Cambridge University Press
978-1-107-13563-5 - Passive Imaging with Ambient Noise
Josselin Garnier and George Papanicolaou
Excerpt
More information

1

Introduction and overview of the book

This book addresses wave-based imaging: that is, imaging of unknown media from recorded wave signals that have propagated through them. The typical problems that we consider are velocity estimation and reflector imaging. In the first case, we recover approximately, that is, we image the background propagation velocity of the medium from estimated travel times between sensors. In the second case, we detect and localize, that is, we image anomalies present in the medium from recorded sensor data. Although established methods to address these imaging problems are presented, the focus is on correlation-based or interferometric imaging techniques using illumination signals generated by uncontrolled ambient noise sources. These techniques have attracted a lot of attention recently because they open new possibilities for imaging, in seismology, in synthetic aperture radar and elsewhere, where illuminating sources are rare and often uncontrolled and the recording sensors are passive. Their analysis involves mathematical methods and results that we introduce here in a systematic way. In the first part of the book (Chapters 2–6) we address correlation-based imaging for homogeneous and smoothly varying media. In the second part of the book (Chapters 7–8) we consider scattering media. In the last chapters (Chapters 9–11) we use the mathematical tools presented and developed in this book to revisit and analyze recent imaging modalities that use correlation-based techniques.

1.1 Why passive, correlation-based imaging?

Let us explain our motivation for studying correlation-based or interferometric imaging and, in particular, passive sensor imaging. We first need to introduce a few basic facts about wave propagation. Throughout the book we only address scalar waves, although some of the applications we have in mind involve vector waves (for instance, elastic or electromagnetic waves), because the main ideas, questions, and techniques that we want to introduce can be described and analyzed quite effectively using a scalar wave model. When a point source at $\boldsymbol{y}$ emits a short pulse $f(t)$, receivers located at the set of points

$(\boldsymbol{x}_j)_{j=1,\dots,N}$ record the signals $(u(t,\boldsymbol{x}_j))_{j=1,\dots,N}$ where u is the solution of the wave equation:

$$\frac{1}{c^2(\boldsymbol{x})}\frac{\partial^2 u}{\partial t^2} - \Delta_{\boldsymbol{x}} u = f(t)\delta(\boldsymbol{x}-\boldsymbol{y}), \quad (t,\boldsymbol{x}) \in \mathbb{R}\times\mathbb{R}^3, \tag{1.1}$$

with zero initial conditions and where $c(\boldsymbol{x})$ is the speed or velocity of propagation of the medium. The signal $u(t,\boldsymbol{x})$ is the convolution (in time) of the source pulse $f(t)$ with the Green's function $G(t,\boldsymbol{x},\boldsymbol{y})$ – that is, the fundamental solution of the wave equation (1.1) – with a source of the form $\delta(t)\delta(\boldsymbol{x}-\boldsymbol{y})$. The estimation of the Green's function from recorded signals is a basic problem in imaging because the Green's function contains information about the properties of the medium; for example, the propagation velocity. Depending on the configuration of the sensors and the overall imaging setup, we may wish to estimate only some specific features. We may, for example, estimate the travel times between an array or a network of sensors, or the travel times between the sensors and the locations of reflectors, which are localized discontinuities of the propagation velocity.

1.1.1 Travel time estimation

We assume that the medium is homogeneous or smoothly varying. Then the signal recorded by a receiver at $\boldsymbol{x}_j$ has a peak centered at the time equal to the travel time $\mathcal{T}(\boldsymbol{x}_j,\boldsymbol{y})$ from $\boldsymbol{y}$ to $\boldsymbol{x}_j$, assuming that the source emission is centered at time zero. When the medium is homogeneous, the travel time is simply $\mathcal{T}(\boldsymbol{x}_j,\boldsymbol{y}) = |\boldsymbol{x}_j-\boldsymbol{y}|/c_0$, the distance between the source and the receiver divided by the homogeneous propagation velocity c_0. The source at $\boldsymbol{y}$ generates a spherical wave, and the signal recorded by the receiver is the source pulse with a time shift equal to the travel time, along with a multiplicative factor that results from the geometric spreading of the spherical wave. When the medium is smoothly varying, the travel time $\mathcal{T}(\boldsymbol{x}_j,\boldsymbol{y})$ can be computed from the propagation velocity $c(\boldsymbol{x})$ by the eikonal equation or by Fermat's principle. It is the minimal time necessary for a particle moving in the velocity field $c(\boldsymbol{x})$ to go from the source point $\boldsymbol{y}$ to the receiver point $\boldsymbol{x}_j$. When the source pulse width is small compared to the travel time, then the source at $\boldsymbol{y}$ generates a wave, a disturbance, that is concentrated on a front that arrives at the receiver at time $\mathcal{T}(\boldsymbol{x}_j,\boldsymbol{y})$.

