3.2.3 Lattice Symmetries in Addition to Translation	29
3.3	Reciprocal Lattices	30
3.4 3.5	X-Ray Scattering from Crystals Effects of Lattice Fluctuations on X-Ray Scattering	34 38
3.6	Notes and Further Reading	41
4	Neutron Scattering	44
• 4.1	Introduction to Neutron Scattering	44
+.1 4.2	Inelastic Neutron Scattering	46
T.∠	metastic recuton Scattering	40



viii	CONTE	NTS	
4.3	Dynamic	cal Structure Factor and $f$ -Sum Rule	50
	4.3.1	Classical Harmonic Oscillator	54
	4.3.2	Quantum Harmonic Oscillator	56
4.4	Single-N	Mode Approximation and Superfluid <sup>4</sup> He	60
5	Dynan	nics of Lattice Vibrations	64
5.1	Elasticit	y and Sound Modes in Continuous Media	64
5.2		c Approximation and Harmonic Expansion of Atomic Potential	68
5.3		l Dynamics of Lattice Vibrations	71
6	Quant	rum Theory of Harmonic Crystals	78
6.1	Heat Ca	pacity	78
6.2		al Quantization of Lattice Vibrations	83
6.3		n Dynamical Structure Factor	88
6.4		Waller Factor and Stability of Crystalline Order	91
6.5	•	uer Effect	93
7	Electr	onic Structure of Crystals	98
7.1	Drude T	heory of Electron Conduction in Metals	98
7.2	Independ	dent Electron Model	104
7.3	Bloch's	Theorem	105
	7.3.1	Band Gaps and Bragg Reflection	114
	7.3.2	Van Hove Singularities	115
	7.3.3	Velocity of Bloch Electrons	116
7.4	Tight-Bi	nding Method	117
	7.4.1	Bonds vs. Bands	122
	7.4.2	Wannier Functions	122
	7.4.3	Continuum Limit of Tight-Binding Hamiltonians	124
	7.4.4	Limitations of the Tight-Binding Model	126
	7.4.5	s-d Hybridization in Transition Metals	129
7.5	Graphen	e Band Structure	133
7.6		tylene and the Su–Schrieffer–Heeger Model	138
	7.6.1	Dirac electrons in 1D and the Peierls instability	138
	7.6.2	Ground-State Degeneracy and Solitons	142
	7.6.3	Zero Modes Bound to Solitons	144
	7.6.4	Quantum Numbers of Soliton States and Spin-Charge Separation	147
7.7	Thermoo	dynamic Properties of Bloch Electrons	148
	7.7.1	Specific Heat	149
	7.7.2	Magnetic Susceptibility	150
7.8	Spin-Or	bit Coupling and Band Structure	153
7.9	Photonic	e Crystals	156
7.10	Optical I	Lattices	159
	7.10.1	Oscillator Model of Atomic Polarizability	160
	7.10.2	Quantum Effects in Optical Lattices	162
8	Semic	lassical Transport Theory	164
8.1	Review of	of Semiclassical Wave Packets	164



