

Index of Notations

- $+$ vector addition, 89, 114
- $+\mathcal{F}$ addition in field \mathcal{F} , 90, 103
- \cdot scalar and matrix multiplication, 89, 114
- $\cdot\mathcal{F}$ multiplication in field \mathcal{F} , 97
- \odot Hadamard product, 115
- \otimes Kronecker product, 115
- \times Cartesian product of two sets, 71
- \circ operator composition, 106
- ∇ graph sum, 21
- \square Cartesian graph product, 71
- \subseteq subset, subgraph, 50
- \triangleq operator graph identity, 155
- \cong, \leftrightarrow graph isomorphism, 50
- $\hat{\cong}$ operator graph isomorphism, 154
- $|\cdot|$ modulus; abs-operator, 115
- $|\mathcal{S}|$ cardinality of set \mathcal{S}
- $\|\cdot\|_p$ ℓ^p -norm, 31
- $\langle q^k \rangle$ k -moment of $q \in \mathbb{Q}$, 32
- $\lfloor \cdot \rfloor$ floor, 95; floor operator, 102
- $\lceil \cdot \rceil$ ceiling, 95; ceiling operator, 102
- ${}^{\top}$ transpose; transposition operator, 115
- $\hat{\mathcal{G}}$ operator representation of \mathcal{G} , 139
- \hat{O} graph observable, 164
- \emptyset empty set, 14
- $\mathbf{0}$ zero operator, 109, 110
- $\mathbf{0}^n$ all-zeroes vector, 109
- $\mathbf{0}^{m \times n}$ all-zeroes matrix, 109
- $\mathbf{1}^n$ all-ones vector, 116
- $\mathbf{1}^{m \times n}$ all-ones matrix, 190
- A total adjacency, 28, 166
- A^d, A_i^d node adjacency, 40
- A, A_{ij} matrix of adjacency relations, 44
- \mathbf{a}, a_{ij} adjacency matrix, 17, 165
- \mathbf{a}^d node degree sequence, 31
- a_i^d node degree, 31
- a_i^{nn} average nearest neighbour degree, 35
- \mathcal{A} algebra, 107; operator algebra, 108
- α asymmetry index, 29
- α_w weighted asymmetry index, 38
- $B[p, q]$ Euler beta function, 279
- \mathbb{B} Boolean set, Galois field $GF(2)$, 17
- \mathbb{B}_{-1} Galois field $GF(3)$, 102
- $C, C_{(k)}^d$ global clustering coefficient, 56
- C^d total global clustering coefficient, 57
- C_i local clustering coefficient, 56
- $\langle C_i \rangle$ clustering coefficient, 56
- C_i connected component, 289
- $C_k^{\text{rand}}(\mathcal{S})$ random k -subset over \mathcal{S} , 161
- $\text{circ}[\cdot]$ circulant matrix, 65, 115
- \mathcal{C} positional chess graph, 294
- \mathcal{C}_n cycle graph, 65, 121
- \mathcal{C}_n^d cycle digraph, 66, 156
- $\mathcal{C}_{n,k}$ k -ring graph, 67, 162
- $\mathcal{C}_{n,k}^d$ k -ring digraph, 69, 161
- $\mathcal{C}_{n,k,p}$ randomised k -ring graph, 208
- $\mathfrak{C}_m(\mathcal{S})$ set of m -combinations over \mathcal{S} , 137
- D geodesic graph distance, 47
- d^d, d_i^d node distance, 47
- d, d_{ij} geodesic distance matrix, 44
- d geodesic graph diameter, 47
- $\text{diag}[\cdot]$ diagonal matrix, 99, 115
- $\delta(x)$ Dirac delta function, 59
- $\delta[q]$ Kronecker delta function, 28
- $\delta[\cdot]$ Kronecker delta operator, 103
- $\delta^{\mathcal{F}}[\cdot]$ Kronecker delta op. in \mathcal{F} , 104
- $\tilde{\delta}[\cdot]$ inverse Kronecker delta operator, 104