The usual method to estimate travel times between sensors requires the use of impulsive sources. However, it may happen that such sources are not available or are rare and uncontrolled. In seismology, earthquakes are the main sources of seismic waves, and are recorded by networks of seismometers. Earthquakes are uncontrolled events. Their frequency of occurrence is low and their spatial distribution is rather limited and concentrated along faults. However, the travel time between two sensors in an inhomogeneous medium can also be estimated by using the signals $(u(t,\boldsymbol{x}_j))_{j=1,\dots,N}$ generated by ambient noise sources and recorded by the sensors at

$(\boldsymbol{x}_j)_{j=1,\dots,N}$, more exactly, by computing the cross correlation of the signals recorded by the sensors:

$$C_T(\tau, \boldsymbol{x}_j, \boldsymbol{x}_l) = \frac{1}{T}\int_0^T u(t, \boldsymbol{x}_j)u(t+\tau, \boldsymbol{x}_l)\mathrm{d}t, \qquad j,l = 1\dots,N, \quad \tau \in \mathbb{R}.$$

Here the signals $(u(t, \boldsymbol{x}_j))_{j=1}^N$ are given in terms of the solution of the wave equation:

$$\frac{1}{c^2(\boldsymbol{x})}\frac{\partial^2 u}{\partial t^2} - \Delta_{\boldsymbol{x}} u = n(t, \boldsymbol{x}), \quad (t, \boldsymbol{x}) \in \mathbb{R} \times \mathbb{R}^3,$$

where the source term $n(t, \boldsymbol{x})$ is a stationary random process that models the ambient noise sources. As we will see in this book, the cross correlation of signal amplitudes contains information about the Green's function of the wave equation. In particular, travel times can be estimated from cross correlations or interferometrically. We can then estimate the background propagation velocity from the travel times between sensors in a network covering the region of interest. We describe in Subsection 1.1.3 how we can also use estimated travel times to image reflectors in the medium.

1.1.2 Applications of travel time estimation

Correlation techniques can be applied in seismology, where the sensors are seismic stations recording vertical ground motion. The noise sources come from the nonlinear interaction of ocean waves with the ocean bottom, which generates seismic surface waves (Longuet-Higgins, 1950; Stehly et al., 2006), and the goal is to obtain a background surface wave velocity map for a large part of the Earth. Earlier, the goal was to localize intense weather patterns over oceans (Walker, 1913; Bernard, 1941). In seismology, correlation-based imaging is often referred to as seismic interferometry. The idea of exploiting the ambient noise and using the cross correlation of noise signals to retrieve information about travel times was considered also in helioseismology and exploration seismology (Claerbout, 1968; Duvall et al., 1993; Rickett and Claerbout, 1999; Schuster et al., 2004; Draganov et al., 2006; Curtis et al., 2009). It has been applied to background velocity estimation from regional to local scales (Yao et al., 2006; Larose et al., 2006; Sabra et al., 2005; Shapiro et al., 2005; Gouédard et al., 2008), volcano monitoring (Sabra et al., 2006; Brenguier et al., 2007, 2008b, 2014), carbon dioxide storage monitoring (Draganov et al., 2012), oil reservoir and petroleum field monitoring (Curtis et al., 2006; Draganov et al., 2013), in ocean acoustics (Jensen et al., 2011), and in indoor radio localization (Callaghan et al., 2011). When the support of the noise sources extends over all space and they are uncorrelated – that is, their spatial correlation function is a delta function – the derivative of the cross correlation of the recorded signals is the symmetrized Green's function between the sensors (Roux et al., 2005). This is also true with spatially localized noise source distributions provided the waves propagate within an ergodic cavity (Colin de Verdière, 2009;

Bardos et al., 2008). At a physical level this result is expected in both open and closed environments provided that the recorded signals result from equipartitioned wave energy (Lobkis and Weaver, 2001; Weaver and Lobkis, 2001; Roux and Fink, 2013; Snieder, 2004; Malcolm et al., 2004). In an open environment this means that the recorded signals are an effectively uncorrelated and isotropic superposition of plane waves coming from all directions. In a closed environment it means that the recorded signals are superpositions of normal modes with random amplitudes that are statistically uncorrelated and identically distributed. All these issues are addressed in this book.

