THE COSMIC MICROWAVE BACKGROUND

The cosmic microwave background (CMB), the radiation left over from the Big Bang, is arguably the most important topic in modern cosmology. Its theory and observation have revolutionized cosmology from an order-of-magnitude science to a precision science. This graduate textbook describes CMB physics from first principles in a detailed yet pedagogical way, assuming only that the reader has a working knowledge of general relativity. Among the changes in this second edition are new chapters on non-Gaussianities in the CMB and on large-scale structure, and extended discussions on lensing and baryon acoustic oscillations, topics that have developed significantly in the past decade. Discussions of CMB experiments have been updated from Wilkinson Microwave Anisotropy Probe (WMAP) data to the new Planck data. The CMB success story in estimating cosmological parameters is then treated in detail, conveying the beauty of the interplay of theoretical understanding and precise experimental measurements.

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To Martin, Florian, Melchior, Anna, and Sacha

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Preface

Cosmology, the quest concerning the Universe as a whole, has been a primary interest of study since the beginnings of mankind. For a long time our ideas about the Universe were dominated by religious beliefs – tales of creation. Only since the advent of general relativity in 1915 have we had a scientific theory at hand that might be capable of describing the Universe. Soon after Einstein's first attempt of assuming a static universe, Edwin Hubble and collaborators discovered that the observable Universe is expanding [Hubble, 1929; see, however, Nussbaumer & Bieri (2009) recounting the discovery of the expansion of the Universe]. This, together with the discovery of the cosmic microwave background (CMB) by Penzias and Wilson (Nobel Prize 1978), has established the theory of an expanding and cooling universe that started in a "big bang."

For a long time observations that have led to the determination of cosmological parameters, such as the rate of expansion, the so-called Hubble parameter, and the mean matter density of the Universe or its curvature, have been very sparse and we could only determine the order of magnitude of these parameters.

Roughly since the beginning of this century, this situation has changed drastically and cosmology has entered an era of precision measurements. This major breakthrough is to a large extent due to precise measurement and analysis of the CMB. In this book I develop the theory that is used to analyze and understand measurements of the CMB, especially of its anisotropies and polarization, but also its frequency spectrum. The 2006 Nobel Prize in Physics was awarded to George Smoot and John Mather, for the discovery of these anisotropies and for precise measurements of the CMB spectrum.

The book is directed mainly toward graduate students and researchers who want to obtain an overview of the main developments in CMB physics, and who want to understand the state-of-the-art techniques that are used to analyze CMB data. I believe that the theory of CMB physics is sufficiently mature for a book on this topic to be useful. I shall not enter into any details concerning CMB experiments.

This is by no means because I consider them less interesting, but rather that they are still in development and will hopefully make significant progress, especially in polarization measurements, in the near future. Of course, my background is also that of a theoretical physicist and my main interest lies in the theoretical aspects of CMB physics. I hope, however, that this book will also be useful to CMB experimentalists, or more precisely analysts, who want to know what happens inside their cosmic parameter estimation routines.

It is assumed that the reader is familiar with undergraduate physics including the basics of general relativity, and has an elementary knowledge of quantum field theory and particle physics. The beauty of cosmology lies in the fact that it employs more or less all fields of physics starting with general relativity over thermodynamics and statistical physics to electrodynamics, quantum mechanics, and particle physics. In this book I do not want to present an introduction to these topics as well, since, first of all, there exist wonderful textbooks on all of them, and second, you have learned them in your undergraduate physics courses.

Before we start, let me sketch the content of the different chapters and provide a guide on how to read this book.

The first chapter is an overview of the homogeneous and isotropic universe. We present and discuss the Friedmann equations, recombination, nucleosynthesis, and inflation. Readers familiar with cosmology may skip this chapter or just skim it to familiarize themselves with the notation used.

In Chapter 2 we develop cosmological perturbation theory. This is the basics of CMB physics. The main reason why the CMB allows such an accurate determination of cosmological parameters lies in the fact that its anisotropies are small and can be determined mainly within first-order perturbation theory. In Fourier space the linear perturbation equations become a series of ordinary linear differential equations, which can be solved numerically to high precision without any difficulty. We derive the perturbations of Einstein's equations and the energy–momentum conservation equations and solve them for some simple but relevant cases. We also discuss the perturbation equation for light-like geodesics. This is sufficient to calculate the CMB anisotropies in the so-called instant recombination approximation. The main physical effects that are missed in such a treatment are Silk damping on small scales and polarization. We then introduce the matter and CMB power spectrum and draw our first conclusions for its dependence on cosmological and primordial parameters. For example, we derive an approximate formula for the position of the acoustic peaks. Section 2.7 discusses fluctuations not laid down at some initial time but continuously sourced by some inhomogeneous component, a so-called source. This section lies somewhat outside the main scope of this book and can be skipped in a first reading. An experimentalist mainly interested in parameter estimation may jump, after Chapter 2, directly to Chapter 9 and skip the more theoretical parts between.

The third chapter is devoted to the initial conditions. Here we explain how the unavoidable quantum fluctuations are amplified during an inflationary phase and lead to a nearly scale-invariant spectrum of scalar and tensor perturbations. We also calculate the small non-Gaussianities generated during single-field inflation and discuss the initial conditions for mixed adiabatic and isocurvature perturbations.