- $\tilde{\delta}^{\mathcal{F}}[\cdot]$ inverse Kronecker delta operator in \mathcal{F} , 104
 δ_k^n Kronecker delta vector, 99
 $\delta_{kl}^{m \times n}$ Kronecker delta matrix, 119
 $\Delta_I^{n \times n}$ gen. Kronecker delta matrix, 204
 \mathcal{E} set of edges, 14
 $\hat{\mathcal{E}}$ set of edge operators, 139
 $\hat{\mathcal{E}}_{cl}$ classical subset of $\hat{\mathcal{E}}$, 139
 $\hat{\mathcal{E}}_{aux}$ auxiliary subset of $\hat{\mathcal{E}}$, 143
 $|\mathcal{E}|$ total adjacency count, 28
 \mathbb{E} algebra of edge operators, 144
 \mathbb{E} edge operator, 139
 ϵ, ϵ_i geodesic node eccentricity, 46
 $F_{\mathcal{H}}^{\mathcal{G}}$ frequency of occurrences, 51
 $\tilde{F}_n^d, \tilde{F}_n^{ud}$ frequency of paths, 54
 $\hat{F}_n^d, \hat{F}_n^{ud}$ frequency of cycles, 54
 f file, 292
 \mathcal{F} set of faces, 23; field, 89
 \mathcal{G} finite graph, 14
 $\bar{\mathcal{G}}$ graph complement, 20
 $\mathbb{G}, \mathbb{G}_{n,k}$ set of finite graphs, 139, 153
 $\hat{\mathbb{G}}$ operator graph generator, 169
 γ Euler–Mascheroni constant, 262
 H_n harmonic number, 63
 \mathcal{H} graph pattern, 51
 I^n identity matrix, 99
 $\text{id}[\cdot]$ identity operator, 116
 \mathcal{I} set of ordered index pairs, 204
 \mathcal{K}_0^d null-digraph, 172
 \mathcal{K}_1 trivial graph, 172, 173
 \mathcal{K}_n complete simple graph, 183
 \mathcal{K}_n^d complete relational digraph, 172
 L characteristic geodesic path length, 48
 l characteristic path length, 48
 ℓ geodesic connectivity length, 48
 \mathcal{L} set of self-looped nodes, 28
 $|\mathcal{L}|$ number of self-loops, 28
 λ_d is directed, 27, 167
 λ_{ew} has Eulerian walk, 138, 168
 λ_l is self-looped, 27, 167
 λ_w is weighted, 27, 167
 λ_{wd} is weight-directed, 27
 $\mathcal{L}_{m,n}^{(4,4)}$ square grid graph, 71
 $\mathcal{L}(\mathcal{G})$ line graph of \mathcal{G} , 91
 $\mathfrak{L}_{m,n}^{(4,4)}$ projection operator, 234
 $\max q$ maximum value of q
 $\max[\cdot]$ max-operator, 116
 $\max q$ minimum value of q
 $\min[\cdot]$ min-operator, 116
 \mathfrak{M} linear map, 95
 $\hat{\mathfrak{M}}$ operator graph transformation, 153
 N_a total adjacency count, 28
 N_{ae} number of asymmetrical edges, 29
 N_{awe} number of asymmetrical weighted edges, 38
 N_c number of connected node pairs, 48
 $N_{ij}^{d-d'}$ number of shared neighbours, 34
 N_e number of edges, 28, 167
 N_e^{\max} maximum number of edges, 29
 N_l number of self-loops, 28, 166
 N_n number of nodes, 27, 166
 N_m^C number of cycles, 272
 \tilde{N}_m^C minimum number of Eulerian cycles, 137
 $N_m^{\mathcal{P}}$ number of paths, 269
 $\tilde{N}_m^{\mathcal{P}}$ number of Eulerian paths, 168
 N_m^{w}, \tilde{N}_m^{w} number of walks, 119, 123, 265
 \hat{N}_m^{w} number of closed walks, 265
 $\tilde{N}_m^{w}, \hat{N}_m^{w}$ number of Eulerian walks, 124
 \tilde{N}_m^{w} minimum number of Eulerian walks, 137
 $N_n^{C_i}$ size of connected component, 289
 N_{se} number of symmetrical edges, 29
 N_{swe} number of sym. weighted edges, 38
 n node, 139
 n_0 null-node, 142
 n_i^d vector of source/target nodes, 33
 \mathcal{N} set of nodes, 14
 $\hat{\mathcal{N}}$ set of nodes of \mathcal{G} , 139
 $\hat{\mathcal{N}}_{cl}$ classical subset of $\hat{\mathcal{N}}$, 139
 $\hat{\mathcal{N}}_{aux}$ auxiliary subset of $\hat{\mathcal{N}}$, 143
 $|\mathcal{N}|$ graph order, 27
 O_{xy} occupation state, 232
 \mathbf{o}, o_i occupation vector, 234
 \mathcal{O} complexity scaling behaviour, 5
 \mathcal{O} algebraic structure, 106
 $\mathcal{O}_{\mathcal{H}}^{\mathcal{G}}$ set of occurrences, 51
 $|\mathcal{O}_{\mathcal{H}}^{\mathcal{G}}|$ frequency of occurrences, 51
 \mathfrak{O} operator, 97
 $p[q; q]$ probability density function, 32
 $p[q; q'|q]$ conditional pdf, 35
 $p[\Sigma(\mathcal{G}); \mu]$ spectral density, 58
 p_c critical probability, 84
 \mathcal{P}_{ij} path, 42
 $\mathcal{P}_{ij}, \mathcal{P}_{ij}^m$ set of paths, 43, 53
 \mathcal{P}_n path graph, 61
 \mathcal{P}_n^d path digraph, 63, 175
 $(q)_n$ Pochhammer symbol, 278
 r rank, 292