In many applications, however, the distribution of noise sources is spatially limited and the recorded signals do not come from equipartitioned wave energy. As a result, the waves recorded by the sensors are dominated by energy flux coming from the direction of the noise sources. The cross correlations of the recorded signals depend on the orientation of the sensor pair relative to the direction of the energy flux. This affects significantly the quality of the estimate for the Green's function. As we will see, it is good when the ray between the sensors is along the direction of the energy flux and bad when it is perpendicular to it (Stehly et al., 2006; Garnier and Papanicolaou, 2009; Godin, 2009).

These results hold for homogeneous or smoothly varying media. It is expected that the situation will become more complex and interesting in scattering media, where scatterers can play the role of secondary sources and can therefore help in promoting isotropic illumination of the objects to be imaged (de Hoop et al., 2011, 2013; Derode et al., 2003; Garnier and Sølna, 2009b, 2011a). However, there is a trade-off between an angular diversity enhancement of the illumination and an increase of the level of fluctuations in the cross correlations due to scattering. It turns out that the use of high-order correlation-based techniques can allow for more efficient travel time estimation for background velocity tomography when the noise source distribution is spatially limited and the medium is scattering (Campillo and Stehly, 2007; Stehly et al., 2008; Garnier and Papanicolaou, 2009).

1.1.3 Reflector imaging

Assume now that the medium is homogeneous, or smoothly and slowly varying, and that there is a point-like reflector at $z_{\rm r}$; that is, a strong and localized anomaly in the propagation velocity. When a point source at $\boldsymbol{y}$ emits an impulse, a receiver at $\boldsymbol{x}_j$ records a first peak at a time equal to the travel time $\mathcal{T}(\boldsymbol{y}, \boldsymbol{x}_j)$ from $\boldsymbol{y}$ to $\boldsymbol{x}_j$, corresponding to the direct spherical wave arrival that has not interacted with the reflector, and a second peak at a time equal to the sum of the travel times $\mathcal{T}(\boldsymbol{y}, z_{\rm r}) + \mathcal{T}(z_{\rm r}, \boldsymbol{x}_j)$ between the source $\boldsymbol{y}$ and the reflector at $z_{\rm r}$, and between the reflector and the receiver $\boldsymbol{x}_j$. This second peak comes from the scattered wave that has interacted with the reflector. The scattered wave will be essentially a spherical wave centered at the reflector if the reflector is small compared to the characteristic wavelength of the pulse.

The detection and location of wave reflectors from recorded wave signals is a central issue in imaging. The established method requires the use of an active array: that is, an array whose sensors can be used as sources as well as receivers. To image reflectors with

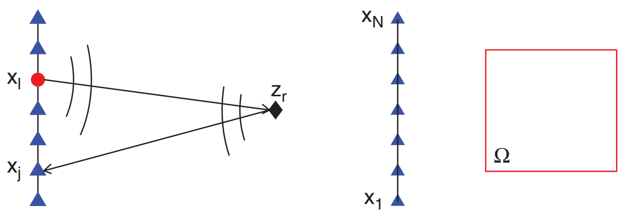

Figure 1.1 Sensor array imaging of a reflector (diamond) located at z_r. Left figure: data acquisition, in which the lth sensor at $\boldsymbol{x}_l$ emits a short pulse and the jth receiver at $\boldsymbol{x}_j$ records the reflected wave. Right figure: search region Ω for the imaging function.

