

## INTRODUCTION TO ACCELERATOR DYNAMICS

How does a particle accelerator work? The most direct and intuitive answer focuses on the dynamics of single particles as they travel through an accelerator. Particle accelerators are becoming ever more sophisticated and diverse, from the Large Hadron Collider (LHC) at CERN to multi-MW linear accelerators and small medical synchrotrons. This self-contained book presents a pedagogical account of the important field of accelerator physics, which has grown rapidly since its inception in the latter half of the last century. Key topics covered include the physics of particle acceleration, collision and beam dynamics and the engineering considerations intrinsic to the effective construction and operation of particle accelerators. By drawing direct connections between accelerator technology and the parallel development of computational capability, this book offers an accessible introduction to this exciting field at a level appropriate for advanced undergraduate and graduate students, accelerator scientists and engineers.

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> To Rachel and Leah



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## **Preface**

This book addresses the single particle dynamics heart of the question; 'How does an accelerator work?', for readers who are accelerator users and operators, who are accelerator physicists or who are interested in real-world linear and nonlinear difference systems. The reader might be a synchrotron light source user, a collider experimentalist, a medical accelerator operator, an engineer in a beam instrumentation group or a controls professional in industry.

The level of the discussion is appropriate for graduate students, final-year undergraduates and practicing accelerator professionals. This is not an exhaustively complete reference handbook, at a high technical level. Rather, it is a pedagogical introduction to the subject, telling a self-contained and accurate story about a field of physics that was born and grew rapidly in the second half of the twentieth century, and which continues to mature by leaps and bounds in the twenty first. The treatment is rigorous enough to be accurate and useful, without letting unnecessary detail obscure the central theme. Deeper investigations of the 'back-stories' are left to other sources.

The central theme is that repetitive motion through an accelerator is a natural, convenient and well-motivated introduction to the generic linear and nonlinear behaviour of highly iterated difference systems. A circular accelerator – or even a linear accelerator – is one answer to another question: 'How do difference systems manifest themselves in the real world?'