- r geodesic graph radius, 47
 $\bar{\mathbf{r}}^k$ column randomisation operator, 161
 \mathbf{r}^p binomial randomisation operator, 150
 ρ connectedness, 29
 \mathcal{R} ring, 105
 $\mathcal{R}_{n,k,n_0,k_0}^{BA}$ Barabási–Albert random graph, 80
 $\mathcal{R}_{n,k}^{ER}$ Erdős–Rényi random graph, 74, 184
 $\mathcal{R}_{n,k}^{dER}$ Erdős–Rényi random digraph, 74
 $\mathcal{R}_{n,k}^G$ Gilbert random graph, 75
 $\mathcal{R}_{n,k}^{dG}$ Gilbert random graph, 74
 $\mathcal{R}_{m,n,p}^{\mathcal{L}}$ randomised square grid graph, 235
 $\mathcal{R}_{n,k,q}^{WS}$ Watts–Strogatz random graph, 78
 S^d global small-worldness index, 261
 $S_{ij}^{d-d'}$ similarity of nearest neighbours, 34
 s^d, s_i^d node strength, 40
 s^n, s_i^n node state vector, 91
 s^e, s_i^e edge state vector, 91
 $\text{sgn}[\cdot]$ signum, 95; signum operator, 102
 $\text{sgn}^{\mathcal{F}}[\cdot]$ signum operator in \mathcal{F} , 103
 $\overline{\text{sgn}}^{\mathcal{F}}[\cdot]$ inverse signum operator in \mathcal{F} , 109
 $S^n(\mathcal{G})$ node state vector space, 91
 $S^e(\mathcal{G})$ edge state vector space, 91
 $\Xi_m(\mathcal{S})$ ordered m -subsets over \mathcal{S} , 123
 \mathcal{S}_n star graph, 176, 174
 $\mathcal{S}_{n,m}$ generalised star graph, 218
 $\mathcal{S}_{n,m,p}$ randomised gen. star graph, 220
 $\Sigma(\mathcal{G})$ graph spectrum, 58
 $\mathbf{T}[\cdot]$ total sum operator, 116
 $\text{Tr}[\cdot]$ trace operator, 104
 $\text{Tr}^{\mathcal{F}}[\cdot]$ trace operator in \mathcal{F} , 105
 $\overline{\text{Tr}}[\cdot]$ inverse trace operator, 110
 $\overline{\text{Tr}}^{\mathcal{F}}[\cdot]$ inverse trace operator in \mathcal{F} , 111
 $\mathbf{t}_{ij}^{\text{reg}}$ regular chess move, 316
 $\mathbf{t}_{ij}^{\text{ep}}$ en passant capture move, 317
 $\mathbf{t}_{ij}^{\text{c}}$ castling move, 317
 $\mathfrak{T}_m(\mathcal{S})$ ordered m -tuples over \mathcal{S} , 123
 τ assortativity, 36
 \mathcal{V} vector space, 89
 \mathbf{w}, w_{ij} weight matrix, 14, 165
 \mathcal{W} potential walk, 98
 \mathcal{W}_{ij} walk, 42
 \mathcal{W}_{ij} set of walks, 265
 \mathcal{W}_{ij}^m set of walks of length m , 265
 $\mathbf{w}[\cdot]$ weight matrix operator, 296

