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FRACTALS IN PROBABILITY AND ANALYSIS

A mathematically rigorous introduction to fractals which emphasizes examples and fundamental ideas. Building up from basic techniques of geometric measure theory and probability, central topics such as Hausdorff dimension, self-similar sets and Brownian motion are introduced, as are more specialized topics, including Kakeya sets, capacity, percolation on trees and the Traveling Salesman Theorem. The broad range of techniques presented enables key ideas to be highlighted, without the distraction of excessive technicalities. The authors incorporate some novel proofs which are simpler than those available elsewhere. Where possible, chapters are designed to be read independently so the book can be used to teach a variety of courses, with the clear structure offering students an accessible route into the topic.

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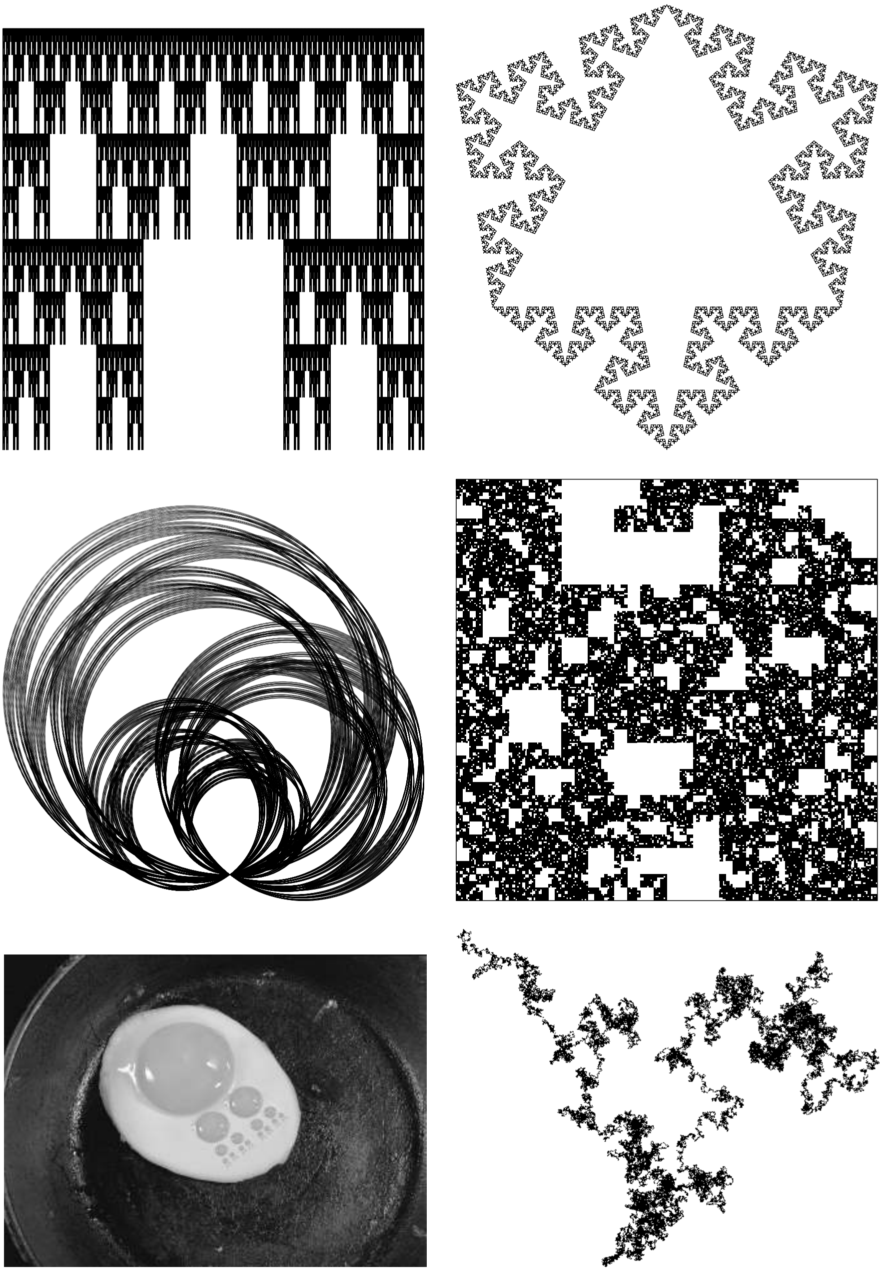

A selection of fractals from the book – and a fractal egg, reproduced with the permission of Kevin Van Aelst.

Fractals in Probability and Analysis

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Preface

The aim of this book is to acquaint readers with some fractal sets that arise naturally in probability and analysis, and the methods used to study them. The book is based on courses taught by the authors at Yale, Stony Brook University, the Hebrew University and UC Berkeley. We owe a great debt to our advisors, Peter Jones and Hillel Furstenberg; thus the book conveys some of their perspectives on the subject, as well as our own.

We have made an effort to keep the book self-contained. The only prerequisite is familiarity with measure theory and probability at the level acquired in a first graduate course. The book contains many exercises of varying difficulty. We have indicated with a "•" those for which a solution, or a hint, is given in Appendix C. A few sections are technically challenging and not needed for subsequent sections, so could be skipped in the presentation of a given chapter. We mark these with a "*" in the section title.

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Finally, we dedicate this book to our families: Cheryl, David and Emily Bishop, and Deborah, Alon and Noam Peres; without their support and understanding, it would have taken even longer to write.