	CONTENTS	ix
8.2	Semiclassical Wave-Packet Dynamics in Bloch Bands	165
	8.2.1 Derivation of Bloch Electron Equations of Motion	169
	8.2.2 Zener Tunneling (or Interband Transitions)	109
8.3	Holes	171
8.4	Uniform Magnetic Fields	171
8.5	Quantum Oscillations	176
8.6	Semiclassical $\vec{E} \times \vec{B}$ Drift	179
8.7	The Boltzmann Equation	181
8.8	Boltzmann Transport	186
	8.8.1 Einstein Relation	191
8.9	Thermal Transport and Thermoelectric Effects	193
9	Semiconductors	198
9.1	Homogeneous Bulk Semiconductors	198
9.2	Impurity Levels	204
9.3	Optical Processes in Semiconductors	207
,,,,	9.3.1 Angle-Resolved Photoemission Spectroscopy	210
9.4	The p-n Junction	212
	9.4.1 Light-Emitting Diodes and Solar Cells	215
9.5	Other Devices	216
	9.5.1 Metal–Oxide–Semiconductor Field-Effect Transistors (MOSFETs)	216
	9.5.2 Heterostructures	217
	9.5.3 Quantum Point Contact, Wire and Dot	220
9.6	Notes and Further Reading	221
10	Non-local Transport in Mesoscopic Systems	222
10.1	Introduction to Transport of Electron Waves	222
10.2	Landauer Formula and Conductance Quantization	225
10.3	Multi-terminal Devices	231
10.4	Universal Conductance Fluctuations	233
	10.4.1 Transmission Eigenvalues	238
	10.4.2 UCF Fingerprints	240
10.5	Noise in Mesoscopic Systems	242
	10.5.1 Quantum Shot Noise	245
10.6	Dephasing	248
11	Anderson Localization	252
11.1	Absence of Diffusion in Certain Random Lattices	253
11.2	Classical Diffusion	256
11.3	Semiclassical Diffusion	258
	11.3.1 Review of Scattering from a Single Impurity	258
	11.3.2 Scattering from Many Impurities	262
	11.3.3 Multiple Scattering and Classical Diffusion	265
11.4	Quantum Corrections to Diffusion	267
	11.4.1 Real-Space Picture	268
	11.4.2 Enhanced Backscattering	269



x	CONTENTS	
11.5	Weak Localization in 2D	271
	11.5.1 Magnetic Fields and Spin–Orbit Coupling	273
11.6	Strong Localization in 1D	275
11.7	Localization and Metal–Insulator Transition in 3D	277
11.8	Scaling Theory of Localization and the Metal–Insulator Transition	279
	11.8.1 Thouless Picture of Conductance	279
	11.8.2 Persistent Currents in Disordered Mesoscopic Rings	282
	11.8.3 Scaling Theory	283
	11.8.4 Scaling Hypothesis and Universality	284
11.9	Scaling and Transport at Finite Temperature	287
	11.9.1 Mobility Gap and Activated Transport	291
	11.9.2 Variable-Range Hopping	292
11.10	Anderson Model	294
11.11	Many-Body Localization	297
12	Integer Quantum Hall Effect	301
12.1	Hall-Effect Transport in High Magnetic Fields	301
12.2	Why 2D Is Important	304
12.3	Why Disorder and Localization Are Important	305
12.4	Classical and Semiclassical Dynamics	306
	12.4.1 Classical Dynamics	306
	12.4.2 Semiclassical Approximation	308
12.5	Quantum Dynamics in Strong B Fields	309
12.6	IQHE Edge States	315
12.7	Semiclassical Percolation Picture of the IQHE	318
12.8	Anomalous Integer Quantum Hall Sequence in Graphene	321
12.9	Magnetic Translation Invariance and Magnetic Bloch Bands	324
	12.9.1 Simple Landau Gauge Example	327
12.10	Quantization of the Hall Conductance in Magnetic Bloch Bands	329
13	Topology and Berry Phase	331
13.1	Adiabatic Evolution and the Geometry of Hilbert Space	331
13.2	Berry Phase and the Aharonov–Bohm Effect	336
13.3	Spin-1/2 Berry Phase	339
	13.3.1 Spin–Orbit Coupling and Suppression of Weak Localization	343
13.4	Berry Curvature of Bloch Bands and Anomalous Velocity	344
	13.4.1 Anomalous Velocity	345
13.5	Topological Quantization of Hall Conductance of Magnetic Bloch Bands	348
	13.5.1 Wannier Functions of Topologically Non-trivial Bands	351
	13.5.2 Band Crossing and Change of Band Topology	352
	13.5.3 Relation Between the Chern Number and Chiral Edge States: Bulk–Edge	
	Correspondence	353
13.6	An Example of Bands Carrying Non-zero Chern Numbers: Haldane Model	356
13.7	Thouless Charge Pump and Electric Polarization	358
	13.7.1 Modern Theory of Electric Polarization	360