Subject Index

- abs-operator, 115
- adjacency matrix, 17
 - graph observable, 165, 193
- algebra, 106
 - associative, 107
 - commutative, 107
 - of operators, 108
- algebraic structure, 106
- appearance of a graph, 50
- approximate degree-matched random digraph, 226–231
 - adjacency matrix observable, 230
 - operator graph generator, 229
 - operator representation, 229
- approximate degree-matched random graph
 - asymmetry index observable, 248
 - number of asymmetrical edges observable, 248
 - number of symmetrical edges observable, 248
 - total adjacency observable, 242
- assortative graph, 36
- assortativity, 36
- asymmetry index, 29
 - graph observable, 243
- average geodesic graph distance, 48
- average geodesic node distance, *see* characteristic path length
- average nearest neighbour degree, 35
- average node adjacency, 40
- average node degree, 33, 277
- average node distance, 47
- average node strength, 41
- average number of shared neighbours, 34
- average similarity of nearest neighbours, 35
- Barabási–Albert random graph, 6, 77, 79–81, 216–226
 - adjacency matrix observable, 225
 - canonical model, 217–220
 - operator graph generator, 224
 - operator representation, 225
 - total adjacency observable, 224, 241, 277, 279
 - total node degree observable, 279
 - total node degree sequence observable, 224
- biclique, *see* bipartite graph
- bilinear product, 107
- binomial randomisation operator, 150, 196, 199
 - addition, 198
 - algebra, 198
 - composition, 150, 197
 - expectation values, 150
 - multiplication, 150, 202
- bipartite graph, 22, 23
 - biclique, 22, 23
- bond, *see* edge
- Boolean set, 17, 90
 - operation, 90
 - truth table, 90
- Boolean vector space, 90
- brain network, 261
- Cartesian graph product, 71
- ceiling function, 95, 102
- ceiling operator, 102
 - algebra, 109
 - composition, 106
- central node, 47
- characteristic geodesic path length, 48
- characteristic path length, 48