an active sensor array located at $(\boldsymbol{x}_j)_{j=1,\dots,N}$ one first records the impulse response matrix $(u(t,\boldsymbol{x}_j;\boldsymbol{x}_l))_{j,l=1,\dots,N,t\in\mathbb{R}}$ of the array and then one gets an image using travel time or Kirchhoff migration (see Figure 1.1). The (j,l)th element $(u(t,\boldsymbol{x}_j;\boldsymbol{x}_l))_{t\in\mathbb{R}}$ of the impulse response matrix is the signal recorded by the sensor at $\boldsymbol{x}_j$ when the sensor at $\boldsymbol{x}_l$ emits an impulse. To form an image, each element of the impulse response matrix is evaluated at the sum of the travel times $\mathcal{T}(\boldsymbol{x}_l,\boldsymbol{z}^S)+\mathcal{T}(\boldsymbol{z}^S,\boldsymbol{x}_j)$ between the emitting sensor $\boldsymbol{x}_l$ and a search point $\boldsymbol{z}^S$ in the search domain Ω, and between the search point $\boldsymbol{z}^S$ and the receiving sensor $\boldsymbol{x}_j$. The Kirchhoff migration imaging function is then the sum of the migrated matrix elements over all emitters and receivers, which is given by

$$\mathcal{I}_{\rm KM}(\boldsymbol{z}^S)=\sum_{j,l=1}^{N}u\big(\mathcal{T}(\boldsymbol{x}_l,\boldsymbol{z}^S)+\mathcal{T}(\boldsymbol{z}^S,\boldsymbol{x}_j),\boldsymbol{x}_j;\boldsymbol{x}_l\big).$$

This produces an image because the (j,l)th element of the impulse response matrix has a peak at the sum of the travel times $\mathcal{T}(\boldsymbol{x}_l,z_r)+\mathcal{T}(z_r,\boldsymbol{x}_j)$ between the emitting sensor $\boldsymbol{x}_l$ and a localized reflector at z_r, and between the reflector and the receiving sensor $\boldsymbol{x}_j$. Therefore, if the search point $\boldsymbol{z}^S$ coincides with, or is very close to, the reflector location z_r, all these peaks add up constructively and the Kirchhoff migration imaging function has a strong peak at this point. Migration in seismic imaging is presented in Biondi, 2006; Claerbout, 1985.

As we will see, information about the reflector is also contained in the cross correlations of signals generated by ambient noise sources and recorded by a passive sensor array: that is, an array of receivers only (see Figure 1.2). These cross correlations can therefore be used for imaging of reflectors imbedded in the medium (Garnier and Papanicolaou, 2009, 2010; Garnier and Sølna, 2011b). The data that is used for imaging is now the cross correlation matrix $(C_T(\tau,\boldsymbol{x}_j,\boldsymbol{x}_l))_{j,l=1,\dots,N,\tau\in\mathbb{R}}$ between pairs of sensors of a passive array located at $(\boldsymbol{x}_j)_{j=1,\dots,N}$. The feasibility of this imaging method was first demonstrated in laboratory experiments with ultrasound in Gouédard et al. (2008). Applications to non-destructive testing, imaging of waste disposal areas in landfills, and in seismology, have been proposed and implemented in Harmankaya et al. (2013), Kaslilar et al. (2013, 2014), and Konstantaki et al. (2013).

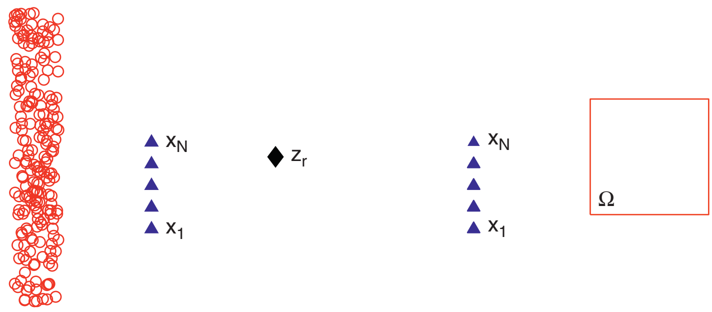

Figure 1.2 Correlation-based imaging with an array of passive sensors (triangles). Left figure: data acquisition, in which the reflector to be imaged (diamond) is located at z_r and is illuminated by noise sources (circles). Right figure: search region Ω for the imaging function.

1.1.4 Auxiliary array or virtual source imaging

Another application of correlation-based imaging is to use data recorded by auxiliary, passive arrays, usually placed near the reflectors to be imaged, while the main active array provides illumination relatively far from the reflectors. The ambient medium between the illuminating array and the reflectors may be homogeneous or scattering. The illumination provided by the main array is with short, asynchronous pulses, which means that the emission times of the pulses are not known, and neither is the emitted pulse form. The illumination from the main array must, in addition, be staggered; that is, the emission times for the pulses issuing from the array elements must be all different. The reason why this type of imaging configuration is interesting and useful is because it can provide images that are essentially unaffected by the scattering inhomogeneities, and may even benefit from them by diversifying the illumination of the reflectors to be imaged. Moreover, this will occur even with strong scattering that would make imaging with data from the main, active array impossible.

