

## PASSIVE IMAGING WITH AMBIENT NOISE

Waves generated by opportunistic or ambient noise sources and recorded by passive sensor arrays can be used to image the medium through which they travel. Spectacular results have been obtained in seismic interferometry, which open up new perspectives in acoustics, electromagnetics, and optics. The authors present, for the first time in book form, a self-contained and unified account of correlation-based and ambient noise imaging. In order to facilitate understanding of the core material, they also address a number of related topics in conventional sensor array imaging, wave propagation in random media, and high-frequency asymptotics for wave propagation. Taking a multidisciplinary approach, the book uses mathematical tools from probability, partial differential equations, and asymptotic analysis, combined with the physics of wave propagation and modeling of imaging modalities. Suitable for applied mathematicians and geophysicists, it is also accessible to graduate students in applied mathematics, physics, and engineering.

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## Preface

In sensor array imaging the objective is to probe an unknown medium with waves, and this can be done in two steps. In the data acquisition step, waves are emitted by a source array, they propagate through the medium being probed, and they are recorded by a receiver array. In the data processing step, this data set is used to extract information about the medium, such as the location of reflectors embedded in it. Recent developments in imaging in complex media and on passive, ambient noise imaging have had a profound impact in many different applied fields. It is these developments that have motivated us to write this book.

Research in wave propagation in complex, scattering media has been active for a long time. It is relevant in many imaging applications in which one wants to probe a medium such as the Earth's lithosphere in seismic imaging, concrete structures in non-destructive testing, the human body in medical imaging, the turbulent atmosphere in optical imaging, or shallow water environments in acoustic imaging. These media are quite complex, while we often want to image only some particular features in them. It turns out that the established imaging methods, which we also describe in this book, may fail when the ambient medium is scattering. It is only recently that wave propagation in complex media, modeled by random media, has been formulated and analyzed in a way that makes it possible to develop new imaging techniques that can mitigate the effects of random scattering.

Array imaging has also been analyzed and used in many applications for a long time. The recent trend to deploy large sensor arrays is due to improved sensor technology, reduced cost in data storage, and increased computational capabilities. In particular, passive sensor array imaging has recently become an area of intense research activity because of the potential impact of its applications. Passive means here that only receiver arrays are used, instead of active source/receiver arrays, and the illumination is provided by unknown, uncontrolled, asynchronous, or opportunistic sources. Imaging with ambient noise sources is one of the main topics of this book. Of course, the structure of the array data is quite different from that of active sensor arrays, and this requires the development of new imaging techniques.

Both in the field of imaging in complex media and in passive, ambient noise imaging, the theoretical analysis shows that the cross correlations between the recorded signals play

an important role. This is because they carry information about the medium through which the waves propagate, and they also reduce spurious noise effects. Interferometric imaging is another commonly used term for correlation-based imaging. The study of the wave field correlations is central to this book. In seismology the emergence of correlation-based imaging using ambient seismic noise has had a profound impact. The use of seismograms generated by earthquakes was previously the only way to image the Earth. With correlation-based imaging, the apparent seismic noise recorded by a distributed network of sensors on the Earth's surface can provide a lot of information about its structure. Beyond seismology, there are many new, emerging areas for correlation-based imaging methods, in passive synthetic aperture radar or in optical speckle intensity correlations for communications and imaging, as discussed in the last chapters of the book.

The subject of this book is multidisciplinary. It uses mathematical tools from probability and stochastic processes, partial differential equations and asymptotic analysis, combined with the physics of wave propagation and modeling of imaging modalities in complex environments. However, the essential results can be obtained at an elementary level by using the multidimensional stationary phase method. This book is aimed at readers with interdisciplinary interests, and in particular students and researchers engaged with imaging methodologies related to wave propagation and sensor arrays.

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