Subject Index

337

- chess, 292
 - castling move, 295
 - checkmate, 292
 - en passant* move, 294
 - regular move, 292, 294
 - special move, 292
- chess graph, *see* positional chess graph
- circulant diagonalisation theorem, 252, 256, 266
- circulant operator, 115
- closed geodesics, 44
- clustering, 55
- clustering coefficient, 56
- column randomisation operator, 161, 184, 188–191, 196, 199
 - addition, 189
 - algebra, 198
 - composition, 189, 196
- complete graph, 19, 22, 85, 296
 - biclique, 22, 23
 - operator representation, 172, 184, 211, 214, 297
 - total global clustering coefficient observable, 253
- computational complexity, 4
- conditional node degree distribution, 35
- configuration model, 227
- connected component, 289
- connected graph, 21, 22
- connected nodes, 22, 44
- connectedness, 29
- critical connectivity threshold, *see* critical probability
- critical percolation threshold, 274
- critical probability, 83, 274
- critical transition, 84, 274
- cycle, 23, 42
 - Eulerian, 131
 - Hamiltonian, 42, 65, 276
- cycle graph, 63–67, 82, 85
 - operator graph generator, 174
 - operator graph transformation, 158
 - operator representation, 158, 165, 175
- degree balance condition, 35
- dense graph, 29
- diagonal operator, 115
- digraph, *see* directed graph
- Dirac delta function, 59
- directed edge, 15
- directed graph, 15
 - multigraph, 17
 - operator representation, 149
 - relational, 17
 - self-looped, 17, 27
 - weight-directed, 37
 - weighted, 17, 27
- distance, 42
- domino graph, 71, 173
- edge, 2, 13
 - directed, 15
 - half-edge, 227
 - multiedge, 17
 - self-loop, 16
 - weighted, 14
- edge operator, 127, 140
- edge-generating operator, *see* edge operator
- edge-to-vertex dual, *see* line graph
- endomorphism, 96
- Erdős–Rényi random graph
 - total global clustering coefficient observable, 259
- Erdős–Rényi random graph, 6, 73–75, 77, 160, 162, 181–200
 - adjacency matrix observable, 187, 193
 - asymmetry index observable, 244
 - generation, 161
 - number of asymmetrical edges observable, 244
 - number of symmetrical edges observable, 244
 - operator graph generator, 184, 193
 - operator representation, 185, 193
 - total adjacency observable, 239
 - total global clustering coefficient observable, 255, 259
- Euclidean distance, 34
- Euler’s formula, 23
- evolving graph, 79
- exact graph, 61
 - cycle graph, 65, 66
 - k -ring graph, 67, 69
 - lattice graph, 70
 - path graph, 61, 63
 - square grid graph, 71
- exhaustive search, 5
- \mathcal{F} -algebra, 106
- \mathcal{F} -linear map, *see* linear map
- \mathcal{F} -linear operator, 97
- \mathcal{F} -operator algebra, 108
- \mathcal{F} -operator graph, 144
- face, 23
- file, 292

- finitary operation, 106
- finite graph, 14, 81
- finite graph theory, 87
- floor function, 95, 102
- floor operator, 102
 - algebra, 109
 - composition, 106
- four-colour problem, 24
- frequency of occurrence
 - cycles, 54
- frequency of occurrence, 51
 - graph patterns, 53
 - paths, 54, 269
- frequent subgraph, *see* recurrent graph pattern
- Galois field, *see* Boolean set, 103
- geodesic connectivity length, 48
- geodesic cycle, 44, 287
- geodesic distance, 44, 287
 - graph observable, 287
- geodesic distance matrix, 44
- geodesic graph diameter, 47
- geodesic graph distance, 47
- geodesic graph radius, 47
- geodesic loop, 44
- geodesic node eccentricity, 46
- geodesic path, *see* shortest path
- giant component, 86, 274
- Gilbert random graph, 74–77, 83, 149, 200–205, 237
 - adjacency matrix observable, 202, 205
 - asymmetry index observable, 245
 - asymptotic limit, 83–87
 - critical percolation threshold, 274, 276
 - number of asymmetrical edges observable, 245
 - number of closed walks observable, 266
 - number of cycles observable, 272
 - number of Hamiltonian cycles, 276
 - number of Hamiltonian paths, 274
 - number of paths observable, 270, 272
 - number of symmetrical edges observable, 245, 273
 - number of walks observable, 266
 - operator graph generator, 201, 203
 - operator representation, 151, 202, 203
 - total adjacency observable, 240, 267, 271
 - total global clustering coefficient observable, 253
- global clustering coefficient, 55, 56
- global small-worldness index, 261
- graph, 13
 - classical representation, 14
 - directed, 15, 17, 27
 - finite, 14
 - multigraph, 17
 - operator graph, 144
 - operator representation, 139
 - self-looped, 16, 17, 27
 - simple, 17
 - weight-directed, 37
 - weighted, 17, 27
- graph complement, 20
- graph inverse, *see* graph complement
- graph isomorphism, 50, 154, 159
- graph measure, 163
 - degree-based, 30–36
 - distance-based, 41–48
 - general, 27–30
 - network motif, 48–57
 - spectral, 57–60
 - weight-based, 36–41
- graph model
 - approximate degree-matched digraph, 229
 - Barabási–Albert model, 6, 80
 - cycle graph, 65, 66
 - Erdős–Rényi model, 6, 74
 - evolving graph, 79
 - exact graph, 61
 - Gilbert model, 74
 - k -ring graph, 67, 69
 - lattice graph, 70
 - of Königsberg’s bridges, 2
 - path graph, 61, 63
 - random graph, 6, 61
 - scale-free graph, 80
 - square grid graph, 71
 - static graph, 79
 - Watts–Strogatz model, 6, 78
- graph observable, 164, 285
- graph pattern, 49, 51, 52
- graph product, *see* Cartesian graph product
- graph spectrum, 58
- graph sum, 20
- graph theory, 1, 14
- graph transitivity, *see* global clustering coefficient
- grid graph, *see* lattice graph
- Hadamard product, 115
- half-edge, *see* stub