This kind of imaging configuration was proposed in exploration seismology by Bakulin and Calvert (2006). The idea is to put passive sensors deep inside boreholes, which form the auxiliary array, and then provide asynchronous illumination from the surface. This form of seismic imaging is also discussed in Schuster (2009) and Wapenaar et al. (2010b).

A more detailed description of the imaging configuration with auxiliary arrays is as follows. We consider a main array of sources located at $(\boldsymbol{x}_s)_{s=1}^{N_s}$. When the array of receivers is coincident with the array of sources, as in Figure 1.3, then the data set is the array response matrix $(u(t,\boldsymbol{x}_r;\boldsymbol{x}_s))_{t\in\mathbb{R},r,s=1,\dots,N_s}$, whose (r,s)th element is the signal recorded by the rth receiver when the sth source emits a short pulse. We obtain an image by migrating the array response matrix to estimate the location of the reflector in the medium (Biondi, 2006). The Kirchhoff migration function at a search point $\boldsymbol{z}^S$ is

$$\mathcal{I}(\boldsymbol{z}^S) = \sum_{r,s=1}^{N_s} u\big(\mathcal{T}(\boldsymbol{x}_s,\boldsymbol{z}^S) + \mathcal{T}(\boldsymbol{z}^S,\boldsymbol{x}_r),\boldsymbol{x}_r;\boldsymbol{x}_s\big),$$

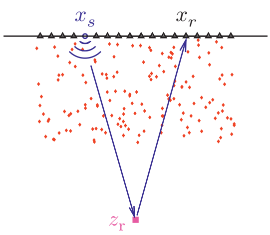

Figure 1.3 Sensor array imaging of a reflector. $\boldsymbol{x}_s$ is a source, $\boldsymbol{x}_r$ is a receiver, and $\boldsymbol{z}_{\rm r}$ is a reflector.

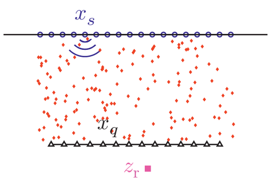

Figure 1.4 Use of an auxiliary passive array for imaging through a scattering medium. $\boldsymbol{x}_s$ is a source, $\boldsymbol{x}_q$ is a receiver located below the scattering medium, and $\boldsymbol{z}_{\rm r}$ is a reflector.

where $\mathcal{T}(\boldsymbol{x}, \boldsymbol{y})$ is a travel time between the points $\boldsymbol{x}$ and $\boldsymbol{y}$ based on a prior model for the propagation velocity of the ambient medium. When the model is a homogeneous medium then $\mathcal{T}(\boldsymbol{x}, \boldsymbol{y}) = |\boldsymbol{x} - \boldsymbol{y}|/c_0$, where c_0 is the constant propagation speed. When, however, the true medium is inhomogeneous then migration based on a homogeneous model may not work well. In weakly scattering media the images can be stabilized statistically by using coherent interferometry (Borcea et al., 2005, 2006a,b, 2007), which is a special correlation-based imaging method that is, however, somewhat different from the one considered in this book. Statistical stability means high signal-to-noise ratio of the image, relative to noise or uncertainty from medium inhomogeneities. In strongly scattering media we may be able to obtain an image by using special signal processing methods (Borcea et al., 2009), but often we cannot get any image at all because the coherent signal from the reflector received at the array is very weak compared to the backscatter from the medium.

Consider now an imaging setup in which there is an auxiliary passive array, located at $(\boldsymbol{x}_q)_{q=1}^{N_{\rm q}}$, and the strongly scattering medium is between it and the surface source-receiver array, as in Figure 1.4. The data set is then the matrix $(u(t, \boldsymbol{x}_q; \boldsymbol{x}_s))_{t \in \mathbb{R},\, s=1,\ldots,N_{\rm s},\, q=1,\ldots,N_{\rm q}}$, where $u(t, \boldsymbol{x}_q; \boldsymbol{x}_s)$ is the signal recorded by the qth receiver when the sth source emits a short pulse. Such a configuration can be realized in seismic exploration, in which sources can be put at the Earth's surface, the lithosphere near the surface is strongly scattering, and auxiliary receivers may be placed in vertical or horizontal boreholes. It is clearly not possible or desirable to place seismic sources in the boreholes (Bakulin and Calvert, 2006; Schuster, 2009; Wapenaar et al., 2010b).

