

WHAT IS A MATHEMATICAL CONCEPT?

Responding to widespread interest within cultural studies and social inquiry, this book addresses the question of what a mathematical concept is by using a variety of vanguard theories in the humanities and post-humanities. Tapping into historical, philosophical, sociological and psychological perspectives, each chapter explores the question of how mathematics comes to matter. Of interest to scholars across the usual disciplinary divides, this book tracks mathematics as a cultural activity, drawing connections with empirical practice. Unlike other books in this area, it is highly interdisciplinary, devoted to exploring the ontology of mathematics as it plays out in different contexts. This book will appeal to scholars who are interested in particular mathematical habits – creative diagramming, structural mappings, material agency, interdisciplinary coverings – that shed light on both mathematics and other disciplines. Chapters are also relevant to social sciences and humanities scholars, as each one offers philosophical insight into mathematics and how we might live mathematically.

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What Is a Mathematical Concept?

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CONTENTS

List	page vii	
Notes on Contributors		
	Introduction	1
	PART I	
1	Of Polyhedra and Pyjamas: Platonism and Induction in Meaning-Finitist Mathematics Michael J. Barany	19
2	Mathematical Concepts? The View from Ancient History <i>Reviel Netz</i>	36
	PART II	
3	Notes on the Syntax and Semantics Distinction, or Three Moments in the Life of the Mathematical Drawing <i>Juliette Kennedy</i>	55
4	Concepts as Generative Devices Elizabeth de Freitas and Nathalie Sinclair	76
	PART III	
5	Bernhard Riemann's Conceptual Mathematics and the Pedagogy of Mathematical Concepts Arkady Plotnitsky	93

V



vi Contents

V 1	Comonic	
6	Deleuze and the Conceptualisable Character of Mathematical Theories Simon B. Duffy	108
	PART IV	
7	Homotopy Type Theory and the Vertical Unity of Concepts in Mathematics David Corfield	125
8	The Perfectoid Concept: Test Case for an Absent Theory <i>Michael Harris</i>	143
	PART V	
9	Queering Mathematical Concepts Heather Mendick	161
10	Mathematics Concepts in the News Richard Barwell and Yasmine Abtahi	175
11	Concepts and Commodities in Mathematical Learning Tony Brown	189
	PART VI	
12	A Relational View of Mathematical Concepts Alf Coles	205
13	Cultural Concepts Concretely Wolff-Michael Roth	223
	PART VII	
14	Ideas as Species Brent Davis	237
15	Inhabiting Mathematical Concepts Ricardo Nemirovsky	251
	PART VIII	
16	Making a Thing of It: Some Conceptual Commentary David Pimm	269
Inde	ex	285



IMAGES

	Cover image by Akiko Ikeuchi, Knotted Thread-Red-hi2ocm
Part I	Elizabeth de Freitas: Partition problems, 2016
Part II	Andy Goldsworthy: Work with Cattails, Installation Pori Art Museum. Photo: Erkki Valli-Jaakola, 2011
Part III	Kazuko Miyamoto: Black Poppy. Installation view at A.I.R. Gallery, NY. Image and artwork. Courtesy Kazuko Miyamoto and EXILE, Berlin, 1979
Part IV	Dick Tahta: Moves about (fragment from his private papers)
Part V	María Clara Cortéz: Tell me what you forget and I will tell you who you are. 2009
Part VI	Kathrin Hilten: Plane lines, Lubec 8/31/10-1, 2010
Part VII	Tania Ennor: Human spirograph, 2016
Part VIII	David Swanson: Eight sixes, 2016



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Notes on Contributors

X

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Notes on Contributors

хi

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xii

Notes on Contributors

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