Subject Index

339

- handshaking lemma, 32
- harmonic number, 63
- has Eulerian walk, 138
- has Eulerian walks
 - observable, 168
- hexagonal grid graph, 71
- identical graphs, 155
- identity operator, 116
- induced subgraph, *see* subgraph
- is directed, 27
 - graph observable, 167
- is self-looped, 27
 - graph observable, 167
- is weight-directed, 37
- is weighted, 27
 - graph observable, 167
- isolated node, 21
 - operator representation, 173
- isomorphic graphs, 50, 154
- k -regular graph, 21, 22
- k -ring graph, 64, 67–70, 78, 206, 210
 - graph complement, 206, 214
 - operator graph generator, 174
 - operator graph transformation, 161
 - operator representation, 161, 175, 214
 - randomised, 208, 210
 - total global clustering coefficient
 - observable, 258
- Königsberg bridge graph, 2, 14, 60
 - assortativity, 36
 - average number of shared neighbours, 34
 - average similarity of nearest neighbours, 35
 - clustering coefficient, 56
 - Eulerian walks, 101
 - geodesic distance matrix, 46
 - geodesic node eccentricity, 47
 - global clustering coefficient, 56
 - graph patterns, 49
 - is directed/weighted/self-looped, 27
 - line graph, 93, 94
 - matrix of adjacency relations, 45, 93
 - node adjacency, 41
 - node degree sequence, 41
 - node strength, 41
 - node/edge state vector, 91
 - node/edge state vector space, 91
 - number of Eulerian cycles, 137
 - number of Eulerian walks, 130, 137
 - number of walks, 45, 130
 - operator graph generator, 177
 - operator representation, 129, 136, 178
 - paths, 43
 - potential walk, 98
 - set of nodes/edges, 15
 - subgraphs, 50
 - walk-generating operators, 99, 100, 111
 - walks, 43
 - weight matrix, 15
- Königsberg bridge problem, 1, 93
 - classical solution, 1
 - graph-theoretical solution, 41
 - operator graph-theoretical solution, 133, 138
 - operator solution, 89, 101, 113
- Kronecker delta function, 28
- Kronecker delta matrix, 119, 188
 - generalisation, 204, 215
- Kronecker delta operator, 103
 - algebra, 110
 - Boolean, 104
 - composition, 106
 - generalisation, 268
 - inverse, 104
 - inverse Boolean, 104
- Kronecker delta vector, 99, 187
- Kronecker product, 115
- ladder graph, 71, 175
 - operator graph generator, 174
 - operator representation, 175
- lattice graph, 70–73
- line graph, 91
- linear map, 95
- linear operator, 97
- linear space, *see* vector space
- link, *see* edge
- local clustering coefficient, 56
- logical operation
 - AND, 90, 289
 - XOR, 90
- Manhattan distance, *see* taxicab distance
- map, *see* operator
- matrix of adjacency relations, 44
- matrix operator, 97
- max-operator, 116
- mesh graph, *see* lattice graph
- min-operator, 116
- module, 105
- multiedge, 17, 176, 227
- multigraph, 17, 176
- nearest neighbour, 34, 92, 289
- network, *see* graph