The main issue now is how the auxiliary passive array data can be used to get an image that is relatively insensitive to the effects of the strong scattering in the ambient medium. By analogy with the situation in which there are N_s uncorrelated point sources at $(\boldsymbol{x}_s)_{s=1,\dots,N_\mathrm{s}}$, we expect that, even in the case of active impulsive sources, the matrix of cross correlations at the auxiliary array

$$C_T(\tau, \boldsymbol{x}_q, \boldsymbol{x}_{q'}) = \int_0^T \sum_{s=1}^{N_\mathrm{s}} u(t, \boldsymbol{x}_q; \boldsymbol{x}_s) u(t+\tau, \boldsymbol{x}_{q'}; \boldsymbol{x}_s) \mathrm{d}t \,, \qquad q, q' = 1, \dots, N_\mathrm{q} \quad (1.2)$$

behaves roughly as if it is the impulse response matrix of the auxiliary array. This means that it can be used for imaging with Kirchhoff migration:

$$\mathcal{I}(\boldsymbol{z}^S) = \sum_{q,q'=1}^{N_\mathrm{q}} C_T\big(\mathcal{T}(\boldsymbol{x}_q, \boldsymbol{z}^S) + \mathcal{T}(\boldsymbol{z}^S, \boldsymbol{x}_{q'}), \boldsymbol{x}_q, \boldsymbol{x}_{q'}\big), \quad (1.3)$$

with the travel time given by $\mathcal{T}(\boldsymbol{x}, \boldsymbol{y}) = |\boldsymbol{x} - \boldsymbol{y}|/c_0$, which corresponds to a homogeneous medium model with propagation speed c_0. A mathematical theory, along with numerical simulations, that clarifies the conditions under which strong random scattering effects are minimized or even eliminated with the imaging function (1.3), is given in Garnier and Papanicolaou (2012, 2014a), and Garnier et al. (2015).

1.1.5 Passive synthetic aperture imaging

Instead of having a passive auxiliary array near the reflectors to be imaged, we can use a single moving receiver that records signals generated by a distant array of sources (see Figure 1.5). With a rich enough illumination, the reflectors can be imaged by migrating the autocorrelation functions of the received signals. How do these images compare with the usual, active synthetic aperture ones? In the usual synthetic aperture imaging (Cheney, 2001; Borcea et al., 2012) the moving receiver is also a transmitter (see Figure 1.6) and imaging is done with the matched filter of the recorded signals along the path of the moving antenna system. Passive synthetic aperture imaging with radar is discussed in Farina and Kuschel (2012).

The question regarding the comparison of the images in passive and active – that is, the usual, synthetic aperture imaging – respectively, in a homogeneous medium is addressed in

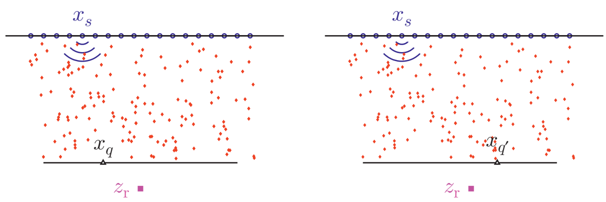

Figure 1.5 Passive synthetic aperture imaging configuration. Two positions $\boldsymbol{x}_q$ and $\boldsymbol{x}_{q'}$ of the moving receiver below the random medium are plotted. $\boldsymbol{x}_s$ is a source and $\boldsymbol{z}_\mathrm{r}$ is a reflector.

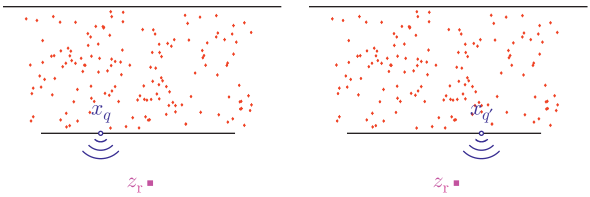

Figure 1.6 Active synthetic aperture imaging configuration. Two positions $\boldsymbol{x}_q$ and $\boldsymbol{x}_{q'}$ of the moving source-receiver are plotted. z_r is a reflector.