	CONTENTS	xi
14	Topological Insulators and Semimetals	362
14.1	Kane–Mele Model	362
14.2	$\mathbb{Z}_2$ Characterization of Topological Insulators	364
14.3	Massless Dirac Surface/Interface States	368
14.4	Weyl Semimetals	371
	14.4.1 Fermi Arcs on the Surface	372
	14.4.2 Chiral Anomaly	373
14.5	Notes and Further Reading	375
15	Interacting Electrons	376
15.1	Hartree Approximation	376
15.2	Hartree–Fock Approximation	378
	15.2.1 Koopmans' Theorem	381
15.3	Hartree–Fock Approximation for the 3D Electron Gas	382
	15.3.1 Total Exchange Energy of the 3DEG in the	
	Hartree–Fock Approximation	384
15.4	Density Functional Theory	385
15.5	Kohn–Sham Single-Particle Equations	387
15.6	Local-Density Approximation	389
15.7	Density–Density Response Function and Static Screening	391
	15.7.1 Thomas–Fermi Approximation	394
	15.7.2 Lindhard Approximation	394
15.8	Dynamical Screening and Random-Phase Approximation	396
15.9	Plasma Oscillation and Plasmon Dispersion	397
	15.9.1 Plasma Frequency and Plasmon Dispersion from the RPA	397
	15.9.2 Plasma Frequency from Classical Dynamics	398
	15.9.3 Plasma Frequency and Plasmon Dispersion from	
	the Single-Mode Approximation	399
15.10	Dielectric Function and Optical Properties	400
	15.10.1 Dielectric Function and AC Conductivity	400
	15.10.2 Optical Measurements of Dielectric Function	401
15.11	Landau's Fermi-Liquid Theory	402
	15.11.1 Elementary Excitations of a Free Fermi Gas	402
	15.11.2 Adiabaticity and Elementary Excitations of an Interacting Fermi Gas	404
	15.11.3 Fermi-Liquid Parameters	407
15.12	Predictions of Fermi-Liquid Theory	409
	15.12.1 Heat Capacity	409
	15.12.2 Compressibility	410
	15.12.3 Spin Susceptibility	411
15.10	15.12.4 Collective Modes, Dynamical and Transport Properties	411
15.13	Instabilities of Fermi Liquids	412
	15.13.1 Ferromagnetic Instability	412
	15.13.2 Pomeranchuk Instabilities	413
	15.13.3 Pairing Instability	414
	15.13.4 Charge and Spin Density-Wave Instabilities	418
	15.13.5 One Dimension	419
	15.13.6 Two-Dimensional Electron Gas at High Magnetic Field	420