- network motif, 51
- node, 2, 13
 - central, 47
 - connected, 22, 44
 - endnode, 42, 272
 - intermediate, 136
 - isolated, 21, 173
 - null-node, 142
 - peripheral, 47
 - terminal, 61
 - transitive, 55
- node adjacency, 40
- node connectivity, *see* node degree
- node degree, 31, 39
- node degree distribution, 32
- node degree sequence, 31
- node distance, 47
- node strength, 40
- null-digraph, 172
- number of asymmetrical edges, 29
 - graph observable, 243
- number of asymmetrical weighted edges, 38
- number of closed walks, 265
 - graph observable, 265
- number of connected node pairs, 48
- number of cycles, 272
 - graph observable, 272
- number of edges, 28
 - graph observable, 167
- number of Eulerian paths observable, 168
- number of Eulerian walks, 124
- number of nodes, 27
 - graph observable, 166
- number of paths, 269
 - graph observable, 269
- number of self-loops, 28
 - graph observable, 166
- number of shared neighbours, 34
- number of symmetrical edges, 29
 - graph observable, 243
- number of symmetrical weighted edges, 38
- number of walks, 119, 123, 265
 - graph observable, 265
- observable, 163
- occupation state, 232
- occupation vector, 234
- occurrence of a graph, 51
- operator, 89, 94, 97
- operator algebra, 108
- operator calculus, 88
- operator composition, 106, 127
- operator graph, 144
- operator graph generator, 169
- operator graph identity, 155
- operator graph isomorphism, 154
- operator graph observable, *see* graph observable
- operator graph seed, 169
- operator graph theory, 9, 117, 139–152
- operator graph transform, 153
- operator graph transformation, 153
- operator representation, 121, 139, 142
 - associative, 127
 - identity, 155
 - isomorphism, 154
 - minimal, 140
 - non-associative, 127
 - operator graph generator, 169
 - transformation, 153
- operator representations, 146
- order, *see* number of nodes
- path, 42
 - endnode, 42
 - Eulerian, 3, 42, 98
 - Hamiltonian, 42, 274
- path graph, 61–64, 82
 - operator graph generator, 174
 - operator representation, 175
- path length, 42
- percolation, 265, 274
- peripheral node, 47
- planar graph, 23, 70
- Platonic graph, 25
- Platonic solids, 25
- positional chess graph, 292, 294
 - castling move, 295, 311, 317
 - check, 295, 317–320
 - checkmate, 295, 323
 - en passant* capture, 294, 309, 317
 - illegal capture move, 302
 - initial pawn move, 307
 - initial position, 292, 307, 322
 - maximum number of edges, 295
 - node states, 293
 - operator graph generator, 296, 320
 - operator graph transformation, 323
 - operator representation, 297