In Chapter 4 we derive the perturbed Boltzmann equation for CMB photons. After a brief introduction to relativistic kinetic theory, we first derive the Liouville equation, that is, the Boltzmann equation without collision term. We also discuss the connection between the distribution function and the energy–momentum tensor. We then derive the collision term, that is, the right-hand side of the Boltzmann equation, due to Thomson scattering of photons and electrons. In this first attempt we neglect the polarization dependence of Thomson scattering. This treatment, however, includes the finite thickness of the last scattering surface and Silk damping. The chapter ends with a list of the full system of perturbation equations for a ΛCDM universe, including massless neutrinos.

In Chapter 5 we discuss polarization. Here we derive the total angular momentum method that is perfectly adapted to the problem of CMB anisotropies and polarization, taking into account its symmetry, which allows a decomposition into modes with fixed total angular momentum. The representation theory of the rotation group and the spin weighted spherical harmonics that are extensively used in this chapter are deferred to an appendix. We interpret some results using the flat sky approximation, which is valid on small angular scales. This is the most technical chapter of this book and may be glanced over by readers not interested in the gory details.

In Chapter 6 we present an introduction to the vast subject of non-Gaussian perturbations. We mainly concentrate on the bispectrum and the trispectrum. We define some standard shapes of the bispectrum in Fourier space and translate them to angular space. For a description of an arbitrary N-point function in the sky we introduce a basis of rotation-invariant functions on the sphere in Appendix 4, Section A4.2.5. This chapter has been added in the second edition.

In Chapter 7 we introduce weak lensing due to foreground structures with the aim of treating lensing of CMB anisotropies and polarization. This second-order effect is especially important on small scales but has to be taken into account for $\ell \gtrsim 400$ if we want to achieve an accuracy of better than 1%. We first derive the deflection angle and the lensing power spectrum. Then we discuss lensing of CMB fluctuations and polarization in the flat sky approximation, which is sufficiently accurate for angular harmonics with $\ell \gtrsim 50$ where lensing is relevant.

In Chapter 8 we present the analysis of the large scale matter distribution within linear perturbation theory in a fully relativistic way. We take into account that only directions and redshifts are observable while lengths scales are always inferred from a cosmological model. We first introduce the traditional density and redshift space distortion contribution to the observed fluctuations and then proceed to discuss the smaller lensing and large-scale relativistic terms. We express the clustering properties of matter in terms of directly observable quantities and study their scale and redshift dependence. We also discuss "intensity mapping," a new observational technique that will hopefully bear fruit in the near future. This chapter has been newly added in the second edition.

Chapter 9 is devoted to parameter estimation. We first discuss the physical dependence of CMB anisotropies on cosmological parameters. After a section on CMB data we then treat in some detail statistical methods for CMB data analysis. We discuss especially the Fisher matrix and explain Markov chain Monte Carlo methods. We also address degeneracies, combinations of cosmological parameters on which CMB anisotropies and polarization depend only weakly. Because of these degeneracies, cosmological parameter estimation also makes use of other, non-CMB related, observations, especially observations related to the large-scale matter distribution. We summarize them and other cosmological observations in two separate sections.

In the final chapter, spectral distortions of the CMB are discussed. We first introduce the three relevant collision processes in a universe with photons and nonrelativistic electrons: Compton scattering, Bremsstrahlung, and double Compton scattering. We derive the corresponding collision terms and Boltzmann equations. For Compton scattering this leads us to the Kompaneets equation, for which we present a detailed derivation. We introduce timescales corresponding to these three collision processes and determine at which redshift a given process freezes out, that is, becomes slower than cosmic expansion. We also discuss the generation of a chemical potential in the CMB spectrum by a hypothetical particle decay and by Silk damping of small-scale fluctuations. Finally, we study the Sunyaev–Zel'dovich effect of CMB photons that pass through hot cluster gas.

All chapters are complemented with some exercises at the end.

In the appendices we collect useful constants and formulas, information on special functions, and some more technical derivations. The solutions to a selection of exercises can be found in Appendix 11 available online.

This book has grown out of a graduate course on CMB anisotropies that I have given on several occasions. Thanks are due to the students of these courses, who have motivated me to write it up in the form of a textbook. I am also indebted to many collaborators and colleagues with whom I have discussed various aspects of the book and who have helped me to clarify many issues. I especially want to mention Camille Bonvin, Chiara Caprini, Martin Kunz, Toni Riotto, Uros Seljak, Norbert Straumann, and Filippo Vernizzi. I am also immensely grateful to students and colleagues who have read parts of the draft and helped me correct numerous typographical errors and other mistakes: Giulia Cusin, Jean-Pierre Eckmann, Jérémie Francfort, Alice Gasparini, Basundhara Ghosh, Goran Jelic-Cizmek, Francesca Lepori, Ermis Mitsou, Sandro Scodeller, Vittorio Tansella, Szabolcs Zakany, and others. Of course any remaining mistakes are entirely my responsibility. Basundhara Ghosh, Francesca Lepori, Vittorio Tansella, Marcus Ruser, and Martin Kunz have also helped me with some of the figures. Finally, I wish to thank Susan Staggs, who provided me with a most useful dataset of the CMB frequency spectrum (which unfortunately had no need to be updated for the second edition of the book).