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978-1-107-12954-2 - Noncommutative Algebraic Geometry

Gwyn Bellamy, Daniel Rogalski, Travis Schedler, J. Toby Stafford and Michael Wemyss

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There are many interactions between noncommutative algebra and representation theory on the one hand and classical algebraic geometry on the other, with important applications in both directions. The aim of this book is to provide a comprehensive introduction to some of the most significant topics in this area, including noncommutative projective algebraic geometry, deformation theory, symplectic reflection algebras, and noncommutative resolutions of singularities.

The book is based on lecture courses in Noncommutative Algebraic Geometry given by the authors at a Summer Graduate School at MSRI in 2012 and, as such, is suitable for advanced graduate students and early postdocs. In keeping with the lectures on which the book is based, a large number of exercises are provided, for which partial solutions are included.

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Preface

These notes are based on the lecture courses in Noncommutative Algebraic Geometry given by Bellamy, Rogalski, Schedler and Wemyss at the Summer Graduate School at the Mathematical Sciences Research Institute (MSRI) in Berkeley, California, in June 2012. This school served in part as an introductory conference to the MSRI program on Interactions between Noncommutative Noncommutative Algebra, Representation Theory and Algebraic Geometry that was held at MSRI in January–May 2013.

We would like to thank Jackie Blue, Riz Mayodong, Megan Nguyen and Stephanie Yurus who made our stay at MSRI so enjoyable, but most especially we extend our thanks to Chris Marshall and H el ene Barcelo who did so much to make the conference run smoothly. We also thank MSRI and the NSF for their financial support; in particular, part of this material is based on work supported by the National Science Foundation under Grant No. 0932078 000, while the authors were in residence at MSRI during the summer of 2012.

Thanks also to all the students who so enthusiastically attended the course and worked so hard. We hope that it was as productive and enjoyable for them as it was for the lecturers.

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[More information](#)

Contents

Preface	vii
Introduction	1
Chapter I Noncommutative projective geometry	13
Introduction	13
1 Review of basic background and the Diamond Lemma	14
2 Artin–Schelter regular algebras	30
3 Point modules	42
4 Noncommutative projective schemes	51
5 Classification of noncommutative curves and surfaces	62
Chapter II Deformations of algebras in noncommutative geometry	71
Introduction	71
1 Motivating examples	75
2 Formal deformation theory and Kontsevich’s theorem	104
3 Hochschild cohomology and infinitesimal deformations	124
4 Dglas, the Maurer–Cartan formalism, and proof of formality theorems	136
5 Calabi–Yau algebras and isolated hypersurface singularities	157
Chapter III Symplectic reflection algebras	167
Introduction	167
1 Symplectic reflection algebras	171
2 Rational Cherednik algebras at $t = 1$	184
3 The symmetric group	197
4 The KZ functor	211
5 Symplectic reflection algebras at $t = 0$	224
Chapter IV Noncommutative resolutions	239
Introduction	239
Acknowledgments	240
1 Motivation and first examples	240
2 NCCRs and uniqueness issues	250
3 From algebra to geometry: quiver GIT	261
4 Into derived categories	270
5 McKay and beyond	285
6 Appendix: Quiver representations	297

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Frontmatter

[More information](#)

x	CONTENTS	
Solutions to the exercises		307
I Noncommutative projective geometry		307
II Deformations of algebras in noncommutative geometry		316
III Symplectic reflection algebras		332
IV Noncommutative resolutions		337
Bibliography		343
Index		353