Subject Index

341

- pawn capture move, 307
- regular move, 294, 304–309, 316
- special move, 309–315
- weight matrix, 294, 320
- preferential attachment, 80
- projection operator, 234, 283
- random graph, 61, 74
 - approximate degree-matched model, 229
 - Barabási–Albert model, 6, 80
 - configuration model, 227
 - Erdős–Rényi model, 6, 74
 - Gilbert model, 74
 - Watts–Strogatz model, 6, 78
- random graph theory, 7, 83
- randomised square grid graph, 231–237
 - adjacency matrix, 235
 - adjacency matrix observable, 236
 - connectedness observable, 283
 - number of paths observable, 283, 285
 - operator graph generator, 236
 - operator representation, 236
 - total adjacency observable, 283
- rank, 292
- recurrent graph pattern, 51
- regular graph, *see k-regular graph*
- regular tiling, *see tessellation*
- relational digraph, 17
- ring graph, *see k-ring graph*
- scale-free graph, 80
- seed, *see operator graph seed*
- self-loop, 16, 19, 227
 - operator representation, 148
- self-looped graph, 16, 17
- shortest path, 44
- signum function, 95
- signum operator, 103
 - \mathbb{B}_1 -algebra, 110
 - algebra, 109
 - Boolean, 103
 - composition, 106, 110
 - inverse Boolean, 109
- similarity of nearest neighbours, 34
- simple graph, 17
 - adjacency matrix observable, 165
- size of connected component, 289
- small-world, 49, 260
- small-world network, *see Watts–Strogatz*
 - random graph
- space complexity, 5
- sparse graph, 29
- spectral density, 58
- square graph, 71
- square grid graph, 64, 71–73, 234
 - operator representation, 235
 - randomised, 231, 282
- star graph, 176, 218
 - generalised, 218
 - operator graph generator, 174
 - operator representation, 176
 - randomised, 220
- state vector, 89, 289, 293
- static graph, 79
- strongly connected digraph, 22
- structural equivalence, 33
- stub, 227
- subcritical phase, 83
- subgraph, 50
 - frequent, 51
 - in Gilbert random graphs, 84
 - induced, 50
- supercritical phase, 83
- supergraph, 50
- symmetrisation, 18, 30, 246
- taxicab distance, 32, 73
- tessellation, 70
- time complexity, 5
- total adjacency, 28
 - graph observable, 166, 239
- total adjacency count, 28
- total global clustering coefficient, 57, 249
 - graph observable, 250
- total node degree, 31
 - graph observable, 277
- total node degree sequence, 31
- total sum operator, 116
- trace operator, 104
 - \mathbb{B} -algebra, 111
 - Boolean, 105, 111
 - composition, 106, 111
 - inverse, 110
 - \mathbb{Q} -algebra, 111
- transform, *see operator graph transform*
- transitive nodes, 55
- transitivity, 55
- transposition operator, 115
- tree graph, 24, 85
- triangle graph, 119
 - multigraph, 125
 - number of Eulerian paths, 124, 129
 - number of Eulerian walks, 124, 129

- triangle graph (cont.)
 - number of walks, 123, 128
 - operator graph transformation, 156
 - operator representation, 121, 126, 156
 - walks, 121
- triangular grid graph, 71
- trivial graph, 173
 - operator graph generator, 172
 - operator representation, 173
- vector of source/target nodes, 33
- vector space, 89
 - Boolean, 90
- vertex, *see* node
- w-regular graph, 21, 22
- walk, 42
 - endnode, 42
 - Eulerian, 3, 42, 98, 123
 - intermediate node, 136
 - potential, 98, 121
- Watts–Strogatz random graph, 6, 76–79, 160, 205–216
 - adjacency matrix observable, 213, 215
 - asymmetry index observable, 246
 - canonical model, 206–210
 - number of asymmetrical edges observable, 246
 - number of symmetrical edges observable, 246
 - operator graph generator, 212, 214
 - operator representation, 212, 214
 - total adjacency observable, 241
 - total global clustering coefficient observable, 257
- weakly connected digraph, 22
- weight matrix, 14
 - graph observable, 165
- weighted asymmetry index, 38
- weighted edge, 14
 - operator representation, 149
- weighted graph, 17
 - directed, 17, 27
 - multigraph, 17
 - operator representation, 149
 - self-looped, 17, 27
 - weight matrix observable, 165
 - weight-directed, 37
- Wood problem, 232, 280–285
- zero operator, 109, 111