xii	CONTENTS	
15.14	Infrared Singularities in Fermi Liquids	420
	15.14.1 Perfect Screening and the Friedel Sum Rule	420
	15.14.2 Orthogonality Catastrophe	422
	15.14.3 Magnetic Impurities in Metals: The Kondo Problem	423
15.15	Summary and Outlook	429
16	Fractional Quantum Hall Effect	430
16.1	Landau Levels Revisited	431
16.2	One-Body Basis States in Symmetric Gauge	433
16.3	Two-Body Problem and Haldane Pseudopotentials	435
16.4	The $\nu = 1$ Many-Body State and Plasma Analogy	438
	16.4.1 Electron and Hole Excitations at $\nu = 1$	441
16.5	Laughlin's Wave Function	442
16.6	Quasiparticle and Quasihole Excitations of Laughlin States	446
16.7	Fractional Statistics of Laughlin Quasiparticles	452
	16.7.1 Possibility of Fractional Statistics in 2D	452
	16.7.2 Physical Model of Anyons	455
	16.7.3 Statistics Angle of Laughlin Quasiholes	457
	<b>(\$</b> )	
16.8	Collective Excitations	460
16.9	Bosonization and Fractional Quantum Hall Edge States	463
	16.9.1 Shot-Noise Measurement of Fractional Quasiparticle Charge	467
16.10	Composite Fermions and Hierarchy States	469
	16.10.1 Another Take on Laughlin's Wave Function	469
	16.10.2 Jain Sequences	470
16.11	General Formalism of Electron Dynamics Confined to a Single Landau Level	470
	16.11.1 Finite-Size Geometries	474
16.12	Relation between Fractional Statistics and Topological Degeneracy	476
16.13	Notes and Further Reading	478
17	Magnetism	480
17.1	Basics	480
17.2	Classical Theory of Magnetism	481
17.3	Quantum Theory of Magnetism of Individual Atoms	481
	17.3.1 Quantum Diamagnetism	482
	17.3.2 Quantum Paramagnetism	485
	17.3.3 Quantum Spin	486
17.4	The Hubbard Model and Mott Insulators	486
17.5	Magnetically Ordered States and Spin-Wave Excitations	491
	17.5.1 Ferromagnets	491
	17.5.2 Antiferromagnets	495
17.6	One Dimension	499
	17.6.1 Lieb–Schultz–Mattis Theorem	501
	17.6.2 Spin-1/2 Chains	502
	17.6.3 Spin-1 Chains, Haldane Gap, and String Order	506
	17.6.4 Matrix Product and Tensor Network States	510



	CONTENTS	xiii
17.7	Valence-Bond-Solid and Spin-Liquid States in 2D and Higher Dimensions	513
	17.7.1 $\mathbb{Z}_2$ Topological Order in Resonating Valence-Bond Spin Liquid	519
17.8	An Exactly Solvable Model of $\mathbb{Z}_2$ Spin Liquid: Kitaev's Toric Code	521
	17.8.1 Toric Code as Quantum Memory	525
17.9	Landau Diamagnetism	528
18	Bose–Einstein Condensation and Superfluidity	531
18.1	Non-interacting Bosons and Bose–Einstein Condensation	531
	18.1.1 Off-Diagonal Long-Range Order	534
	18.1.2 Finite Temperature and Effects of Trapping Potential	535
	18.1.3 Experimental Observation of Bose–Einstein Condensation	536
18.2	Weakly Interacting Bosons and Bogoliubov Theory	539
18.3	Stability of Condensate and Superfluidity	542
18.4	Bose–Einstein Condensation of Exciton-Polaritons: Quantum Fluids of Light	545
19	Superconductivity: Basic Phenomena and Phenomenological Theories	549
19.1	Thermodynamics	549
	19.1.1 Type-I Superconductors	550
	19.1.2 Type-II Superconductors	552
19.2	Electrodynamics	553
19.3	Meissner Kernel	556
19.4	The Free-Energy Functional	558
19.5	Ginzburg-Landau Theory	559
19.6	Type-II Superconductors	566
	19.6.1 Abrikosov Vortex Lattice	568
	19.6.2 Isolated Vortices	569
19.7	Why Do Superconductors Superconduct?	573
19.8	Comparison between Superconductivity and Superfluidity	576
19.9	Josephson Effect	579
17.7	19.9.1 Superconducting Quantum Interference Devices (SQUIDS)	585
19.10	Flux-Flow Resistance in Superconductors	587
19.11	Superconducting Quantum Bits	587
20	Microscopic Theory of Superconductivity	592
20.1	Origin of Attractive Interaction	592
20.2	BCS Reduced Hamiltonian and Mean-Field Solution	594
	20.2.1 Condensation Energy	598
	20.2.2 Elementary Excitations	599
	20.2.3 Finite-Temperature Properties	602
20.3	Microscopic Derivation of Josephson Coupling	603
20.4	Electromagnetic Response of Superconductors	606
20.5	BCS–BEC Crossover	609
20.6	Real-Space Formulation and the Bogoliubov-de Gennes Equation	611
20.7	Kitaev's p-Wave Superconducting Chain and Topological Superconductors	614