Chapter 9. Using the methods developed in this book it is shown that when the illumination in the passive case is rich enough, then there is no loss of resolution.

When the ambient medium between the sources and the moving recording antenna is randomly inhomogeneous then, as with virtual source imaging, correlation-based synthetic aperture imaging mitigates almost entirely the effects of the inhomogeneities. In fact, random inhomogeneities can even have a beneficial effect because they tend to diversify the illumination from the sources. This is also a result presented in this book for the first time, in Chapter 10.

1.1.6 Imaging with intensity cross correlations

We have so far exploited cross correlations of signals that are recorded fully resolved as functions of time, even if they are rapidly oscillating. Therefore, cross correlations provide interferometric information, which can be used to form images. Fully resolved recording of signals in seismic imaging is easily accessible with standard electronic equipment. This is also the case in acoustics, even up to ultrasonic regimes. However, as the central frequency of the signals increases, for example in radar above 10 GHz or in optics, we reach the limit of the ability of electronic instruments to sample signals fast enough so as to resolve them. The technology of recording instruments is constantly improving, of course, but there seems to be a hardware limit to the applicability of correlation-based imaging methods.

Sampling issues for the recorded signals become a limiting factor at high frequencies, but there are other sampling issues in imaging that we do not address in this book. One is the inter-sensor distance of the receiver array, which we assume throughout the book is small enough (about half a central wavelength) that the array can be replaced by a continuum in the analysis. We also assume that the sources of illumination are spaced close enough that they can be described with a continuous density. The only place in the book where we consider the consequences of a sampling issue is in Chapter 11 where we assume that only signal intensities can be recorded. By signal intensities we mean here local time averages of the square of the real-valued wave field.

The question arises, therefore, whether it is possible to image with intensity-only measurements. It turns out that this can be done with correlation-based imaging provided that, as is often the case in this book, the sources are space–time incoherent. This means that the illumination comes, in effect, from noise sources. This is perhaps surprising since correlation-based imaging uses interferometric information which may be lost when only intensity measurements are available. In the ghost imaging experiments (Cheng, 2009; Li et al., 2010; Shapiro and Boyd, 2012), analyzed here in Chapter 11, this is not the case.

1.2 Chapter-by-chapter description of the book

In this book we give a systematic exposition of travel time estimation and imaging of reflectors by cross correlation of signals generated by ambient noise sources and recorded by a passive sensor array. We introduce a self-contained theoretical framework for the analysis of correlation-based or interferometric imaging techniques with ambient noise illumination. We will next give a brief description of the contents of each chapter.

Chapter 2: Green's function estimation from noise cross correlations

In Chapter 2 we present different approaches that establish the relation between the Green's function for the wave equation and the cross correlations of ambient noise signals. The first approach is based on an explicit and simple calculation, but it is only valid when the medium is homogeneous and open, and when the sources are uniformly distributed throughout the medium. The second approach is based on the Helmholtz–Kirchhoff identity that results from the divergence theorem and the Sommerfeld radiation condition, presented in Subsection 2.1.3. This approach is simple and elegant, but it requires particular configurations for the spatial distribution of the noise sources: the sources must surround completely the region of interest that contains the receivers. One of the objectives of this book is to show that although these special conditions are necessary for this approach to be applicable, they are not necessary in order to have a relation between the Green's function and the cross correlation of ambient noise signals. The third approach is proposed in the physics literature (Lobkis and Weaver, 2001; Weaver and Lobkis, 2001) and it is based on an equipartition of energy principle. We present it in the case of a bounded cavity. Finally, a fourth approach follows from the concept of daylight imaging proposed by Claerbout (1968, 1985, 1999) and we analyze it in the case of a one-dimensional medium. At the end of this chapter it becomes clear that there is a need to have an approach that is based on weaker hypotheses so that it can be applied to more realistic imaging situations.

Chapter 3: Travel time estimation from noise cross correlations using stationary phase

In Chapter 3 we develop a general approach, based on high-frequency asymptotic analysis, for establishing a relation between the Green's function and the cross correlation of ambient noise signals (Garnier and Papanicolaou, 2009). We use the multi-dimensional stationary phase method to analyze the estimation of the travel time between two sensors