xiv	CONTENTS	
20.8	Unconventional Superconductors	61′
	20.8.1 General Solution of Cooper Problem	617
	20.8.2 General Structure of Pairing Order Parameter	619
	20.8.3 Fulde–Ferrell–Larkin–Ovchinnikov States	620
20.9	High-Temperature Cuprate Superconductors	62
	20.9.1 Antiferromagnetism in the Parent Compound	622
	20.9.2 Effects of Doping	624
	20.9.3 Nature of the Superconducting State	624
	20.9.4 Why <i>d</i> -Wave?	627
Append	ix A. Linear-Response Theory	632
A.1	Static Response	632
A.2	Dynamical Response	634
A.3	Causality, Spectral Densities, and Kramers–Kronig Relations	630
Append	ix B. The Poisson Summation Formula	640
Append	ix C. Tunneling and Scanning Tunneling Microscopy	642
C.1	A Simple Example	642
C.2	Tunnel Junction	643
C.3	Scanning Tunneling Microscopy	645
Append	ix D. Brief Primer on Topology	647
D.1	Introduction	64
D.2	Homeomorphism	648
D.3	Homotopy	648
D.4	Fundamental Group	650
D.5	Gauss-Bonnet Theorem	651
D.6	Topological Defects	654
Append	ix E. Scattering Matrices, Unitarity, and Reciprocity	657
Append	ix F. Quantum Entanglement in Condensed Matter Physics	659
F.1	Reduced Density Matrix	659
F.2	Schmidt and Singular-Value Decompositions	661
F.3	Entanglement Entropy Scaling Laws	662
F.4	Other Measures of Entanglement	663
F.5	Closing Remarks	664
Append	ix G. Linear Response and Noise in Electrical Circuits	665
G.1	Classical Thermal Noise in a Resistor	665
G.2	Linear Response of Electrical Circuits	668
G.3	Hamiltonian Description of Electrical Circuits	670
	G.3.1 Hamiltonian for Josephson Junction Circuits	672
Append	ix H. Functional Differentiation	673



	CONTENTS	xv
Appendix I. Low-Energy Effective Hamiltonians		675
I.1	Effective Tunneling Hamiltonian	675
I.2	Antiferromagnetism in the Hubbard Model	677
I.3	Summary	679
Append	ix J. Introduction to Second Quantization	680
J.1	Second Quantization	680
J.2	Majorana Representation of Fermion Operators	683
	References	685
	Index	692



#### Preface

This textbook is intended for both introductory and more advanced graduate-level courses in condensed matter physics and as a pedagogical reference for researchers in the field. This modern textbook provides graduate students with a comprehensive and accessible route from fundamental concepts to modern topics, language, and methods in the rapidly advancing field of quantum condensed matter physics.

The field has progressed and expanded dramatically since the publication four decades ago of the classic text by Ashcroft and Mermin [1], and its name has changed from Solid State Physics to Condensed Matter Physics, reflecting this expansion. The field of inquiry is vast and is typically divided into two halves. The first, often called "soft matter," covers the classical statistical physics of liquid crystals, glassy materials, polymers, and certain biological systems and materials. This area is nicely addressed in the textbook of Chaikin and Lubensky [2]. The second area, often called "hard matter" or "quantum matter," primarily covers the quantum physics of electrons in solids but these days also includes correlated quantum states of ultra-cold atomic gases and even photons. While a number of good textbooks [3–5] address various aspects of hard matter, the present text offers broader and more in-depth coverage of the field and provides physical intuition through many deep phenomenological descriptions, in addition to introducing the required mathematical background.

The present text is aimed primarily at graduate students and researchers in quantum condensed matter physics and provides encyclopedic coverage of this very dynamic field. While sharing a similar starting point with Ashcroft and Mermin, we have attempted to cover the aforementioned new developments in considerably greater depth and detail, while providing an overarching perspective on unifying concepts and methodologies. Chapters 1-9 cover traditional introductory concepts, but we have made considerable effort to provide a modern perspective on them. The later chapters introduce modern developments both in theory and in experiment. Among the new topics are coherent transport in mesoscopic systems, Anderson and many-body localization in disordered systems, the integer and fractional quantum Hall effects, Berry phases and the topology of Bloch bands, topological insulators and semimetals, instabilities of Fermi liquids, modern aspects of quantum magnetism (e.g. spinons, the Haldane gap, spin liquids, and the toric code), quantum entanglement, Bose–Einstein condensation, a pedagogical introduction to the phenomenology of superfluidity and superconductivity, superconducting quantum bits (qubits), and finally a modern review of BCS theory that includes unconventional pairing, high-temperature superconductivity, topological superconductors, and majorana fermions. We have also attempted to make contact with other fields, in particular ultra-cold atomic gases, photonic crystals, and quantum information science, emphasizing the unifying principles



#### xviii PREFACE

among different branches of physics. For this reason the text should also be of interest to students and practitioners outside condensed matter physics.

The text is intended to be accessible and useful to experimentalists and theorists alike, providing an introduction both to the phenomenology and to the underlying theoretical description. In particular, we provide the mathematical background needed to understand the topological aspects of condensed matter systems. We also provide a gentle and accessible introduction to scaling and renormalization group methods with applications to Anderson localization, the Kondo problem, and the modern approach to the BCS problem. The text assumes prior knowledge of quantum mechanics and statistical physics at the level of typical first-year graduate courses. Undergraduate preparation in condensed matter physics at the level of Kittel [6] would be useful but is not essential. We make extensive use of harmonic oscillator ladder operators but almost completely avoid second quantization for fermions until Chapter 17. In addition, we provide a pedagogical appendix for the reader to review second quantization.

Recent decades have seen the application of advanced methods from quantum field theory which provide effective descriptions of "universal" features of strongly correlated many-body quantum systems, usually in the long-wavelength and low-energy limit [7–15]. The present text provides a pedagogical gateway to courses on these advanced methods by introducing and using the language of many-body theory and quantum field theory where appropriate.

This book has evolved over several decades from course notes for graduate condensed matter physics taught by the authors at Indiana University and Florida State University, respectively. The content exceeds the amount of material which can be covered in a one-year course but naturally divides into an introductory portion (Chapters 1–10) which can be covered in the first semester. For the second semester, the instructor can cover Chapters 11–15 and then select from the remaining chapters which cover the fractional quantum Hall effect, magnetism, superfluidity, and superconductivity.



### Acknowledgments

We are grateful to many people for kindly taking the time to provide feedback on the manuscript as it was developing. We would particularly like to acknowledge Jason Alicea, Collin Broholm, Jack Harris, Alexander Seidel, A. Douglas Stone, and Peng Xiong. SMG thanks KY for carrying the bulk of the writing load during the decade that SMG was serving as deputy provost for research at Yale. SMG also thanks Diane Girvin for extensive assistance with proofreading. KY would like to thank the students who took his course PHZ5491-5492 at Florida State University for their comments, in particular Shiuan-Fan Liou, Mohammad Pouranvari and Yuhui Zhang who have also helped draw many figures. He is also grateful to Li Chen for help proofreading several chapters. Over the years our research in condensed matter and quantum information theory has been supported by the NSF, DOE, ARO, the Keck Foundation, Yale University, Florida State University and the National High Magnetic Field Laboratory. This work was begun at Indiana University.

We are grateful to the staff of Cambridge University Press and especially to our editor Simon Capelin whose patient encouragement over the span of two decades helped us reach the finish line.

Finally, we are most grateful for the infinite patience of our families over the many years that this project was underway.